THE HUMAN HIP STRESS ANALYSIS: A BALL-SOCKET ELASTIC CONTACT MODELLING

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ABSTRACT

The peak stress, the weight bearing area and the stress distribution of the articular contact at the human hip joint is the most important factors, which determine the location of the degenerative foci that later result in degenerative damage and in the development of osteoarthritis. In the paper, a ball-socket articular elastic contact modelling of the human hip joint is successfully established. This modelling can be used to predict the peak stress, the weight bearing area and the stress distributions of the articular contact at the human hip joint. In order to verify the validation of current modelling, an example is examined. The results show that the predictions from the current modelling are basically agreement with those reported in literatures. This shows the validity of the current modelling.

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Keywords: Hip, modelling, peak stress, weight bearing area, stress distribution

1. INTRODUCTION

The hip joint is one of the biggest ball-socket and bearing-weight joint in the human body [1]. The clinical research shows that the hip joint appears particularly vulnerable to osteoarthritis, and the evidence suggests conditions which increase contact stresses also substantially reduce its durability [2-4]; on the other hand, procedures to reduce contact stresses may improve its longevity [5-7]. It is well recognized that the intrinsic pathomechanical changes in the hip joint depend upon local stress levels rather than upon global joint loading. Therefore, the estimation of hip joint contact stress is very useful for

both preoperative planning and postoperative rehabilitation. In addition, it is also very helpful to know the hip contact stress distributions for better understanding the mechanics of the normal and pathologic hip joint.

Basically, there are two ways to approach hip contact stress distributions. One is an experimental approach, including directly [8-11] or indirectly [12-13] hip contact stress measurements. The newer experimental approach uses the thin pressure sensitive film [14]. Certainly, these measurements give insight into the features of the hip. However, these measurements can only obtain contact stresses at the discrete locations, rather than overall. Even as pressure sensitive films, there are still some problems, but are typically thicker, the spherical nature of the hip would obviously create problems for flat sensors. Other is an analytic [15-24], either numerical or using finite element models [25-27].

The earlier modellings to study hip joint contact stresses were static and semi-qualitative [15-17]. Such models took into account statically estimated loads; approximate joint surface areas, determined average

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stresses, and then estimated stress distribution onto a planar projection of a joint. Although these approaches illustrated important concepts, they did not contribute materially to knowledge of actual stress levels or distribution. The recent modellings to study hip joint contact stress are based on an important assumption, which adopted a cosine function form as the stress distribution at the hip joint [18-24]. With regard to the contact finite element modelling [25-27], although they provide complete stress distributions, however, it is always difficult and time consuming in analyzing a three dimensional model. Therefore this method is not suitable to acquire information for treatment in individual case.

In this paper, the classical elastic contact theory [28-30] will be employed to study the peak stress, the contact stress distributions and the contact area at the hip joint. Firstly a ball-socket (hip head-acetabulum) articular elastic contact modeling at the hip joint is established. The femoral head and the acetabular shell are taken to be spherical, while the thickness of the cartilage layer before the deformation is assumed to be constant. Then the classical contact theory [28-30] is applied to analyze the peak stress, the contact stress distributions and the contact area at the hip joint. The results show that the present modelling predictions are basically agreement with those reported in the literatures.

2. METHODS

2.1 Geometric Relationships on the Modelling

The coordinate system for ball-socket elastic contact modelling at the hip joint is constructed as shown in Fig. 1. The initial contact point O between the femoral head and the acetabulum acts as the origin of the rectangular Cartesian coordinate system. The coordinate system is oriented so that the x - y plane is the public tangent one of the two contact surfaces; while the z -axis is perpendicular to the public tangent plane and through the centers of contact surfaces. The acetabulum is described as a hemisphere concave (socket) contact surface with the radius R_1 , while femoral head is described as a hemisphere convex (ball) one with the radius R_2 . It is assumed that the articular cartilage is a homogenous and linear elastic solid with a uniform thickness. And it is assumed that the frictional force between the femoral head and acetabulum is neglected since the fact of synovia in the hip joint [31]. The elastic modulus and Poisson ratios for the articular cartilages of the acetabulum and the femoral head are, respectively, E_1 , μ_1 , and E_2 , μ_2 . The point m and n (see Fig.1) are two arbitrary points on the two hemispherical surfaces, which are very

close to the public tangent plane, and the distance from the point m and n to the z axis is r . While the distances from the public tangent plane to the points are, respectively, z_1 and z_2 . Thus, we have

$$\begin{cases} (R_1 - z_1)^2 + r^2 = R_1^2 \\ (R_2 - z_2)^2 + r^2 = R_2^2 \end{cases} \quad (1)$$

Or

$$\begin{cases} z_1 = \frac{r^2}{2R_1 - z_1} \\ z_2 = \frac{r^2}{2R_2 - z_2} \end{cases} \quad (2)$$

It is because of the fact that points m and n are very close to the public tangent plane, thus we have,

$$\begin{cases} 2R_1 - z_1 \approx 2R_1 \\ 2R_2 - z_2 \approx 2R_2 \end{cases} \quad (3)$$

Furthermore, the equation (2) can be rewritten as following,

$$\begin{cases} z_1 = \frac{r^2}{2R_1} \\ z_2 = \frac{r^2}{2R_2} \end{cases} \quad (4)$$

Fig 1. The coordinate system for the ball-socket elastic contact modelling.

Or

$$z_2 - z_1 = \frac{r^2}{2} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) = \frac{R_1 - R_2}{2R_1 R_2} r^2 \quad (5)$$

If the displacements of the points m and n at the z -axis direction are, respectively, w_1 and w_2 , then the shorten quantity between any two points at the z -axis, δ , can be written as following,

$$\delta = z_2 - z_1 + w_1 + w_2 \quad (6)$$

Substituting Eq.(5) into (6), we have,

$$w_1 + w_2 = \delta - \beta r^2 \quad (7)$$

where $\beta = \frac{R_1 - R_2}{2R_1 R_2}$. This is the displacement equation or geometric relationship of the ball-socket elastic contact modelling at the hip joint.

2.2 Physical Relationships and Equilibrium Equations on the Modelling

It is assume that the displacement at any point M on the contact area is w due to the contact pressures. If the element area is taken as $s d\varphi ds$, here the s is the distance from the point M to the element area, while the φ is the angle between the secant line through the point M and the oy axis (see Figure 2). Then, the contact pressure acting on the element area can be written as following,

$$qsd\varphi ds$$

where the q is the contact pressures on the contact area.

Thus, according to the Boussinesq Problem's solutions [30], the displacement at any point M due to the pressures $qsd\varphi ds$ can be written as following,

$$w = \frac{1 - \mu^2}{\pi E} \iint_A q d\varphi ds \quad (8)$$

If the two elastic bodies contact each other, then we have,

$$\begin{cases} w_1 = \frac{1 - \mu_1^2}{\pi E_1} \iint_A q d\varphi ds \\ w_2 = \frac{1 - \mu_2^2}{\pi E_2} \iint_A q d\varphi ds \end{cases} \quad (9)$$

Fig 2. The coordinate system for calculating pressures on the contact area.

where the A is the elastic contact area. Substituting Eq.(9) into (7), we have,

$$(k_1 + k_2) \iint_A q ds d\varphi = \delta - \beta r^2 \quad (10)$$

$$\text{where } k_1 = \frac{1 - \mu_1^2}{\pi E_1}, \quad k_2 = \frac{1 - \mu_2^2}{\pi E_2}.$$

It is because of the fact that the elastic contact area between the hip head and acetabulum is the circular one, therefore, at the boundary of the contact area, the following relationship must be held,

$$w_1 + w_2 = 0 \quad (11)$$

Thus, from Eq.(7) or (10), the radii of the elastic contact area can be determined as following,

$$r_c = \sqrt{\delta / \beta} \quad (12)$$

The integral equation (10) can be solved by the inverse method. The results are given as following:

$$\begin{cases} r_c = \left[\frac{3\pi P(k_1 + k_2)R_1 R_2}{4(R_1 - R_2)} \right]^{\frac{1}{3}} \\ q_0 = \frac{3P}{2\pi r_c^2} \\ q(\zeta, \eta) = \frac{3P}{2\pi r_c^2} \left(1 - \frac{r^2}{r_c^2} \right)^{\frac{1}{2}}, \quad r^2 = \zeta^2 + \eta^2 \\ \delta = \frac{3\pi P(k_1 + k_2)}{4r_c} \end{cases} \quad (13)$$

Where the r_c is radii of the contact area; the q_0 is the maximum contact stress; the q is the stress distributions of the contact area; the P is the resultant force acting on the hip joint.

3. RESULTS

In the previous section, a ball-socket contact elastic modelling of the human hip joint was successfully established. On other words, if the radius and the material constants of the human femoral head and acetabulum and the resultant force acting on the hip are known, the peak stress and stress distributions and the articular contact area can be directly determined from Eq. (13). At the following, as an example, the formulae are examined. The parameter values are taken as following[32]:

$$R_1=25\text{mm}, R_2=24\text{mm},$$

$$E_1=E_2=2.989\text{GPa}, \mu_1=\mu_2=0.29$$

$$P=70\text{Kg}$$

Then from Eq (13), we have,

$$r_c \approx 5.74\text{mm}$$

$$q_0 \approx 9.74\text{MPa} \quad (14)$$

$$q = (\zeta, \eta) = \frac{3P}{2\pi r_c^2} \left(1 - \frac{r^2}{r_c^2} \right)^{\frac{1}{2}}$$

It is, in the case, that the peak stress and the radii of the articular elastic contact at the human hip joint are, respectively, 9.74MPa and 5.74mm . While, the stress distributions of the articular contact area at the hip joint is determined by the Eq (14). Basically, these results are basically agreement with those reported in the literatures for peak contact stress, such as [33]: 10MPa ; [34]: $4.9\text{-}9.6$; [35]: $6\text{-}10\text{MPa}$; [36]: $7.5\text{-}9.0\text{MPa}$; etc. This shows the validity of the current modelling.

4. DISCUSSIONS

The peak stress, the weight bearing area and the stress distributions of the articular contact at the human hip joint are thought to be most important factors, which determine the location of the degenerative foci that later result in degenerative damage and in the development of osteoarthritis. However, so far these key factors are, mainly, determined by measured in vitro or vivo. It is noticeable that the human bone is inhomogeneous. And also, the mechanical properties

of bone are different due to the age, race, sex, dry bone and fresh bone etc. Therefore, it is necessary to develop a predictive modelling for clinical practice. In this paper, a ball-socket articular contact elastic modelling of the human hip joint was successfully established. This modelling can be used to predict the peak stress, the weight bearing area and the stress distributions of the articular contact at the human hip joint. As a test for the validation of modelling, an example is examined. The results show that the predictions from the current modelling are basically agreement with those reported in literatures. This shows the validity of the current modelling.

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