1. Errata and small complements

Where	**As it is**	**Should be**
x, footnote 8	have been	has been
4_{10}	associates to each subset A, a map	associates with each subset A maps
$18^{12,13}$	then C is also an almost intersection	then D is also an almost intersection
30, before Exercise II.2.5	Insert: This filter is called *cofinite filter on* X and is denoted $(X)_0$	
30^{9}	Recall (Section I.5)	Recall (Section I.4)
44_4	$\bigcup_{\mathcal{U}\in\mathbb{G}} U_{\mathcal{U}} \notin \mathcal{F}$	$\bigcup_{\mathcal{U}\in\mathbb{D}} U_{\mathcal{U}} \notin \mathcal{F}$
44_3	$[\beta(\mathcal{F}]^{<\omega}$	$[\beta(\mathcal{F})]^{<\omega}$
63, Figure III.3, line 4	indicates the that the	indicates that the
69^{5}	$P \in \mathcal{F} \cap \mathcal{P}$	$P \in \mathcal{F}^{\uparrow} \cap \mathcal{P}$
71^{14}	convergence ξ on X the set	convergence ξ on X, the set
71, Definition III.7.1	on X the finest	on X, the finest
116, Propositions V.1.4	any family $\mathbb{D}$ of filter	any family $\mathbb{D}$ of filters
121, before section V.2	The sentence "Consequently...Bourdaud space." should be after the end-of-proof symbol	
123	Example V.2.11 should be placed after Example V.2.7	
131	Exercise V.3.8 should be placed after Exercise V.3.10	
133, Example V.3.16	see Example III.2.1	see Section III.2.1
$159^{7,8}$	the pavement	a pavement
159^{8}	τ -converges	τ -converge
159_5	the distance d	a distance d
162, Definition VI.1.7	$\{\mathcal{G}_i :\in I\}$	$\{\mathcal{G}_i : i \in I\}$
163^{1}	the family	a family
233, Exercise IX.1.9	Omit part (2) (same as Exercise IX.1.2 (1)) and thus omit "and let $B \subset X$"	
239	Shift Exercise IX.2.3 at the end of Section IX.11	
244_{11}	Lemma V.4.41 that	Lemma V.4.41 says that
247_5	the analog	an analog
249, Exercise IX.5.7	$\mathbb{F}X$	$\mathbb{F}X$
249, Exercise IX.5.7	a subset $\mathbb{D}$	a non-empty subset $\mathbb{D}$
249, Exercise IX.5.8	for each non-empty subset $\mathbb{A}$	for each non-empty subset $\mathbb{D}$
273, Exercise IX.10.5	elements	subsets
299, Proposition XI.1.9	If X is a subset	If X is an open subset
	$\mathcal{P}$ is a cover of X	$\mathcal{P}$ is a cover of Y
	a regular refinement	a Y-regular refinement
299, Proof (line 4)	$F_{\mathcal{F}} \in \mathcal{F} \cap \mathcal{P}$	$F_{\mathcal{F}} \in \mathcal{F}$ and $P \in \mathcal{P}$
	Add "As X is open, there exists "$F_X \in \mathcal{F}$ such that $\operatorname{adh}_Y F_X \subset X$."	
299_7	be collection	be a collection
307, Exercise XI.5.5 $(1)^{3}$	each $n \in \mathbb{N}$.	for each $n \in \mathbb{N}$.
309, Exercise XI.6.6	map	map onto a Hausdorff space
311^{11}	$\lim_Y \mathcal{U} \subset V_\alpha$.	$\lim_Y \mathcal{U} = \lim_Y \left(\operatorname{adh}_Y^{\natural} \mathcal{U}\right) \subset \operatorname{adh}_Y U$ for each $U \in \mathcal{U}$, hence, by Proposition XI.1.9, $\lim_Y \mathcal{U} \subset V_\alpha$.
378, Example XIV.6.15	intersection which	intersection with
383_7	$(h : \xi\| \to \|\tau\|)_{h \in C(\xi,\tau), \tau \in \mathbf{W}}$	$(h : \|\xi\| \to \|\tau\|)_{h \in C(\xi,\tau), \tau \in \mathbf{W}}$
385^{2}	every convergences ξ	every convergence ξ
400, Definition XV.3.1	subset A of X	subset A of $\|\xi\|$
422, Exercise XVI.5.3	Theorem has	Theorem XVI.5.2 has
452, Lemma XVII.5.4	§	$\mathcal{S}$
Proof Lemma XVII.5.4	§	$\mathcal{S}$

Where	As it is	Should be
452_{1-2}	$S\xi$	$\mathrm{S}\,\xi$
$453^{7,8}$	§	$\mathcal{S}$
479, Exercise XVIII.3.2 (3)	ε	r
498_6	can rephrased	can be rephrased
507^{15} and 507^{22}	tertiary	ternary
511, Exercise A.7.3 (1)	completely distributive	a completely distributive
519, Proposition A.10.4	there exists	there exist
519_2	$\omega_1$the first	ω_1 the first
542	injective map(one-to-one)	injective map (one-to-one)

2. Incomplete proof of Theorem V.4.21

The proof of Theorem V.4.21 is not complete. An alternative proof uses Corollary V.4.22, which is proved below.

Corollary (V.4.22)**.** *For each convergence ξ, there exists the finest topology that is coarser than ξ.*

Proof. Define

$$\lim_\eta \mathcal{F} := \bigcap\nolimits_{A\in\mathcal{F}^\#} \mathrm{cl}_\xi A,$$

and observe that η is a convergence coarser than ξ. Indeed, $\mathcal{F}_0 \leq \mathcal{F}_1$ implies that $\lim_\eta \mathcal{F}_0 \subset \lim_\eta \mathcal{F}_1$, and $x \in \lim_\eta \{x\}^\uparrow$. Moreover, if $x \in \lim_\xi \mathcal{F}$ then $x \in \mathrm{cl}_\xi A$ for every $A \in \mathcal{F}^\#$. We notice that $\mathcal{C}_\eta = \mathcal{C}_\xi$. Indeed, $\mathcal{C}_\eta \subset \mathcal{C}_\xi$, because $\xi \geq \eta$. Conversely, if $C \in \mathcal{C}_\xi$ and $C \in \mathcal{F}^\#$, then $\lim_\eta \mathcal{F} \subset \mathrm{cl}_\xi C = C$, by the definition of η, and thus $C \in \mathcal{C}_\eta$. Therefore,

$$\lim_\eta \mathcal{F} = \bigcap\nolimits_{A\in\mathcal{F}^\#} \mathrm{cl}_\eta A,$$

that is, η is a topology, coarser than ξ, and such that $\mathcal{C}_\eta = \mathcal{C}_\xi$. If now τ is any topology coarser than ξ, then $\mathcal{C}_\tau \subset \mathcal{C}_\xi = \mathcal{C}_\eta$, hence $\tau \leq \eta$, showing that η is the finest among the topologies that are coarser than ξ. □

We denote by $\mathrm{T}\xi$ the finest topology among all topologies that are coarser than ξ, and call it the *topologization* of ξ. It follows that $\mathrm{T}\xi = \eta$ from the proof above, hence

$$\lim_{\mathrm{T}\xi} \mathcal{F} = \bigcap\nolimits_{A\in\mathcal{F}^\#} \mathrm{cl}_\xi A.$$

Theorem (V.4.21)**.** *A supremum of a non-empty set of topologies is a topology.*

Proof. Let Ξ be a non-empty set of topologies on a set X. As $\bigvee \Xi \geq \xi$ for each $\xi \in \Xi$, also $\mathrm{T}(\bigvee \Xi) \geq T\xi$ for each $\xi \in \Xi$, hence

$$\mathrm{T}(\bigvee \Xi) \geq \bigvee\nolimits_{\xi\in\Xi} T\xi = \bigvee\nolimits_{\xi\in\Xi} \xi = \bigvee \Xi,$$

because $\mathrm{T}\xi = \xi$ for each $\xi \in \Xi$. □

3. Complements of indices

Add a symbol entry under **Maps and map operations**:

$\operatorname{osc} f(x)$ oscillation of f at x, page 292

Add an index entry for "*cofinite set*" referencing page 13
Add an index entry for "*cofinite filter*" referencing page 30
Add an index entry for "*refinement*" referencing page 28

References to add

[1] S. Dolecki. *Analyse Fondamentale: espaces métriques, topologiques et norrmés*. Hermann, 2nd ed edition, 2013.
[2] Z. Frolík. Generalizations of the G_δ-property of complete metric spaces. *Czechoslovak Math. J.*, **10**:359–379, 1960.
[3] K. Kuratowski. *Topologie*. 1958. 4th edition.
[4] E. K. Van Dowen. The integers and topology. In K. Kunnen and J. E. Vaughan, editors, *Handbook of set-theoretic topology*, pages 111–167. North-Holland, 1984.