Fusion of BWM and AHP MCDM Methods to Choose the Most Suitable Secondary School for an Individual in the Context of Bangladesh

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Choosing the best schools from a group of schools is a multi-criteria decision-making (MCDM) problem. In this paper, we have represented a method that uses the fusion of two multi-criteria decision-making methods, Best–Worst Method (BWM) and Analytic Hierarchy Process (AHP), to rank some of the user preferred alternatives. The system considers the choice of the user and the quality of the alternatives to rank them. User preferences on the criteria are taken as inputs in the form of best–worst comparison vectors to measure the choice of the user. These values are applied to calculate the numeric weights of each of the criteria. These weights reflect the preference of the user. A dataset of secondary schools in Bangladesh has been compiled and used for automatic quantitative pairwise comparison on the alternatives to calculate the score of each alternative in every criterion, which reflects its quality in that criterion. These scores are calculated using AHP. The weights of the criteria as well as the scores of these alternatives in those criteria are then used to calculate the final score of the alternatives and to rank them accordingly. An extensive experimental analysis and comparative study is reported at the end of this paper.

Keywords: Analytic hierarchy process; best–worst method; multi-criteria decision-making.

1. Introduction

Multi-criteria decision-making (MCDM) problem arises in many sectors, such as business, agriculture, industrial production, technology and so on. Such problems

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can be solved by using comparison among criteria and solving those comparison values to obtain the weights of each criterion. If the scores of the alternatives in those criteria are not available or calculable as crisp numbers, the solution process also includes methods to calculate them.

To solve those problems, a number of methods have been introduced in the literature. Best–Worst Method (BWM) is one of such recently proposed methods. In this method the user chooses the most important criterion and the least important criterion. After picking up those a comparison has been made of the most important criterion with the other criteria, as well as the least important criterion with the other criteria. These importance values are used to calculate the weights of the criteria.

Another popular and reliable method to calculate the weights of criteria is Analytic Hierarchy Process (AHP). Like BWM, this method also assigns a number from a range for comparisons. But unlike BWM, comparison is done for every possible pair of criteria. These comparisons are mapped to numbers in a range, which are used to form a square matrix. From this matrix, weights of the criteria are calculated. In our previous paper,¹ Fuzzy AHP (FAHP) was used for MCDM problems. In this paper we present how BWM and AHP can be used in conjunction to obtain the weights of criteria and scores of alternatives in those criteria. It is demonstrated by solving the problem of selection of secondary school. In a developing country like Bangladesh, with overflowing population and very limited resources, it is truly important to select a good institute for a student's future prospects and professional success. The optimum selection of institution can help him or her to build a successful career, whereas a less suitable selection can lead the student into peril in professional life. Currently the decisions for mass students are made by their guardians. The conventional way of this decision-making includes considering some key factors, such as the school's location, how old it is whether it is a co-education school or a girls-only/boys-only school, the socioeconomic status of the students, the average age of the students in the intended grade, etc. Unfortunately, the outcome of such decision-making could not pick the most suitable selection, due to the lack of a systematic approach. This paper aims to propose and demonstrate techniques to solve this issue.

2. Literature Review

MCDM problems are very common in everyday life. Many methods have been proposed to solve the problem, such as Technique for Order of Preference by Similarity to Ideal Solution or TOPSIS,² Analytic Network Process or ANP,³ Elimination and Choice Expressing Reality or ELECTRE⁴ and Gray Decision-Making or GDM.⁵ One popular MCDM method is the AHP. Originally proposed by Saaty,^{6,7} this is used to solve MCDM problems, especially when calculating the weights of the criteria in consideration. This trait can be observed significantly in approaches to solve facility location selection problems. This functionality of AHP was demonstrated by Hwang and Yoon in Ref. 2 and Vaidya and Kumar in Ref. 8. Ayhan⁹ proposed Fuzzy

AHP approach to find out the most suitable supplier for a gear motor manufacturing company. The selection involved three alternatives that were to be judged based on five criteria: quality, origin, cost, delivery and after-sales. Two of the company's executives took part in a survey that determined the relative importance of these criteria. Then Fuzzy AHP was applied to determine crisp weights of each criterion. The alternatives were further pairwise compared based on all the criteria to determine alternative-specific scores in those criteria.

Boltürk *et al.*¹⁰ proposed a new method called Hesitant Fuzzy AHP in order to deploy warehouses for humanitarian relief organizations. They compared five alternatives using five criteria: geographical location, cost, transport connectivity, labor availability and stable government. After constructing a linguistic scale for Hesitant the authors used triangular fuzzy number (TFN) and trapezoidal fuzzy number for Fuzzy AHP, a pairwise comparison matrix was generated. This matrix was used to calculate fuzzy comparison scores for the alternatives.

Singh¹¹ proposed Extent Fuzzy AHP to locate a suitable facility. He presented a case study of an automobile manufacturing company that wanted to open a new manufacturing plant. For four location alternatives, four criteria were considered: proximity, infrastructure, economic factors and business environment — each having multiple sub-criteria. In this case, a matrix was used to determine the relative importance of these criteria. To find out the criteria weights, this matrix was used later on.

Another similar approach was proposed by Chou *et al.*¹² that integrated Fuzzy Set Theory with Factor Rating System and Simple Additive Weighting. This method was used to evaluate the facility location alternatives for a supply chain management problem — optimization of logistic network configuration of facilities like factories, warehouses, retail centers, etc. The authors categorized the criteria into three categories: critical, objective and subjective. They used both triangular and trapezoidal fuzzy numbers for relative evaluation of criteria. In this method fuzzy numbers denoted relative importance of each pair of criteria as well the absolute importance of each criterion. These data were used to construct a fuzzy rating matrix, from which a crisp value of the weight was obtained through defuzzification.

Trivedi and Singh¹³ proposed a combination of Fuzzy AHP and Goal Programming that could be used for emergency shelter location and projects' relocation. The authors minimized a number of criteria that include distance, risk, number of sites and uncovered demand and simultaneously maximized the suitability. The suitability depended on some other factors such as demand, capacity, utilization and budgetary constraints. This was a multi-objective problem and the authors proposed a solution by combining all objectives into a single objective function by Goal Programming. Preferences from experts, responses from decision makers and comparison of subjective criteria were taken as inputs in this process. Fuzzy AHP was used to calculate the priority vectors of criteria and locations, which were used in Goal Programming to come up with optimal solution for location relocation.

Yang and Lee¹⁴ showed how crisp numbers could be used for AHP. The paper presented an example of four key factors (market, transportation, labor and community) and 12 sub-factors being considered for selecting the facility location.

Best–Worst Method was proposed by Rezaei.¹⁵ He demonstrated MCDM problems could be solved by a very low number of pairwise comparisons. In this method at first the user chooses the most important criterion and the least important criterion. The most important criterion, or the “best” criterion, is compared to each other criterion and assigned a number in a range, for example, between 1 and 9. In this case, 1 represents that the criterion is as important as the “best” criterion, and 9 means that the “best” criterion is 9 times more important than the “worst” criterion. Similarly, a comparison is done of the least important criterion, or the “worst” criterion, with each of the remaining criteria. These numbers are used for an optimization process. The goal of this optimization process is to gain the maximum degree of consistency, via minimizing the maximum difference between the best-criterion-to-any-criterion comparison value and the ratio of the weights of the “best” criterion and that criterion. It also targets to minimize the maximum difference between the any-criterion-to-worst-criterion comparison value and the ratio of the weights of that particular criterion and the “worst” criterion. This optimization is done by some more value constraints, such as the maximum possible value of the sum of the weights.

Later in another paper¹⁶ the author proposed the linear model of BWM with a unique optimal solution. Rezaei *et al.*¹⁷ employed this simpler linear method to solve a supplier selection problem. The approach has three steps: pre-selection, selection and aggregation. In the pre-selection phase, a set of potential suppliers were selected through conjunctive screening. In the second phase, BWM was applied on a set of criteria. Finally, qualitative, quantitative, traditional business and environmental criteria were used in the third phase to obtain the most suitable supplier.

Guo and Zhao¹⁸ combined BWM and fuzzy logic in their paper by proposing a Fuzzy BWM procedure. In this method, triangular fuzzy numbers were used in BWM instead of crisp numbers. The rest of the BWM process is the same as described earlier.

3. Fuzzy Analytic Hierarchy Process

For the methodology described in Ref. 1, a database of secondary schools in Bangladesh was compiled. A few datasets were selected from Bangladesh Open Data.¹⁹ These datasets were combined to form a single database on schools. This database contains 20,000 schools with different information such as institute ID, name of the institution, division, district, thana, mauza, post, institute type, number of total teachers, number of female teachers, number of total students, number of female students, number of students by age, number of students categorized by parent's or guardian's profession, results at different public examinations, etc. The methodology is described in the following sub-sections.

3.1. User inputs

The user had to give the following crisp inputs about the student:

- (1) intended class for admission;
- (2) student's age;
- (3) student's sex;
- (4) student's residential location: division, district, thana, union/ward.

Personalization of results was an important factor in the methodology of Ref. 1. One of the key targets of the methodology is to make sure that two users do not get the same results only because of their close location or same age and grade. For this reason, the user compares the relative importance of the following criteria with each other: (1) Teacher–Student Ratio (TSR); (2) Male–Female Ratio (MFR); (3) Socioeconomic Status (SES) of the guardians; (4) Age of School (AS); (5) Distance from Home (DIST); (6) Difference between the Average Age of Students in the intended class and the user's inputted age (ADS). The user compares each possible pair of these criteria. For the six criteria, the user compares 15 pairs and assigns a linguistic term to denote the relative importance. These linguistic terms are given in Table 1.

Table 1. Linguistic values and their corresponding TFNs.

Criteria	Best/Worst
Equally important	(1, 1, 1)
Moderately important	(2, 3, 4)
Fairly important	(4, 5, 6)
Very important	(6, 7, 8)
Absolutely important	(9, 9, 9)

Here, each linguistic term is mapped to a TFN value. A TFN is denoted as a tuple $(l \ m \ u)$, for which the membership functions are described as $f(l) = 0$, $f(m) = 1$ and $f(u) = 0$.

3.2. Calculation of weights of criteria

The user inputs are used to form a comparison matrix compiled of TFNs. This matrix is of $n \times n$ in dimension, where n is the number of criteria. The value of TFN in the i th row and j th column represents how much important the i th criterion is compared to the j th criterion. Therefore, this TFN would be an inverse of the TFN in the j th row and the i th column. This comparison matrix is shown in Eq. (1).

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix}. \quad (1)$$

A column matrix R was composed with the geometric mean of each row of this comparison matrix:

$$R = \begin{bmatrix} r_1 \\ \vdots \\ r_n \end{bmatrix}. \quad (2)$$

Here, each member of R can be calculated by $r_i = (\prod_{j=1}^n c_{ij})^{1/n}$. Each of these r_i values is a triangular fuzzy number, (l_i, m_i, u_i) . From each of these members, we obtain $L = \sum_{i=1}^n l_i$, $M = \sum_{i=1}^n m_i$ and $U = \sum_{i=1}^n u_i$. These three values are used to form a vector $[1/U \ 1/M \ 1/L]$. Each member of the column matrix R is scalar-multiplied with this vector. This process results in the fuzzy weight vector W :

$$W = \begin{bmatrix} l_1/U & m_1/M & u_1/L \\ \vdots & \vdots & \vdots \\ l_n/U & m_n/M & u_n/L \end{bmatrix}. \quad (3)$$

The members of this vector W are the fuzzy weights of the criteria. These fuzzy weights are defuzzified by the center of area (COA) method:

$$v_i = \frac{1}{3} (l_i + m_i + u_i). \quad (4)$$

Finally, the values are normalized by their sum to obtain the weights of the criteria:

$$w_i = \frac{v_i}{\sum_{p=1}^n v_p}. \quad (5)$$

In this way, by applying FAHP on the comparison matrix C , the weights of n criteria $w_1, w_2, \dots, w_n$ are obtained.

3.3. Calculation of scores of alternatives

After calculating the weights of criteria, we calculate the scores of each alternative school in those criteria. For this process, we have used a dataset of secondary schools in Bangladesh. This dataset was compiled from multiple datasets available in Bangladesh Open Data,¹⁹ a government-maintained portal of datasets for public domain.

For SES a socioeconomic index is assigned based on the school's location in terms of administrative and socioeconomic levels (i.e. rural area, urban area or metropolitan area) and the profession of the guardians of the students studying in that school. In the dataset the schools are classified into seven administrative levels in terms of locations. How the location-based numeric values are assigned to schools is shown in Table 2.

Similarly, in the dataset the schools are classified into 15 occupations in terms of guardians' occupations. How the profession-based numeric values are assigned to

Table 2. Administrative levels and their status.

Administrative level	Area-based value (A)
Metropolitan	10.0
City corporation	9.0
District sadar municipality	7.0
Upazila sadar municipality	5.0
Other municipality area	4.0
Upazila sadar but not municipality	2.0
Rural area	1.0

Table 3. Occupations and their status.

Occupation	Weight of occupation (p_j)
Lawyer, doctor, engineer, businessman	10.0
Government job, private job, teacher	7.5
Expatriate, cultivation, small business owner	5.0
Worker, fisherman, weaver, potter, blacksmith	2.5

schools is shown in Table 3. The weights of these occupations from Table 3 are multiplied with the number of students whose guardians work in that occupation. The average of the 15 products is used to gain occupation-based weight value.

The socioeconomic index, S , is then calculated by taking the weighted average of the area-based value and the occupation-based value. The weights of area-based value W_A and occupation-based value W_P are assigned 10 and 5, respectively. N is the total number students and n_j is the number of students whose guardians work in the j th occupation:

$$S = \left(\frac{1}{N} \sum_{j=1}^{15} p_j n_j \right) \times W_P + A \times W_A. \quad (6)$$

Now, all the scores of schools must be between 0 and 1, inclusive. For some of these criteria, we use the cumulative distribution function (CDF) of the standard normal distribution. This is given by Eq. (7)

$$\text{cdf}(x, \mu, \sigma) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{x - \mu}{\sigma \sqrt{2}} \right) \right]. \quad (7)$$

Here, μ is the mean and σ is the standard deviation. For calculating mean and standard deviation, we consider the data of all the schools available in our dataset. Here, $\text{erf}(x)$ is the error function, which can be described by the following equation:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (8)$$

For example, “Teacher–Student Ratio” is an important criterion for selecting a good educational institute. For the criterion “Teacher–Student Ratio” we can apply

the formula described by Eq. (7). For the criterion Teacher–Student Ratio we apply Eq. (9) to get the score:

$$\text{score}_{\text{tsr}} = 1 - \text{cdf}(\text{tsr}, \mu_{\text{TSR}}, \sigma_{\text{TSR}}). \quad (9)$$

Here, tsr means the ratio of the number of students and the number of teachers for a specific school we are currently considering (e.g. educational alternative), $\text{score}_{\text{tsr}}$ means Teacher–Student Ratio score on the criterion tsr for that specific school, μ_{TSR} denotes the mean of all the TSR values of schools recorded in our database and σ_{TSR} denotes the standard deviation in TSR values recorded in database

The cumulative distributive function value is used to calculate the score of the school in Socioeconomic Status using Eq. (10):

$$\text{score}_{\text{SES}} = \text{cdf}(S, \mu_S, \sigma_S). \quad (10)$$

For Male–Female Ratio the score is calculated by the following Eq. (11):

$$\text{score}_{\text{MFR}} = \frac{m}{N}. \quad (11)$$

Here, m represents the number of male students in the school if the user’s inputted sex is male; otherwise it represents the number of female students in the school. N is the total number of students in that school.

For the criterion “distance” a variable $d = 10$ is assigned initially. If the school is in the same division as the user’s inputted division, 4 is subtracted from d . If the school is in the same district, thana or union/ward, 3, 2 or 1 is subtracted, respectively. Then the score is calculated by Eq. (12):

$$\text{score}_{\text{DIST}} = 1 - \frac{d}{10}. \quad (12)$$

For the criterion “age of school” the age of the schools from the dataset is taken and cumulative distributive function is applied [see Eq. (13)]:

$$\text{score}_{\text{AS}} = \text{cdf}(as, \mu_{\text{AS}}, \sigma_{\text{AS}}). \quad (13)$$

Finally, for the age-difference criterion, the absolute value of the difference between the user’s given age and the average age of students in the class, ads , is taken. Then CDF is applied and its complement is used for the score [see Eq. (14)]:

$$\text{score}_{\text{ADS}} = 1 - \text{cdf}(\text{ads}, \mu_{\text{ADS}}, \sigma_{\text{ADS}}). \quad (14)$$

3.4. Calculation of final compatibility score

After getting the scores of each school in every criterion, the user selects m schools to compare with each other in every criterion. Among these schools, for comparing i th school to j th school on n th criterion, an integer $s_{n_{ij}}$ is assigned as a comparative score according to Eq. (15). In this equation, score_{n_i} and score_{n_j} are the score values of the

Table 4. Comparative scores $s_{n_{ij}}$ and the corresponding TFNs $a_{n_{ij}}$.

Comp. score $s_{n_{ij}}$	TFN $a_{n_{ij}}$	Comp. score $s_{n_{ij}}$	TFN $a_{n_{ij}}$
$s_{n_{ij}} \leq -9$	(1/9 1/9 1/9)	2	(1 2 3)
-8	(1/9 1/8 1/7)	3	(2 3 4)
-7	(1/8 1/7 1/6)	4	(3 4 5)
-6	(1/7 1/6 1/5)	5	(4 5 6)
-5	(1/6 1/5 1/4)	6	(5 6 7)
-4	(1/5 1/4 1/3)	7	(6 7 8)
-3	(1/4 1/3 1/2)	8	(7 8 9)
-2	(1/3 1/2 1)	$s_{n_{ij}} \geq 9$	(9 9 9)
$-1 \leq s_{n_{ij}} \leq 1$	(1 1 1)		

i th alternative and the j th alternative, respectively, for the n th criterion:

$$s_{n_{ij}} = \begin{cases} 10 \times \lfloor |\text{score}_{n_i} - \text{score}_{n_j}| \rfloor, & \text{if } \text{score}_{n_i} \geq \text{score}_{n_j}, \\ 10 \times \lceil |\text{score}_{n_i} - \text{score}_{n_j}| \rceil, & \text{otherwise.} \end{cases} \tag{15}$$

Every comparative score $s_{n_{ij}}$ is mapped to a TFN $a_{n_{ij}}$ according to Table 4. These TFNs form a comparison matrix for each criterion, as shown in Eq. (16).

$$C_n = \begin{bmatrix} a_{n11} & a_{n12} & \cdots & a_{n1m} \\ \vdots & & \ddots & \vdots \\ a_{nm1} & a_{nm2} & \cdots & a_{nmm} \end{bmatrix}. \tag{16}$$

These comparison matrices are used for Fuzzy AHP as described earlier. Applying the processes denoted by Eqs. (2)–(5) on all n comparison matrices denoted by Eq. (16) yields crisp scores for schools in each criterion. This means that for matrices $C_1, C_2, \dots, C_n$ the process produces criteria-wise scores $\{v_{11}, v_{21}, \dots, v_{m1}\}, \{v_{12}, v_{22}, \dots, v_{m2}\}, \dots, \{v_{1n}, v_{2n}, \dots, v_{mn}\}$ for n criteria and m schools. Finally, the final score of these schools Z_i is calculated by taking their weighted sum, as denoted by Eq. (17):

$$Z_i = \sum_{p=1}^n w_p v_{ip}. \tag{17}$$

4. Fusion of BWM and AHP

The proposed method is a combination of BWM and AHP. The weights of the criteria are calculated by BWM. The scores of the alternatives in those criteria are calculated by AHP. In this methodology, Crisp AHP is used instead of Fuzzy AHP described earlier. This procedure can be described in the following steps.

- (1) Calculation of weights of criteria using BWM.

(2) Calculation of scores of alternative schools using AHP.

(3) Final score calculation of alternatives.

4.1. Calculation of weights of criteria using BWM

AHP requires the user to provide the relative importance value of criterion for each possible pair of criteria in the form of pairwise comparison. Here exactly $(n \times (n - 1) \div 2)$ comparisons are required for n number of criteria. In Ref. 1 it was shown how, for six criteria, 15 comparative values in linguistic terms for 15 possible pairs of criteria were obtained, and how they were assigned to TFNs to construct a comparison matrix. In this paper, how BWM can be applied with crisp numbers to obtain the same goal is shown. BWM requires less comparisons ($2n$ for n number of criteria) than what AHP requires and also yields results with good consistency ratio (CR).

Golany and Kress²⁰ proposed three evaluation measures of MCDMs, namely minimum variation (MV), total deviation (TD) and conformity (C). Rezaei¹⁵ clearly shows that the comparison results of three of the important measures favor BWM as compared to AHP. The ultimate goal of the criteria weight calculation is to find such weights which reflect the intuition of the taste of an individual user and require less human interaction to find a consistent result. Here, using BWM would be more reasonable than AHP because of better scores of BWM in MV, TD and C.

In this proposed method, the linear BWM, as described by Rezaei,¹⁶ is used for the sake of simplicity. The steps of BWM are as follows:

4.1.1. Selection of criteria

We determine the set of six criteria: $C = \{\text{TSR, SES, MFR, DIST, AS, ADS}\}$.

4.1.2. Choosing the “best” and the “worst” criteria

The user chooses the “best” criterion (denoted by BEST) and the “worst” criterion (denoted by WORST) from the set C .

4.1.3. Comparison of the best and the worst criteria to all the other criteria

The user is asked to provide the relative importance values (within the range 1–9) of BEST criterion over others in the form shown in Table 5. Here, $a_{\text{BEST,TSR}} = 1$ means

Table 5. Comparison format of BEST criterion to all other criteria.

Criterion	BEST
TSR	$a_{\text{BEST,TSR}}$
SES	$a_{\text{BEST,SES}}$
MFR	$a_{\text{BEST,MFR}}$
AS	$a_{\text{BEST,AS}}$
DIST	$a_{\text{BEST,DIST}}$
ADS	$a_{\text{BEST,ADS}}$

Table 6. Comparison format of all other criteria to WORST criterion.

Criterion	WORST
TSR	$a_{\text{TSR,WORST}}$
SES	$a_{\text{SES,WORST}}$
MFR	$a_{\text{MFR,WORST}}$
AS	$a_{\text{AS,WORST}}$
DIST	$a_{\text{DIST,WORST}}$
ADS	$a_{\text{ADS,WORST}}$

equal importance of BEST and TSR and $a_{\text{BEST,TSR}} = 9$ means the highest order of relative importance of BEST over TSR.

Similarly the user provides the relative importance values (within the range 1–9) of each criterion over the WORST one in the form shown in Table 6. Here, $a_{\text{WORST,TSR}} = 1$ means equal importance of the WORST and TSR and $a_{\text{WORST,TSR}} = 9$ means the highest order of relative importance of TSR over the WORST.

4.1.4. Assignment of values and optimization

It is assumed that W_{TSR} , W_{SES} , W_{MFR} , W_{DIST} , W_{AS} and W_{ADS} are the weights of the criteria TSR, SES, MFR, DIST, AS and ADS, respectively. It is also assumed that W_{BEST} and W_{WORST} are the weights of the BEST and the WORST criteria from the set of criteria, respectively. Now, the optimal weights for the criteria are obtained by $\forall j \in C$, ($\frac{W_{\text{BEST}}}{W_j} = a_{\text{BEST},j}$ or $W_{\text{BEST}} - a_{\text{BEST},j}W_j = 0$) and ($\frac{W_j}{W_{\text{WORST}}} = a_{j,\text{WORST}}$ or $W_j - a_{j,\text{WORST}}W_{\text{WORST}} = 0$).

According to the linear model described in Ref. 16, to satisfy these conditions for $\forall j \in C$, the target is to find a solution with the maximum absolute differences $|W_{\text{BEST}} - a_{\text{BEST},j}W_j|$ and $|W_j - a_{j,\text{WORST}}W_{\text{WORST}}|$ minimized for $\forall j \in C$. This term is defined as consistency ratio, denoted by ξ . Lower consistency ratio means the resulting weights are more consistent, and hence more acceptable. Now, an optimization problem can be designed to minimize this value ξ with all the constraints described below:

$$\begin{aligned}
|W_{\text{BEST}} - a_{\text{BEST,TSR}}W_{\text{TSR}}| &\leq \xi, \\
|W_{\text{BEST}} - a_{\text{BEST,SES}}W_{\text{SES}}| &\leq \xi, \\
|W_{\text{BEST}} - a_{\text{BEST,MFR}}W_{\text{MFR}}| &\leq \xi, \\
|W_{\text{BEST}} - a_{\text{BEST,DIST}}W_{\text{DIST}}| &\leq \xi, \\
|W_{\text{BEST}} - a_{\text{BEST,AS}}W_{\text{AS}}| &\leq \xi, \\
|W_{\text{BEST}} - a_{\text{BEST,ADS}}W_{\text{ADS}}| &\leq \xi, \\
|W_{\text{TSR}} - a_{\text{WORST,TSR}}W_{\text{WORST}}| &\leq \xi, \\
|W_{\text{SES}} - a_{\text{WORST,SES}}W_{\text{WORST}}| &\leq \xi, \\
|W_{\text{MFR}} - a_{\text{WORST,MFR}}W_{\text{WORST}}| &\leq \xi,
\end{aligned}$$

$$\begin{aligned} &|W_{\text{DIST}} - a_{\text{WORST,DIST}}W_{\text{WORST}}| \leq \xi, \\ &|W_{\text{AS}} - a_{\text{WORST,AS}}W_{\text{WORST}}| \leq \xi, \\ &|W_{\text{ADS}} - a_{\text{WORST,ADS}}W_{\text{WORST}}| \leq \xi, \\ &\sum_{i \in C} W_i = 1, \\ &W_{\text{TSR}}, W_{\text{SES}}, W_{\text{MFR}}, W_{\text{DIST}}, W_{\text{AS}} \text{ and } W_{\text{ADS}} \geq 0. \end{aligned}$$

Solving this optimization problem yields individual weights of criteria W_{TSR} , W_{SES} , W_{MFR} , W_{DIST} , W_{AS} and W_{ADS} .

4.2. Calculation of scores of alternatives

The member of the matrix C_n is given by

$$cs_{n_{ij}} = \begin{cases} \text{round}(|\text{score}_{n_i} - \text{score}_{n_j}| \times 8 + 1) & \text{if } \text{score}_{n_i} \geq \text{score}_{n_j}, \\ 1 \div \text{round}(|\text{score}_{n_i} - \text{score}_{n_j}| \times 8 + 1) & \text{otherwise.} \end{cases} \tag{18}$$

With these values, one comparison matrix for each criterion is constructed to compare m number of schools. Each of the comparison matrices $C_1, C_2, \dots, C_n$ is $m \times m$ in dimension, where each member of the matrix C_k is a $cs_{k_{ij}}$ value. Applying AHP, as described in Refs. 6 and 7, on these matrices gives the scores of alternatives in all the criteria.

4.3. Calculation of final scores of alternatives

After obtaining the scores from AHP, their weighted sum is used to calculate the final value. For this purpose, the weights of criteria that are described in Sec. 4.1 and the scores of alternatives that are described in Sec. 4.2 are used.

5. Demonstration/Experiments

A demonstration of a user’s inputs on class, age, sex and address is shown in Table 7. The same demo crisp inputs shown in Ref. 1 are used here. The same schools are also chosen as described in Ref. 1.

Table 7. User details.

Field	Input
Class	7
Age	15
Sex	Female
Division	RAJSHAHI
District	NAWABGANJ
Thana	CHAPAI NAWABGANJ SADAR
Union/Ward	WARD NO-03

Table 8. Scores of alternative schools.

Criterion	Alternatives		
	A1	A2	A3
TSR	0.00	0.46	0.62
SES	0.98	1.00	1.00
MFR	0.51	0.36	0.23
AS	0.46	0.46	0.45
DIST	0.90	0.40	0.40
ADS	0.73	0.39	0.51

The scores of the alternative schools per criteria are shown in Table 8. The scores are calculated from the database using the procedure described in Sec. 3.3.

Now in order to calculate the criteria weights we use the BWM method. Let us assume that the user selected BEST and WORST criterion as DIST and ADS, respectively. The relative preference values are shown in Tables 9 and 10.

Using the relative preference values of criteria given by the user we calculate the criteria weights using the procedure described in Sec. 4.1. We have calculated the alternative weights from the alternative scores that are shown in Table 8 using the procedure described in Sec. 3.4. Both the criteria weights and alternative weights per criteria obtained are shown in Table 11.

Table 9. Comparison values of BEST criterion to all other criteria.

Criterion	BEST (DIST)
TSR	4
SES	2
MFR	5
AS	7
DIST	1
ADS	9

Table 10. Comparison values of all other criteria to WORST criterion.

Criterion	WORST (ADS)
TSR	5
SES	7
MFR	4
AS	2
DIST	9
ADS	1

Table 11. Weights of criteria after BWM and weights of alternatives after AHP.

Criterion	Weights (obtained from BWM)	Alternatives		
		A1	A2	A3
TSR	0.123331	0.094454	0.421948	0.483598
SES	0.246662	0.333333	0.333333	0.333333
MFR	0.098665	0.404703	0.323239	0.272058
DIST	0.421921	0.333333	0.333333	0.333333
AS	0.070475	0.713464	0.143268	0.143268
ADS	0.038947	0.542988	0.208938	0.248074
Total score		0.486498	0.253934	0.259568

The result shown in Table 11 has a consistency ratio of criteria weights $\xi = 0.07140208$ which is quite good as $\xi = 0.1$ is considered to be a good consistency ratio. This BWM–AHP fusion method shows that the first alternative (A1) is the most suitable school for the user.

6. Result Analysis and Discussion

In order to analyze the results of our proposed method (i.e. BWM+AHP) we have taken our previous Fuzzy AHP method with automatic pairwise comparison of alternatives as a reference for the analysis. We have used the same example that we used in our previous work¹ for analyzing the result of our proposed method. The order of preference of criteria in Ref. 1 is preserved here. But the linguistic inputs of pairwise comparison from Ref. 1 are not preserved. In order to demonstrate the preference of criteria for two different types of comparison, the user’s choice of criteria is sorted by the degree of importance (from most important to least important), which is shown in Table 12. Since Fuzzy AHP described in Ref. 1 had four degrees of relative importance in linguistic terms, and BWM applied in this paper makes the user assign a number between 0 and 9 in relative importance inputs, the criteria are assigned a number between 0 and 4 for AHP and a number between 0 and 9 for BWM.

For the sake of simplicity, in this example, each criterion has the same relative importance difference with its neighboring criteria on both directions, i.e. the

Table 12. Criteria and relative importance.

Rank of importance	Criterion	Relative importance for FAHP	Relative importance for BWM+AHP
1	DIST	4.0	9.0
2	SES	3.2	7.2
3	TSR	2.4	5.4
4	MFR	1.6	3.6
5	AS	0.8	1.8
6	ADS	0.0	0.0

Table 13. Fuzzy linguistic inputs for pairwise comparison.

Absolutely important	Strongly important	Fairly important	Weakly important	Criterion	Equally important	Criterion	Weakly important	Fairly important	Strongly important	Absolutely important
(9,9,9)	(6,7,8)	(4,5,6)	(2,3,4)	TSR	(1,1,1)	SES	(2,3,4) ✓	(4,5,6)	(6,7,8)	(9,9,9)
			✓	TSR		MFR				
		✓		TSR		AS				
				TSR		DIST		✓		
		✓		TSR		ADS				
		✓		SES		MFR				
		✓		SES		AS				
				SES		DIST	✓			
	✓			SES		ADS				
			✓	MFR		AS			✓	
				MFR		DIST				
		✓		MFR		ADS				
				AS		DIST			✓	
			✓	AS		ADS				
✓				DIST		ADS				

Table 14. Example comparison values of BEST criterion to all other criteria.

Criterion	BEST (DIST)
TSR	4
SES	2
MFR	5
AS	7
DIST	1
ADS	9

Table 15. Example comparison values of all other criteria to WORST criterion.

Criterion	WORST (ADS)
TSR	5
SES	7
MFR	4
AS	2
DIST	9
ADS	1

criterion that is i th ($1 \leq i \leq 6$) by importance has the same difference of relative importance with both $(i + 1)$ th and $(i - 1)$ th criteria. In the table, higher relative importance difference means more importance. The differences of these relative importance values for FAHP are rounded to the nearest integer and used to emulate fuzzy inputs in linguistic terms, as shown in Table 13. For example, the difference of the relative importance values between SES and AS for FAHP is $(3.2 - 0.8) = 2.4$, which is rounded to 2.0 that refers to the second degree (i.e. Fairly Important) of linguistic value of relative importance. That is why we put tick mark on the Fairly Important column of Table 13 while comparing SES to AS.

Similarly we have calculated the comparison values of BEST to all others and all others to WORST and represented them in Tables 14 and 15 for our proposed method (i.e. BWM+AHP). For example, the difference of the relative importance values between BEST (DIST) and TSR for our proposed method (i.e. BWM+AHP) is $(9.0 - 5.4) = 3.6$ which is rounded to 4.0 referring to the relative importance value of BEST (DIST) over TSR.

Table 13 is then used as the input for FAHP and Tables 14 and 15 are used as the inputs for our proposed method (BWM+AHP). We have used the same three schools that we have used in our previous work.¹ The results of the alternative schools are shown in Fig. 1.

From Fig. 1 we can see that the choice of school has not changed from Ref. 1. Both methods show the same choices of schools: first A1, then A3 and finally A2. This

Fig. 1. BWM+AHP versus FAHP alternative weights comparison chart.

shows that both methods are capable of showing a personalized output as per user's choice and scores are quite similar to each other. However, since BWM+AHP requires less number of comparisons than what FAHP does, BWM+AHP is more suitable for the case and efficiency.

7. Conclusion

Selecting a secondary school in a developing country like Bangladesh can have a huge impact on student's future. In our previous paper¹ we showed how we can use Fuzzy AHP to take user's inputs for pairwise comparison of all pairs of criteria in linguistic terms. In this paper, instead of all possible pairs of criteria and linguistic terms, the user selects the BEST and the WORST criteria and assigns a number between 1 and 9 while comparing the other criteria with those two criteria, separately. Then instead of Fuzzy AHP, we used Crisp AHP to calculate the scores of alternatives in those criteria. This new method can be used to solve any type of MCDM problems, given a dataset.

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