The Income *Gini* of Fairness

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We are in favor of wealth accumulation in a free and democratic nation. Yet, the question is whether the mechanism of wealth accumulation is the same to all the people or are there different mechanisms to different socioeconomic-groups, therefore unfair biases in the nation. Here, we would like to develop ways to quantify and fight there unfair distortions. Utilizing methods from socioecono-physics, we find that in a democratic and a free nation, a fair society is such that the income distribution's *Gini* is in the range, 25% and 33%, 34%. (*Gini* is the departure of the distribution from the "all are equal" distribution). The value, $Gini \approx 40\%$, is indeed the transition from the intermediate stage nation to the corrupt nation, where $Gini \leq 40\%$ requires that the income distribution is smooth in the top percentiles (rather than a jump like formed). Therefore, the effect of the 0.1% excessive wealth is pivotal to the problem; such nations have *Gini* values beyond 40%. The wealth distribution's properties further strengthen these findings.

We develop a dynamical model that is emphasizing the difference in the environment of the two camps: the top and the rest, yet find that interactions create a smoother *income density*.

We therefore think that the income distribution's $Gini \approx 40\%$ is the alarm stage, yet the *Gini* should better follow the middle range of the fair society stage (25% through 33%) rather than the upper bound in that stage, where ways to realize that are elaborated on.

Keywords: *Fairness*; *Gini*; income distribution; wealth distribution; 1% biased economics; crony economics.

1. Introduction

Since the 2007, 2008, 2009 worldwide economical crisis occurred, the discussion and debate about fairness in nations and about "how distorted the economy is" and about "crony economics" and other economical biases have been appearing everywhere, from the main stream media to social networks and the scientific world^{1–21,a};

^aHere the reader is encouraged to see Ref. 4 appearing in the media from familiar economists about the structure of 1% and 99%.

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clearly, people want to benefit from their capabilities, relative advantages and efforts^{22–25} yet the question is whether there are biases in the society that interfere with the basic feeling (and presumably the reality) of fairness and success based on these.

The income distribution “tells all the stories” of the society^{26–28}: “who is rich??”, “who is poor??”, about fairness and about opportunities, versus corruption and crony economics^{1–7} (see footnote a). Clearly, there are other economical and sociological functions and indicators that are valuable in describing nations^{36–53} and freedom⁸ and democracy⁶ are influencing lives much more. Yet, the income distribution, and the distributions derived from that, the “wealth distribution” and the “control on the flow of wealth distribution”, are clearly among the basic most influential quantities and should be researched on. The income density function $f(c)$ is the probability density function to earn wages c per month; from that, we compute the cumulative income distribution $F(c)$ (Lorenz curve in economics) and the “society income curve”, $\tilde{F}(y)$; $F(c) = \int_0^c f(x)dx = y \rightarrow \tilde{F}(y) = c$. Fairness in a nation can also be computed based on the properties of the income density function.¹ For example the *Gini*^{9,26–28} of that distribution tells about the distortion in the income distribution, therefore, suggests to research on whether the opportunities are based on abilities and efforts rather than on “crony economics” and other biases (e.g., corruption). *Gini* is a number, $0 \leq Gini \leq 1$, where the *Gini* formula is^{26–36}

$$Gini = \frac{1}{\langle c \rangle} \int_0^\infty F(c)(1 - F(c))dc \quad (1)$$

(here $\langle c \rangle$ is the average of f and is finite). The value $Gini = 1$ is for a society, where only a few are very rich and the rest are poor; $Gini = 0$ is for “all are the same” society, where we expect that fairness in a society can create the reality, where those with capabilities that take more efforts can progress and benefit more than others. Such society’s dynamics create the exponential income density function,^{27–30} where the *Gini* upper bound is 0.5. Clearly, those that “have money” can make more money, nonetheless, they also face competition that is based on their situation. Yet, the problems with biased economy are with these opportunities that are unjustly blocked; such economy creates an inverse power law income density function (known as Pareto density in economics), where the *Gini* upper bound is 1, and is “built” (many think) from biases in banks to the rich and the top bankers’ social circles, monopolies, crony economics,^{4,5} biased governmental bids, and other economical biases in business and the public sector.

Here, we study the income density’s dynamics and *Gini* and show that the value $Gini \approx 40\%$ is the transition from a fair to a corrupt nation (here what is not fair is corrupt with various levels) and that $Gini \leq 40\%$ requiring smoothed form of the income density at the top percentiles, where ways to realize that are elaborated on. Among the results in this work, we are yet researching on the wealth density to identify how features in this density influence fairness in society. We develop a model to explain $f(c)$ which emphasizes mathematically what the biases that “build” the

0.1% excessive wealth are, and how the model facilitates in treating these biases. Examples to create fairness in nations and to create the income density with $Gini < 40\%$, and to fight unjust economic biases that build the biased dynamics and the unjust 0.1% excessive wealth include founding a “bank of the citizens” in the nation that will fight “crony” banks, advancing special taxation on the top 0.1% also relative to the poor 20% conditions, fighting corruption and breaking monopolies that build the 1% biased economy, yet also creating economical circle with socio-economic strength to form economical interactions with the top 1% that tend to concentrating on the control on the flow of wealth there (in that top circle).

2. The Income Exponential Density

In this work, we start basically by analyzing $f(c)$ form to determine the *Gini*’s range of fairness, treating the simplest form seen in nations, and that is of the exponential income density,

$$f(c) = \begin{cases} 0, & c < \mu, \\ e^{-(c-\mu)/\theta}/\theta, & c \geq \mu, \end{cases} \quad (2)$$

f in Eq. (2) appears in many cases to describe the poorer 90% of the population.^{27–30} Here in Eq. (2), μ is the minimal wages and $\mu + \theta$ is the average wages, where the fluctuations around the average, σ , equal θ . See Fig. 1. The income exponential density is derived from “fair economics dynamics”, where people rewarding chances’ opportunities and benefits are based on their capabilities and efforts, see, e.g., Refs. 26–30, and this is equal to $f(c)$ diffusion equation with a linear potential, Eq. (10), saying that those with income c face potential proportional to c . What is pretty interesting is that the resulting society income curve $\tilde{F}(y)$ (that shows “how much each socioeconomic fraction earning”) is smoothed at the top, where those from the top one in a thousand circle earn just nine times the minimal wages, see Fig. 1.

The *Gini* of the exponential income density is computed from the general formula, e.g., Ref. 26,

$$Gini = \frac{1}{2(1 + \frac{\mu}{\theta})} \leq 50\%, \quad (3)$$

yet the following analysis is pretty unique. Clearly, when θ is much larger than the minimal wages, the poor are very poor and are far away from the average. In many nations, the average wages are twice or three times the minimal wages, $\theta = \mu, \dots, 2\mu$. In that θ range, the *Gini* is in the range,

$$25\% \leq Gini \leq 33\%, \quad \mu \leq \theta \leq 2\mu, \quad \text{fair stage.} \quad (4)$$

Therefore, presumably, most nations are fair. The problem is that the income distribution is not an exponential in the upper class, where f is the inverse power law and the *Gini* beyond 40%; see the next paragraph.

Fig. 1. (a) A *log linear* plot of the exponential income density function. The first vertical curve is μ , the second is $\theta + \mu$ (with cumulative probability 63%), the third is 3μ (the fluctuations σ from the average) (with cumulative probability 86%). (b) The *Gini* for the exponential income density function as a function of θ , wages, in terms of μ , therefore with normalization *wages/minimal wages* (*x*-axis). The fair *Gini* is within the light blue horizontal lines. The three other regimes are highlighted, I (departure from fairness), II (corruption built nation), Cor (corrupt). (c) The society income curve on a *log linear* scale, with the normalization, *wages/minimal wages* (*y*-axis). This function is the income of every fraction in the society. The form is a multi exponential with a slope one and two for the whole group of 95% of the society (blue and green curves), and then a slope 5 to the deciders top 4% before the top 1% (red curve), and a slope 50 to the top 1% (light green). The function is continuous at the top, where those from the top 0.1% earn just five times the “90%”.

When θ is within 2μ – 3μ , the *Gini* values are in the intermediate stage of 38%,

$$33\% \leq Gini \leq 38\%, \quad 2\mu \leq \theta \leq 3\mu. \quad (5)$$

Next, *Gini* increases to $\frac{1}{2}$, with $\theta, 3\mu \leq \theta \leq \infty$, i.e., in the exponential distribution, the maximal *Gini* is $\frac{1}{2}$.

Yet, we can identify another regime in the *Gini* of the exponential distribution and that is in the range 38% to 45%; we term that stage, “corruption built nation” and after “corrupt” nation, see Fig. 1 (the figure shows that the *Gini* as a function of

θ has three regions, fast, intermediate and slow varying function regions, where the intermediate regime is further defined with the two regions, I and II).

The wealth distribution for the exponential case: We also research on the wealth density to compute the function from the income density, saying that just those with the income C_{save} at least can save, e.g., the exponential wealth density is

$$f_w(c) = \begin{cases} p\delta(c), & c < C_{\text{save}}, \\ qe^{-\frac{(c-C_{\text{save}})}{\theta}}/\theta, & c \geq C_{\text{save}}, \end{cases}$$

where $p + q = 1$, and $C_{\text{save}} = \mu + \theta \ln \frac{1}{q}$. The average wealth, $\langle \text{wealth} \rangle$, is $q(\theta + C_{\text{save}})$ and the *Gini* is

$$Gini = p \frac{\theta(1 + \frac{q}{2p}) + C_{\text{save}}}{\theta + C_{\text{save}}} = p + \frac{q}{2} \frac{1}{1 + \frac{\mu}{\theta} + \ln(1/q)},$$

where with $2/3$ of the population that can save, $q = 0.67$, $C_{\text{save}} = \mu + \theta \ln \frac{1}{0.67} \approx \mu + 0.4\theta \leq \text{the average wages}$, i.e., the average wages can save, yet those around q are just in the first steps in saving, therefore the *Gini* is above 40%.

We can also compute other functions from $f(c)$, e.g., the “control on the flow of wealth” density function (derived in a similar way to the computations of the wealth density function from the income density function), where in that density function the situation is much more severe and just the top class appears on the chart ($q_{\text{control}} \approx \text{the top fraction}$). The top class is traditionally the top 10% (yet we can define that the top class is the top 5% based on the form of f seen in many societies), still in any case, we can further define groups within the top group and these are the top 1% and, in particular, the top 0.1%, what we defined in Ref. 1 the “deciders”. We emphasize that the size of the group within a group affects C_{save} : smaller and richer and more influential such subgroup increase C_{save} (see Ref. 1).

3. The Income Pareto Distribution

We now analyze the income Pareto density, Eq. (6),

$$f(c) = \begin{cases} 0, & c < \mu, \\ \left(\frac{\gamma}{\mu}\right) \left(\frac{c}{\mu}\right)^{-1-\gamma}, & c \geq \mu, \end{cases} \quad (6)$$

where this $f(c)$ is based on biased dynamics: the “rich are getting richer”,⁵³ or on a logarithmic potential in the $f(c)$ diffusion equation, see Eq. (10), also emphasizing the biased environment, the top are living in (this is seen from the ratio U/c or the force absolute value $|U'|$ equal to $1/c$, i.e., the force decreases with c and therefore to the rich “things are relatively simpler”).

Fig. 2. (a) The *Gini* for the *Pareto* income density function as a function of the power γ , where the fair *Gini* is within the light blue horizontal lines. The three other regimes are highlighted, I (departure from fairness), II (corruption built nation), Cor. (corrupt). (b) The society income curve on a *log linear* scale, with normalization, *wages/minimal wages* (*y*-axis). This function is the income of every percentile in the society. Red curve is the exponential income curve (lower curve, stars), the blue curve is the Pareto income curve with $\gamma = 2$ (circles) and the black curve is the Pareto income curve with $\gamma = 1$ (upper stars); it is clear that those in the top 0.1% are very rich just in the Pareto $\gamma = 1$ curve, where in the other cases, each curve is continuous and the rich are just moderately rich.

In Eq. (6), μ is the minimal wages, where with $\gamma > 1$ the average wages are finite, and equal to, $\mu \frac{1}{1-\frac{1}{\gamma}}$. Clearly, just when $2 < \gamma$, the fluctuations σ are finite ($\sigma = \mu \frac{\gamma}{(\gamma-2)(\gamma-1)^2}$) and we are seeing a fair nation where the rich are just moderately rich in that case. See Fig. 2.

The Pareto *Gini* is computed from the general formula, e.g., Ref. 26,

$$Gini = \frac{1}{2\gamma - 1} \leq 1, \quad \gamma > 1. \quad (7)$$

We emphasize that from Eq. (7), we are seeing here also that the fair *Gini* bound is $Gini \leq 0.34$ ($\gamma \approx 2$), where the average is just twice the minimal wages (like in the exponential case with $\theta = \mu$) and the fluctuations are finite and therefore the rich are just moderately rich. (See, Fig. 2 panel A, again showing that the *Gini* as a function of γ in that case, has three regions, fast, intermediate and slow varying function regions, where the intermediate regime is further defined with the two regions, I and II). The definitions, “fair” and “corrupt”, that are derived from the *Gini* simply refer to the flow of opportunities in the nation: a nation with distorted wealth density at the top biases opportunities toward the top, and that we define corrupt.

The society Pareto income curve is very interesting (see Fig. 2). The case where γ is about 1 has a jump like formed distribution at the top: advancing from the “1%” to the “0.1%”, the income is 10 times larger, and it is similar to the jump from the “10%” and the “1%”. Many capitalist nations have these features in the society income curve. Yet, in the limit $\gamma = 2$, the distribution is smooth at the top 1% and top 0.1%, what we define a fair income curve; see Fig. 2.

We also emphasize that in the symmetric Theta function income density function $f(c) = \theta(c)/\text{interval size}$, the *Gini*'s upper bound is 33% (see supplementary information on the website), where although there are many rich people in that society, this result is of the equal unbiased case, therefore valuable to compare with.

4. The Combined Case

Here in the combined case, there are two camps in the society (say the top and the others or the deciders and the society) each having a unique $f(c)$. In many cases that are seen in the world, the income density for the 90% group (or any other group with top income $\tilde{c}$) is the exponentially shaped $f(c)$, where the top of the society has the Pareto density $f(c)$,

$$f(c) = \begin{cases} 0, & \mu > c, \\ \frac{1}{\#} e^{-\frac{c-\mu}{\theta}}, & \tilde{c} \geq c \geq \mu, \\ \frac{1}{\#} e^{-(\tilde{c}-\mu)/\theta} \left(\frac{c}{\tilde{c}}\right)^{-1-\gamma}, & c \geq \tilde{c}, \end{cases} \quad (8)$$

where $\#$ is the normalization, $\#^{-1} = 1 - e^{-\frac{\tilde{c}-\mu}{\theta}} + \frac{\tilde{c}}{\theta} e^{-\frac{\tilde{c}-\mu}{\theta}} \frac{1}{\gamma} = p + q \frac{\tilde{c}}{\theta\gamma}$. Here, $\tilde{c} = \mu + \theta \ln \frac{1}{1-p/\#}$, $f(c)$ in Eq. (8) is a continuous function, see Fig. 3, where the integral $\int f(c)dc$ is unity, showing that $f(c)$ with normalization; also, from Eq. (8) we deduce that the upper class influences the others through the value of $\tilde{c}$, and causes a slight increase in their income where $\frac{\tilde{c}}{\theta\gamma} > 1$, yet also an increase in the *Gini* (relative to the case where $\#^{-1} = 1$, seen from the computations) where the reason is that, causing the increase in the income mostly in the area of the top of that circle.

Fig. 3. (Axes' captions are within the pictures to facilitate the presentation). (a) A log linear plot of the combined case income density function. The vertical curve is μ , the light blue curve (circles) is the exponential income curve with $\theta = \mu$, and the black curve (stars) is the Pareto curve with $\gamma = 1$, (b) the *Gini* for the combined income density function as a function of the γ , with $\theta = 1.4 \mu$, where the fair *Gini* is just where $2 \leq \gamma$.

We can say that $\#^{-1} = 1$ in Eq. (8) and rewrite the third equation in Eq. (8),

$$f(c) = \frac{\gamma}{\tilde{c}} e^{-(\tilde{c}-\mu)/\theta} \left(\frac{c}{\tilde{c}}\right)^{-1-\gamma}, \quad c \geq \tilde{c}, \quad (9)$$

where this $f(c)$ may show a jump at the value $c = \tilde{c}$, yet that depends on the $\tilde{c}$ and γ values (this is the case where $q \leq 11\%$ and $\gamma \leq 3$, where these are the relevant values here to these quantities). Here, the top circle does not influence the “90%” normalization and $\tilde{c}$ is independent of the top circle’s wages, $\tilde{c} = \mu + \theta \ln \frac{1}{q}$. (This function is presented in the supplementary information with comparison to Eq. (8) $f(c)$.)

A logarithmic potential in the $f(c)$ diffusion equation of motion will result in a Pareto density. This potential’s shape is to compare with the linear potential in the $f(c)$ diffusion equation of motion that results in the exponential density function, further emphasizing the difference between the two camps. In the combined case, each camp’s equation of motion is with the unique per case potential, where the two camps can interact, see Eqs. (10)–(14). Due to the boundary at $\tilde{c}$, the potential shapes change there, yet also in that case, there is a very visible difference between the two camps (perhaps even more relative to other cases), and therefore the sociological economic arguments (that are presented in this part) are still valid.

We compute the average and fluctuations of f and the *Gini* (see supplementary information). We find that in the case where $\gamma \approx 2$, the *Gini* is in the 40% area, again showing that *Gini* $\approx 40\%$ is the transition from fairness to corruption since that is also the value where the fluctuations in the income density are finite, see Fig. 3. In the case of $f(c)$ top circle is Eq. (9), the *Gini* is just about 3% smaller than the *Gini* of $f(c)$ from Eq. (8).

The value of the *Gini*, where $\gamma \approx 2$, is a function also of θ/μ , where with the increase of θ/μ from 1 to 3, the increase in the *Gini* is from 35% to 45%. In a society, where the “90% economics” is not so fair and the average wages ($\mu + \theta$) are far from the minimal wages (μ), we should put higher pressure on the deciders to see fairness in the nation (just higher values of γ can result in *Gini* $\leq 40\%$, and higher values of γ basically take income from the deciders top earners to the rest). These matters are also seen in the case of $f(c)$ top circle as shown in Eq. (9).

Again, this analysis on the combined case can show that a society with a smooth income distribution, in particular at the top, is required for realizing fairness.

About the dynamics for the combined case: The equation of motion for $f(c)$ can serve in explaining completely (within the model) how the situation in the society is formed and how to influence the society. Various dynamics were proposed for the combined case that are based on stochastic dynamics.^{27–30} Here, we develop a dynamic equation to the density $f(c)$, where similar to the work about fairness in nations,¹ two camps or circles of society are defined, where these are termed, “society” and “deciders” (SC for society and the DC for deciders) and these interact through the same temperature (general interactions) yet here also directly. The

deciders are the top 10% (or the top 5%, also with inner camps in the top “1%” and the top “0.1%”) and the rest is the society. The equation of motion for the income probability density $f_{SC}(c, t)$ and $f_{DC}(c, t)$ is as follows:

$$\partial_t f_{SC}(c, t) = \partial_c (U'_{SC} f_{SC}(c, t)) + D \partial_c^2 f_{SC}(c, t) + \text{interactions term}, \quad (10)$$

here, $\mu \leq c \leq \tilde{c}$, with reflecting boundaries, where,

$$\partial_t f_{DC}(c, t) = \partial_c (U'_{DC} f_{DC}(c, t)) + D_{DC} \partial_c^2 f_{DC}(c, t) + \text{interactions term}, \quad (11)$$

with $\tilde{c} \leq c$, and a left reflecting boundary. The potential (up to a free constant) in the society case is

$$U_{SC} = (c - \mu)/\theta, \quad (12)$$

where in the deciders' case it is,

$$U_{DC} = (1 + \gamma) \ln(c/\tilde{c}). \quad (13)$$

Therefore, this model shows further the problem in the society that has a distorted income density, as the rich diffuse almost without any opposition where the poor face difficulties that increase with the income c (seen from the various $|U'|$ forms).

The interactions term in Eqs. (10) and (11) is quadratic in the wealth function, showing that people from the top interact with those from the society circle, wherein the rate functions determine the flow of wealth, here, e.g., is the society interactions term equation,

$$\text{interactions} = \sum_y (w_{DC}(y \rightarrow c) f_{DC}(y, t) f_{SC}(c, t) - w_{SC}(c \rightarrow y) f_{SC}(c, t) f_{DC}(y, t)). \quad (14)$$

Here, $w_{SC}(y \rightarrow c) = v e^{-|y-c|/\tilde{l}}$, with $\tilde{l}$ is a fraction of μ (say, 33% $\mu \leq \tilde{l} \leq 55\% \mu$, depending on the model design, corresponding to the circle of size of 3% through 8% of the society, respectively). $w_{DC}(y \rightarrow c) = w_{SC}(y \rightarrow c)$ is in the range $\tilde{c} \leq y \leq \tilde{c} + \tilde{l}_*$, ($\tilde{c} + \tilde{l}_*$ are the wages to enter the 1%), yet beyond, $w_{DC}(y \rightarrow c) = v e^{-|c-\tilde{c}|/\tilde{l}} \equiv \tilde{v}$. The form of $w_{DC}(y \rightarrow c)$ is explained with the following; here, we deal with the case, where the wealthy can choose with whom to interact within the top society circle, yet there are various economic reasons that impose or create these interactions (e.g., pressure from the society circle). Therefore, Eq. (14) is computed to yield

$$\sum_{y=\tilde{c}+\tilde{l}_*} (\tilde{v} f_{DC}(y, t) f_{SC}(c, t)) \leq q \tilde{v} f_{SC}(c, t).$$

In the first approximation $v^0 \rightarrow 0$, therefore, in equilibrium,

$$f_{SC}(c) = \tilde{f}_{SC} \exp\{-(c - \mu)/\theta\}, \quad (15)$$

and,

$$f_{DC}(c) = \tilde{f}_{DC} \exp\{-(1 + \gamma) \ln(c/\tilde{c})\}, \quad (16)$$

where $\tilde{f}_{SC}$ and $\tilde{f}_{DC}$ are the normalizations.

Fig. 4. The representation of the wages c , the society's fractions, and the interaction figure. Here the wealth to enter the top 1% is just five times the average wages.

The correction term is about various percents (3%) of $f_{SC}(c)$ in the area of the interaction (circle of about 5% of the society top) (see supplementary information and Fig. 4). This is a very valuable result, showing how interactions with the top circle can create a smoother $f(c)$ also in a society where two camps exist. The computations and simulations, see supplementary information, showed that the top 1%, when interacting with the society circle, should also have a left reflecting boundary at $\tilde{c}$ and at the top 1% wages, with either (#) free diffusion with a diffusion coefficient that is twice that of the society, or (§) diffusion with the logarithmic potential yet with a diffusion coefficient that is 3, 4 times larger than that of the society. Otherwise, the 1% lost wealth due to the interactions (will lose more than the wealth that is shifting to the top society circle, and that is due to the “spring effect” caused from these interactions, on the top circle). These results form the comparison point. The reason for the interactions is the socioeconomic strength, that the top society circle presumably have created, with the formation of unique organizations, (see (7) in Sec. 5), and therefore these results just suggest how to press on the top 1% rather than to “bribe” these (from the top 1%) to interact with the top society circle.

To combine this model with “agents’ model” is possible (similar to the model in Ref. 1) and might generalize the results, yet this is of value to emphasize that the structure is unique here and should characterize the model also in generalizations; any model that ignores the fact that in all world nations the base structure is: 99% and then the 1%, is “too naïve” to work and fight biases.

5. Discussion and Conclusions

Utilizing methods from socioecono-physics, we showed that in a democratic and a free nation, a fair society income distribution's *Gini* values are in the range, $25\% < Gini < 34\%$, and for a *Gini* beyond that, we can define various “intermediate society” stages, showing the departure from a fair nation to a corrupt nation: I, $33\% \leq Gini \leq 38\%$ (“just departure from fairness”), II, $38\% \leq Gini \leq 45\%$ (“corruption built nation”), and then, Cor. $45\% \leq Gini \leq 50\%$ (“corrupt nation”); a *Gini* beyond 50% is for extremely corrupt nations. This characterization is based on analyzing various cases and forms of the income density function and dynamics. In all cases, there are three regimes in the plot of the *Gini* suggesting the definitions of fair, intermediate and corrupt, where the intermediate region has two sub-regions. For

example zooming on the *Gini* in the combined case seen in many nations (where the income density is exponentially shaped for the most and then a Pareto density), the value $Gini \approx 40\%$ is indeed the transition from the intermediate stage nation to the corrupt nation, where $Gini \leq 40\%$ requiring that the income distribution is smooth in the top percentiles (rather than a jump like formed). The properties of the wealth distribution and the control on the flow of wealth distribution further strengthen these findings. We therefore think that the income distribution's $Gini \approx 40\%$ is the alarm, yet the *Gini* should better follow the middle range of the fair society's stage (25% through 33%) rather than the upper bound in that stage. The evidence, implications and suggestions to make this is presented in the following paragraphs.

- (1) The exponential income density shows that, $Gini \approx 33\%$ is the bound for the fair society, and $Gini > 38\%$ is the transition from the "intermediate fair" society to a corrupt society as the average wages are too far from the minimal wages in the "fair society model" (the exponential income density is derived (among various possibilities) from "a linear potential acting on the density" diffusion equation of motion; that is to say that also those that are rich face competition that is proportional to their situation, therefore, it is a fair economy). In most societies, the average wages are twice the minimal wages, and the *Gini* is beyond 40%. The problem is the distortion in the income density at the top percentiles, in particular, the top 0.1% (there the distribution is not exponential).
- (2) The income density tells only a part of the "story" of the situation of the people, where complementing the description are properties from the wealth and the "control on the flow of wealth" densities. In comparison to the income density, the (distorted) situation is more severe in the wealth density and the "control on the flow of wealth distribution". In terms of the wealth density, we treat the question "how many can save?". The wealth density's *Gini* is a function of the amount of those that can save, $Gini \approx 1 - \text{fraction those that save}$. With $2/3$ that can save, the (exponential wealth distribution) *Gini* is about 40% and (these from) the average wages can save and that's (we can define) fair.

Form analyzing the "control on the flow of wealth" density function, we see that when the richer the top "1%" are, the created society has fewer saving percentiles.

- (3) The income *Pareto* density shows that $Gini \approx 33\%$ is the fair society *Gini* bound, where the reason is that the income fluctuations are finite just through $Gini \approx 33\%$, where the rich are then just moderately rich.
- (4) The equal symmetric density has a maximal *Gini* of 33%. Although there are many rich people in that society, it is the unbiased scenario, forming valuable bound to compare with.
- (5) The combined case, where the "society" has the exponential income density and the "deciders" have the Pareto income density explicitly shows that $Gini > 40\%$

is the departure from fairness since the f fluctuations are finite just through there ($Gini \leq 40\%$).

- (6) Writing the equation of motion of the combined case, we developed here the dynamical model with two interacting camps, each having a different potential form: a linear potential in the “society” equation of motion, and a logarithmic potential in the “deciders camp” equation of motion. This emphasizes the different environment, the two camps are living in, where to the rich “things are simpler” also in a relative way (as U/c or the force absolute value is a constant in the society’s camp and a decreasing function in the deciders’ camp).

Interactions create another area, the combined area of size of about 3%, 4%, 5% of the population just before the deciders’ camp, and a smoother f in the top percentiles.

- (7) The effect of the 0.1% excessive wealth is therefore pivotal to the problem; such nations have $Gini$ values beyond 38%. We developed a dynamical model that emphasizes the difference in the environment (the potential forms) of the two camps: the top and the rest, yet find that interactions create a smoother income density. The activities to create fairness in nations (and a smoother f) fight unjust biases and the unjust 0.1% excessive wealth. The possible activities to create a fair society are presented in the following matters, these include,

- (1) *Citizens’ activities to replace the “top 0.1% money”*: Replacing the “top 0.1% money” with 99% money is central to the 99% camp’s ability to fight unjust biases from that top “0.1%” camp. This is possible when citizens organize to work together to create crowdfunding and unbiased banks, a bank with a clear “pro 99%-based management”, that work with citizens in all areas of life. Banks’ crony behavior builds the excessive wealth of top 0.1%. Founding a “bank of the citizens” in the nation can help shifting wealth in the direction of the citizens from the “1%” (in the wealth density: see Figs. 1 and 3), in particular wealth that is not based on abilities and contributions to the citizens/society. Clearly, the banks’ stability is important in society; yet valuable to emphasize that recent years showed that “banks stability” is a matter of the public intervention. In other words, the banks depend on the public, not the opposite. Various related ideas appear in the recent economic literature involving the debate on “who should control the money production?”^{55,56}.
- (2) *Special taxation on the top 0.1%*: Taxation, although does not change the power structure of the society like the other matters here, is the simplest way to fight unjust excessive 0.1% wealth. (These taxation policies can compute the particular profession and the relative top “1%” wealth, also to compensate through taking into account long-term effects). There are various European nations that applied similar measures, e.g., Ref. 57. We can also create these taxation rates relative to the poor 20% conditions. In general, policies that help those 20% poor will advance fairness probably beyond that category. Yet also, there are fairness arguments regarding the “1%” wealth versus the poor

20% situation: in a nation where the relative success depends on contributions and abilities, we should take into account the 20% poor situation relative to the top 1% situation; e.g., this ratio can determine the taxation rate on the top 1%. Policies that will utilize this ratio when e.g., computing taxation on the top 0.1% might form the convergence area among socialism and capitalism. The “poverty line” is an absolute definition, yet the ratio, $\frac{\text{wealth top } 1\%}{\text{wealth poor } 20\%}$, is a relative one. The importance of the relative definition is evident when talking on the flow of opportunities and fairness.

- (3) *Fighting corruption and monopolies that build the 1% biased economy:* Fighting corruption in general is an important matter when advancing fairness in nations, yet from the information regarding many nations it appears that the 1% is built from corruption in many branches of the society. Policymakers should help corruption fighters with special policies.

Breaking monopolies that build the 1% crony economics is also central to fairness; clearly, there are various governmental agencies that are focusing on monopolies, yet there is a lot of work, since many monopolies are adjusted-effective-monopolies that prevent competition and bypass the regulation with more than implicit help from the regulators in many cases.

- (4) *Interactions of the top circle with the others in the top third:* The model showed that creating another circle of socioeconomic fraction just before the top camp that can interact with the top, will create the smoother f . This result is very valuable and differs from the other three, since the other three just fight the top 0.1% yet the approach here is to interact with the top circle. Clearly, the upper circle tends to concentrate the control on the wealth in that circle, and therefore interactions are created just when the control on the flow of wealth is moved from the top to the others. This is possible when the citizens organize to work together and then also approach to interact with the top 1%, to compete with others from the top circle, showing where the public organization with public pressure can make the difference.

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