Electric-Field Control of Magnon Gaps in a Ferromagnet using a Spatially-Periodic Electric Field

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The frequencies and linewidths of spin waves in one-dimensional (1D) and two-dimensional (2D) periodic superlattices of magnetic materials are found, using the Landau–Lifshitz–Gilbert equations. The form of the exchange field from a surface-torque-free boundary between magnetic materials is derived, and magnetic-material combinations are identified which produce gaps in the magnonic spectrum across the entire superlattice Brillouin zone for hexagonal and square-symmetry superlattices. The magnon gaps and spin-wave dispersion properties of a uniform magnetic material under the influence of a periodic electric field are presented. Such results suggest the utility of magnetic insulators for electric-field control of spin-wave propagation properties.

Keywords: Magnonics; voltage control; Dzyaloshinskii–Moriya interaction.

1. Introduction

Advancements in the control of spin-wave propagation and dynamics¹ have led to the demonstration of magnonic bose condensation² and coupling of electronic spin currents to spin waves in hybrid systems.³ Such effects, along with theoretical proposals^{4,5} and experimental demonstrations⁶ of electrical control of spin-wave propagation in insulators, may provide the foundation for an information-processing technology based on spin waves.^{7,8} Any such technology would benefit from magnetic materials with designed spin-wave dispersion relations,

group velocities, and linewidths. A common method of designing such features is the fabrication of a superlattice of different constituent materials, used to design electronic band structures in semiconductor superlattices,⁹ and photonic band structures^{10–12} in dielectric superlattices.

Passive control of spin current propagation by opening gaps in the spin-wave dispersion, such as with a magnonic crystal,^{13–22} has been achieved, especially for quasi-one-dimensional structures. Active manipulation of the spin currents, especially within insulating materials such as ferrites, has

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proved much more difficult, and efforts have focused on modifications from internal currents,^{17,23} external magnetic fields,^{19,20,24} or spin currents.²⁵ A simple active method of controlling the propagation of spin currents, especially without substantial intrinsic dissipation, would have broad utility in the fundamental studies of spin-wave propagation, such as by allowing time-dependent studies of spin current propagation and dissipation that would illuminate many of the fundamental processes involved in spin current dynamics. Recent predictions⁴ and demonstrations⁶ of a modified spin-wave dispersion in yttrium iron garnet (YIG) by an electric field due to the spin-orbit interaction (via the Dzyaloshinskii-Moriya interaction, or DMI) produced phase shifts in propagating spin waves, but would not open or close a spin-wave gap.

Here we focus on the effect of one-dimensional (1D) and two-dimensional (2D) superlattices of magnetic materials on the magnonic frequencies and linewidths, obtained from a reciprocal-space solution to the Landau-Lifshitz-Gilbert (LLG) equation.²⁶ We consider two significant examples to illustrate the range of potential scenarios that might be possible for spin wave control. First, we consider infinite cylinders of one magnetic material that are embedded in a second magnetic material, in a periodic arrangement corresponding to a 2D square lattice or hexagonal lattice. Large gaps within the spin-wave spectrum are obtained when the exchange constants and saturation magnetization of the two materials differ greatly; thus the gaps are considerably larger for cylinders of iron embedded within YIG than for iron embedded within nickel. For iron embedded in YIG, we demonstrate the existence of a gap throughout the superlattice Brillouin zone in the magnon spectrum for both square and hexagonal symmetry magnonic crystals. In photonic crystals, such a feature forms an essential element of photonic band gap materials,¹⁰⁻¹² and permits the control of spontaneous emission of emitters embedded within the photonic crystal; here similarly the spontaneous emission of magnons from a source such as a spin-torque nano-oscillator could be suppressed by embedding this spin-wave emitter in a fully-gapped magnonic crystal. For the second example, we consider a uniform magnetic material (YIG) under the influence of a periodic electric field. The presence of this periodic electric field opens a substantial and tun-

able magnon gap within the dispersion and thus permits on/off voltage-based control of spin-wave propagation. The spatially-varying electric fields can be configured to generate an artificial 1D magnonic crystal structure with sufficient influence on the spin waves to open a gap orders of magnitude larger than the linewidth of the spin waves, corresponding to quality factors in excess of 100. Advancements in fundamental studies of spin-wave dynamics are enabled by this theory and these predictions, including electrically-modulated spin-wave pump-probe techniques. In addition, this work may improve spin-wave interconnects and other spin-wave devices.^{27,28}

In both of the examples considered here a careful consideration of the boundary conditions between the two effective magnetic materials, whether they consist of different materials or of the same material with a differing electric field, is required to obtain the proper spin-wave properties. For a magnonic superlattice the exchange field that enters into the LLG equations is discontinuous at the boundary, and that discontinuity strongly influences the spin wave dynamics. Two distinct forms for this exchange field in the presence of inhomogeneous material parameters (saturation magnetization and exchange constant) have been described in the literature,^{14,29-34} although to our knowledge it has not been pointed out that the differences in the solutions of the LLG equation obtained from the two effective fields produce large quantitative differences in the spin-wave dispersion and linewidths, nor has a derivation been presented of the correct form. In Sec. 2 we present an explicit derivation of the correct form of the exchange field, followed by spin wave frequencies and linewidths for several magnetic material combinations in Sec. 3. In Sec. 4 we show that solutions to the LLG equations for the incorrect forms of the exchange field differ greatly from those for the correct form. We note that for both the correct and the incorrect forms of the exchange field the presence of the dipolar field generated by the periodic magnetic material breaks the anticipated point-group symmetry of the 2D lattice. The extent of this symmetry breaking, however, differs greatly for the two forms of the boundary conditions. In Sec. 5 we present solutions for a magnetic material, YIG, with a periodically-alternating electric field and describe the spin-wave dispersion relations for the system.

2. LLG Formalism for a Quasi Two Dimensional Magnonic Crystal

We consider a magnonic crystal composed of an array of infinitely long cylinders of ferromagnetic material A embedded in a second ferromagnetic material B in a square or hexagonal lattice; the structures are shown in Fig. 1 and have lattice constant a and cylinder radius R_{cyl} . The cylinders are aligned parallel to a static external magnetic field $\mathbf{H}_0 = H_0 \hat{z}$, and the magnetization of both materials is assumed to be parallel to $\mathbf{H}_0$. The equation of motion for this system is the LLG equation²⁶:

$$\frac{\partial}{\partial t} \mathbf{M}(\mathbf{r}, t) = \gamma \mu_0 \mathbf{M}(\mathbf{r}, t) \times \mathbf{H}_{\text{eff}}(\mathbf{r}, t) + \frac{\alpha(\mathbf{r})}{M_s(\mathbf{r})} \mathbf{M}(\mathbf{r}, t) \times \frac{\partial}{\partial t} \mathbf{M}(\mathbf{r}, t). \quad (1)$$

Here, γ is the gyromagnetic ratio, $M_s(\mathbf{r})$ is the spontaneous magnetization, $\alpha(\mathbf{r})$ is the Gilbert damping parameter, and $\mathbf{r}$ is the three-dimensional (3D) position vector. The effective magnetic field

$$\mathbf{H}_{\text{eff}}(\mathbf{r}, t) = \mathbf{H}_0 + \mathbf{h}(\mathbf{r}, t) + \mathbf{H}_{\text{ex}}(\mathbf{r}, t) \quad (2)$$

acting on the magnetization $\mathbf{M}(\mathbf{r}, t)$ consists of three terms: the external field $\mathbf{H}_0$, the dynamic dipolar field $\mathbf{h}(\mathbf{r}, t)$, and the exchange field $\mathbf{H}_{\text{ex}}(\mathbf{r}, t)$.

2.1. Derivation of the effective magnetic field

We wish to derive the correct form of $H_{\text{eff}}(\mathbf{r}, t)$ to use in Eq. (1) for our magnonic crystal. As shown by Gilbert,²⁶ the exchange field can be obtained by

Fig. 1. Physical structure of the magnonic crystals. The ferromagnetic material B is the host for infinitely long cylinders of a different ferromagnetic material A arranged in either a square (left) or hexagonal (right) lattice. The lattice constant of the lattice is a and the cylinder radius is R_{cyl} .

taking the functional derivative of the exchange energy. For a homogeneous material, the exchange energy is³⁵

$$U_{\text{ex}}[\mathbf{M}(\mathbf{r})] = \frac{A}{M_s^2} \int [(\nabla m_x(\mathbf{r}))^2 + (\nabla m_y(\mathbf{r}))^2 + (\nabla m_z(\mathbf{r}))^2] d\mathbf{r}, \quad (3)$$

where A is the exchange stiffness constant. This yields the following exchange field:

$$\mathbf{H}_{\text{ex}}(\mathbf{r}) = -\frac{1}{\mu_0} \frac{\delta U_{\text{ex}}[\mathbf{M}(\mathbf{r})]}{\delta \mathbf{M}(\mathbf{r})} = \frac{2A}{\mu_0 M_s^2} \nabla^2 \mathbf{M}(\mathbf{r}). \quad (4)$$

For the inhomogeneous crystal considered here the values of the exchange constant and the spontaneous magnetization will differ for the two ferromagnets, so A and M_s become spatially dependent quantities:

$$A(\mathbf{r}) = A_B + \Theta(\mathbf{r})(A_A - A_B), \quad (5)$$

$$M_s(\mathbf{r}) = M_{s_B} + \Theta(\mathbf{r})(M_{s_A} - M_{s_B}),$$

where $\Theta(\mathbf{r}) = 1$ in material A and $\Theta(\mathbf{r}) = 0$ in material B. The exchange energy for this inhomogeneous situation is

$$U_{\text{ex}}[\mathbf{M}(\mathbf{r})] = \int A(\mathbf{r}) \left\{ \left[\nabla \left(\frac{m_x(\mathbf{r})}{M_s(\mathbf{r})} \right) \right]^2 + \left[\nabla \left(\frac{m_y(\mathbf{r})}{M_s(\mathbf{r})} \right) \right]^2 + \left[\nabla \left(\frac{m_z(\mathbf{r})}{M_s(\mathbf{r})} \right) \right]^2 \right\} d\mathbf{r}. \quad (6)$$

By approximating the energy with U_{ex} we have neglected nonexchange terms that would give rise to a surface torque (such as terms in the energy associated with surface-induced magnetic anisotropy).

The total magnetization will consist of both a time-dependent term and a time-independent term: $\mathbf{M}(\mathbf{r}, t) = M_s(\mathbf{r}) \hat{z} + \mathbf{m}(\mathbf{r}, t)$. Using the linear magnon approximation we assume that the time-dependent magnetization is small compared to $M_s(\mathbf{r})$, and therefore we only keep terms up to first order in $\mathbf{m}(\mathbf{r}, t)$. With these assumptions the inhomogeneous exchange field derived from Eq. (6) is

$$\begin{aligned} \mathbf{H}_{\text{ex}}(\mathbf{r}, t) = & \frac{2}{\mu_0} \left(\nabla \cdot \frac{A(\mathbf{r})}{M_s^2(\mathbf{r})} \nabla \right) \mathbf{M}(\mathbf{r}, t) \\ & + \frac{2\mathbf{M}(\mathbf{r}, t)}{\mu_0 M_s(\mathbf{r})} (\nabla \cdot A(\mathbf{r}) \nabla) \frac{1}{M_s(\mathbf{r})} \\ & - \frac{2\mathbf{m}(\mathbf{r}, t)}{\mu_0 M_s^2(\mathbf{r})} \cdot \left[(\nabla \cdot A(\mathbf{r}) \nabla) \frac{\mathbf{m}(\mathbf{r}, t)}{M_s(\mathbf{r})} \right] \hat{z}. \end{aligned} \quad (7)$$

The exchange field enters the LLG equation only as a cross-product with the magnetization $\mathbf{M}(\mathbf{r}, t)$. The second term is parallel to $\mathbf{M}(\mathbf{r}, t)$ and thus will not contribute to Eq. (1). The third term of Eq. (7), which is proportional to $\mathbf{m}(\mathbf{r}, t)$ and parallel to $\mathbf{M}_0 = M_s(\mathbf{r})\hat{z}$, will only produce terms of second order in $\mathbf{m}(\mathbf{r}, t)$ in Eq. (1) and can safely be dropped. Therefore, we can approximate

$$\mathbf{H}_{\text{ex}}(\mathbf{r}, t) = \frac{2}{\mu_0} \left(\nabla \cdot \frac{A(\mathbf{r})}{M_s^2(\mathbf{r})} \nabla \right) \mathbf{M}(\mathbf{r}, t), \quad (8)$$

which produces an LLG equation from Eq. (1) that is correct to first order in $\mathbf{m}(\mathbf{r}, t)$.

We now have the following equation for the effective field:

$$\begin{aligned} \mathbf{H}_{\text{eff}}(\mathbf{r}, t) = & H_0 \hat{z} + \mathbf{h}(\mathbf{r}, t) \\ & + \frac{2}{\mu_0} \left(\nabla \cdot \frac{A(\mathbf{r})}{M_s^2(\mathbf{r})} \nabla \right) \mathbf{M}(\mathbf{r}, t). \end{aligned} \quad (9)$$

This form is a generalization of the boundary condition obtained at the interface between a ferromagnet and vacuum³⁶ in the absence of any surface torque, later derived for the boundary condition between dissimilar magnetic materials.^{37,38} It is also the form used in Refs. 29–32.

2.2. Plane-wave solution to LLG equation for quasi two dimensional magnonic crystal

When solving for magnons of a specific frequency ω , we write $\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r}) \exp(-i\omega t)$ and the dipolar field, $\mathbf{h}(\mathbf{r}, t) = -\nabla \Psi(\mathbf{r}) \exp(-i\omega t)$, with $\Psi(\mathbf{r})$ the magnetostatic potential. With the form of the effective field in Eq. (9), the LLG equation (Eq. (1)) can be written

$$\begin{aligned} i\Omega m_x(\mathbf{R}) + M_s(\mathbf{R}) \nabla \cdot (Q(\mathbf{R}) \nabla m_y(\mathbf{R})) \\ - m_y(\mathbf{R}) \nabla \cdot (Q(\mathbf{R}) \nabla M_s(\mathbf{R})) - m_y(\mathbf{R}) \\ - \frac{M_s(\mathbf{R})}{H_0} \frac{\partial \Psi(\mathbf{R})}{\partial y} + i\Omega \alpha(\mathbf{R}) m_y(\mathbf{R}) = 0, \end{aligned} \quad (10)$$

$$\begin{aligned} i\Omega m_y(\mathbf{R}) - M_s(\mathbf{R}) \nabla \cdot (Q(\mathbf{R}) \nabla m_x(\mathbf{R})) \\ + m_x(\mathbf{R}) \nabla \cdot (Q(\mathbf{R}) \nabla M_s(\mathbf{R})) + m_x(\mathbf{R}) \\ + \frac{M_s(\mathbf{R})}{H_0} \frac{\partial \Psi(\mathbf{R})}{\partial x} - i\Omega \alpha(\mathbf{R}) m_x(\mathbf{R}) = 0, \end{aligned} \quad (11)$$

where $\Omega = \omega/(|\gamma|\mu_0 H_0)$ and $Q(\mathbf{R}) = 2A(\mathbf{R})/(\mu_0 H_0 M_s^2(\mathbf{R}))$. Additionally, since there is no z dependence in the above equations, the 3D position vector $\mathbf{r}$ has been replaced with the 2D position vector, $\mathbf{R} = (x, y)$.

This system of equations can be efficiently solved with a plane-wave method.^{14,29–33} We take advantage of the crystal's periodicity and use Bloch's theorem to write the magnetization and magnetostatic potential as an expansion of plane waves:

$$\mathbf{m}(\mathbf{R}) = e^{i\mathbf{k} \cdot \mathbf{R}} \sum_i \mathbf{m}_{\mathbf{k}}(\mathbf{G}_i) e^{i\mathbf{G}_i \cdot \mathbf{R}}, \quad (12)$$

$$\Psi(\mathbf{R}) = e^{i\mathbf{k} \cdot \mathbf{R}} \sum_i \Psi_{\mathbf{k}}(\mathbf{G}_i) e^{i\mathbf{G}_i \cdot \mathbf{R}}. \quad (13)$$

Here, $\mathbf{G}_i$ represents a 2D reciprocal lattice vector of the crystal and $\mathbf{k}$ is a wave vector in the first Brillouin zone. The magnetostatic potential can be rewritten in terms of the magnetization by using one of Maxwell's equations:

$$\nabla \cdot (\mathbf{h}(\mathbf{R}) + \mathbf{m}(\mathbf{R})) = 0. \quad (14)$$

Replacing $\mathbf{h}(\mathbf{R})$ with $-\nabla \Psi(\mathbf{R})$, substituting in Eqs. (12) and (13), and solving for the potential yield

$$\Psi(\mathbf{G}) = -i \frac{m_{x,\mathbf{k}}(\mathbf{G})(\mathbf{G}_x + k_x) + m_{y,\mathbf{k}}(\mathbf{G}_y + k_y)}{(\mathbf{G} + \mathbf{k})^2}. \quad (15)$$

Next, we need to be able to write the material properties $M_s(\mathbf{R})$, $Q(\mathbf{R})$, and $\alpha(\mathbf{R})$ in reciprocal space. Since these have the same periodicity as the crystal lattice, this can be done with a Fourier series expansion:

$$\begin{aligned} M_s(\mathbf{R}) &= \sum_i M_s(\mathbf{G}_i) e^{i\mathbf{G}_i \cdot \mathbf{R}}, \\ Q(\mathbf{R}) &= \sum_i Q(\mathbf{G}_i) e^{i\mathbf{G}_i \cdot \mathbf{R}}, \\ \alpha(\mathbf{R}) &= \sum_i \alpha(\mathbf{G}_i) e^{i\mathbf{G}_i \cdot \mathbf{R}}. \end{aligned} \quad (16)$$

The Fourier coefficients are obtained by an inverse Fourier transform:

$$M_s(\mathbf{G}) = \frac{1}{S} \int_S M_s(\mathbf{R}) e^{-i\mathbf{G} \cdot \mathbf{R}} d^2 \mathbf{R}, \quad (17)$$

where S is the area of the 2D unit cell.

Performing the integration for $\mathbf{G} = 0$ gives the average

$$M_s(\mathbf{G} = 0) = M_{s_A} f + M_{s_B} (1 - f), \quad (18)$$

where f is the fractional space occupied by a cylinder in the unit cell. For $\mathbf{G} \neq 0$, we have

$$M_s(\mathbf{G} \neq 0) = (M_{s_A} - M_{s_B})2f \frac{J_1(|\mathbf{G}|R_{\text{cyl}})}{|\mathbf{G}|R_{\text{cyl}}}. \quad (19)$$

Here J_1 is a Bessel function of the first kind, and R_{cyl} is the radius of the cylinders. The following infinite system of equations in reciprocal space is obtained by substituting Eqs. (12)–(16) in Eqs. (10) and (11):

$$\begin{aligned} i\Omega \sum_j (m_{x,\mathbf{k}}(\mathbf{G}_i)\delta_{ij} + \alpha(\mathbf{G}_i - \mathbf{G}_j)m_{y,\mathbf{k}}(\mathbf{G}_j)) \\ = \sum_j \left\{ M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{x,j} + k_x)(G_{y,j} + k_y)}{H_0(\mathbf{G}_j + \mathbf{k})^2} \right. \\ \times m_{x,\mathbf{k}}(\mathbf{G}_j) + \left[\delta_{ij} + M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{y,j} + k_y)^2}{H_0(\mathbf{G}_j + \mathbf{k})^2} \right. \\ + \sum_l (M_s(\mathbf{G}_i - \mathbf{G}_l)Q(\mathbf{G}_l - \mathbf{G}_j) \\ \times ((\mathbf{k} + \mathbf{G}_j) \cdot (\mathbf{k} + \mathbf{G}_l) \\ - (\mathbf{G}_i - \mathbf{G}_j) \cdot (\mathbf{G}_i - \mathbf{G}_l))) \left. \right] m_{y,\mathbf{k}}(\mathbf{G}_j) \left. \right\}, \quad (20) \end{aligned}$$

$$\begin{aligned} i\Omega \sum_j (m_{y,\mathbf{k}}(\mathbf{G}_i)\delta_{ij} - \alpha(\mathbf{G}_i - \mathbf{G}_j)m_{x,\mathbf{k}}(\mathbf{G}_j)) \\ = - \sum_j \left\{ M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{x,j} + k_x)(G_{y,j} + k_y)}{H_0(\mathbf{G}_j + \mathbf{k})^2} \right. \\ \times m_{y,\mathbf{k}}(\mathbf{G}_j) + \left[\delta_{ij} + M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{y,j} + k_y)^2}{H_0(\mathbf{G}_j + \mathbf{k})^2} \right. \\ + \sum_l (M_s(\mathbf{G}_i - \mathbf{G}_l)Q(\mathbf{G}_l - \mathbf{G}_j) \\ \times ((\mathbf{k} + \mathbf{G}_j) \cdot (\mathbf{k} + \mathbf{G}_l) \\ - (\mathbf{G}_i - \mathbf{G}_j) \cdot (\mathbf{G}_i - \mathbf{G}_l))) \left. \right] m_{x,\mathbf{k}}(\mathbf{G}_j) \left. \right\}. \quad (21) \end{aligned}$$

We solve this by limiting the number of reciprocal lattice vectors in the sum and expressing it as a matrix equation:

$$i\Omega \begin{bmatrix} \delta_{ij} & \alpha(\mathbf{G}_i - \mathbf{G}_j) \\ \alpha(\mathbf{G}_i - \mathbf{G}_j) & \delta_{ij} \end{bmatrix} \begin{bmatrix} m_{x,\mathbf{k}}(\mathbf{G}_1) \\ \vdots \\ m_{x,\mathbf{k}}(\mathbf{G}_N) \\ m_{y,\mathbf{k}}(\mathbf{G}_1) \\ \vdots \\ m_{y,\mathbf{k}}(\mathbf{G}_N) \end{bmatrix}$$

$$= \begin{bmatrix} B_{ij}^{xx} & B_{ij}^{xy} \\ B_{ij}^{yx} & B_{ij}^{yy} \end{bmatrix} \begin{bmatrix} m_{x,\mathbf{k}}(\mathbf{G}_1) \\ \vdots \\ m_{x,\mathbf{k}}(\mathbf{G}_N) \\ m_{y,\mathbf{k}}(\mathbf{G}_1) \\ \vdots \\ m_{y,\mathbf{k}}(\mathbf{G}_N) \end{bmatrix}, \quad (22)$$

$$B_{ij}^{xx} = -B_{ij}^{yy} = M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{x,j} + k_x)(G_{y,j} + k_y)}{H_0(\mathbf{G}_j + \mathbf{k})^2}, \quad (23)$$

$$\begin{aligned} B_{ij}^{xy} = \delta_{ij} + M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{y,j} + k_y)^2}{H_0(\mathbf{G}_j + \mathbf{k})^2} \\ + \sum_l M_s(\mathbf{G}_i - \mathbf{G}_l)Q(\mathbf{G}_l - \mathbf{G}_j)[(\mathbf{k} + \mathbf{G}_j) \\ \cdot (\mathbf{k} + \mathbf{G}_l) - (\mathbf{G}_i - \mathbf{G}_j) \cdot (\mathbf{G}_i - \mathbf{G}_l)], \quad (24) \end{aligned}$$

$$\begin{aligned} B_{ij}^{yx} = \delta_{ij} + M_s(\mathbf{G}_i - \mathbf{G}_j) \frac{(G_{x,j} + k_x)^2}{H_0(\mathbf{G}_j + \mathbf{k})^2} \\ + \sum_l M_s(\mathbf{G}_i - \mathbf{G}_l)Q(\mathbf{G}_l - \mathbf{G}_j)[(\mathbf{k} + \mathbf{G}_j) \\ \cdot (\mathbf{k} + \mathbf{G}_l) - (\mathbf{G}_i - \mathbf{G}_j) \cdot (\mathbf{G}_i - \mathbf{G}_l)]. \quad (25) \end{aligned}$$

The LLG equation is now reduced to finding the eigenvalues and eigenvectors for the above equation. We increase the number of Fourier components in the calculation until the result converges.

3. Results

From Eq. (22), we calculate the complex eigenvalues Ω corresponding to the frequencies of magnons in 2D magnetic superlattices of Fe, Co, Ni, and YIG. The real part of $\Omega_n(\mathbf{k})$ is the magnon frequency of branch n for the wave vector $\mathbf{k}$ and the imaginary part is the inverse spin-wave lifetime. To focus on the dependence of these properties on magnetic material combinations, we consider superlattices with a lattice constant $a = 10$ nm, an external field $\mu_0 H_0 = 0.1$ T, and a filling fraction $f = 0.5$. The material properties, M_s , A , and α , are listed in Table 1, and are obtained from Refs. 39–42.

Figure 2 shows the empty-lattice band structures obtained from the LLG equation for homogeneous crystals of Fe, Co, Ni, and YIG in a square lattice.

Table 1. Properties of the different materials considered for the magnonic crystals.

	M_s (A/m)	A (pJ/m)	α
Fe	$1.711 \cdot 10^6$	8.3	0.0019
Co	$1.401 \cdot 10^6$	10.3	0.011
Ni	$0.485 \cdot 10^6$	3.4	0.064
YIG	$0.14 \cdot 10^6$	4.15	0.0014

Fig. 2. Empty square lattice band structure obtained from the LLG equation for a homogeneous crystal of Fe (upper left), Co (upper right), Ni (lower left), and YIG (lower right) with lattice constant $a = 10$ nm. Frequencies are in units of THz.

Figure 3 shows the same for a hexagonal lattice. As the empty-lattice features are governed by the lattice symmetry and the material's spin wave velocity, these plots depend on material only in setting the frequency scale of the features, which is demonstrated in both Figs. 2 and 3.

Figure 4 shows spin-wave dispersions when Fe is embedded in a square lattice within a host of Co, Ni

Fig. 3. Empty hexagonal lattice band structure obtained from the LLG equation for a homogeneous crystal of Fe (upper left), Co (upper right), Ni (lower left), and YIG (lower right) with lattice constant $a = 10$ nm. Frequencies are in units of THz. Fig. 4. Magnonic band structures for a square lattice magnonic crystal with lattice spacing $a = 10$ nm and filling fraction $f = 0.5$. On the left is Fe cylinders embedded in Co (top), Ni (middle), and YIG (bottom). The right is for an Fe host with Co (top), Ni (middle), and YIG (bottom) cylinders. Frequencies are in units of THz.

or YIG, and also for Co, Ni or YIG cylinders embedded in Fe. The change in band structure from the homogeneous case (upper left of Fig. 2) is more substantial when there is a greater difference in the

Fig. 5. Magnonic band structures for a hexagonal lattice magnonic crystal with lattice spacing $a = 10$ nm and filling fraction $f = 0.5$. On the left is Fe cylinders embedded in Co (top), Ni (middle), and YIG (bottom). The right is for an Fe host with Co (top), Ni (middle), and YIG (bottom) cylinders. Frequencies are in units of THz.

Fig. 6. The lowest nine spin wave frequencies (in THz) for a square lattice magnonic crystal composed of Fe cylinders in Ni with a filling fraction of $f = 0.5$, and a lattice constant of $a = 10$ nm. These results were obtained using the exchange field in Eq. (8). The square figures show the entire Brillouin zone of the lattice.

spontaneous magnetization between the two materials. For example, the magnetic properties of Fe and Co are fairly similar, and so for a crystal composed of these materials, the band structure differs little from the homogeneous case, with only some small splittings of the spin-wave dispersion curves occurring. However, the magnetization of YIG differs from that of Fe by more than a factor of ten, so for crystals of YIG embedded in Fe the magnonic

modes are almost completely different from the homogeneous crystal. Furthermore, wider band gaps are opened in the spin-wave dispersion in these structures if the magnetization is larger in the cylinders than it is in the host. For a crystal with Fe cylinders embedded in YIG in a square lattice (bottom left of Fig. 4), there are four gaps occurring within the lowest nine spin-wave modes, whereas if instead YIG cylinders are embedded in Fe (bottom

Fig. 7. The spin wave relaxation rate (in units of GHz) corresponding to the lowest nine spin wave modes from Fig. 6. The square figures show the entire Brillouin zone of the lattice.

Fig. 8. The lowest nine spin-wave frequencies (in THz) for a hexagonal lattice magnonic crystal composed of Fe cylinders in Ni with a filling fraction of $f = 0.5$, and a lattice constant of $a = 10$ nm. These results were obtained using the exchange field in Eq. (8). The figures include the entire hexagonal Brillouin zone of the lattice.

right of Fig. 4), there is only one small gap between the first and second spin-wave modes. Similar influence on cylinder composition was reported in Ref. 29 and ascribed to both the larger exchange constant and larger magnetization of Fe versus YIG; we find it is due to the larger magnetization, and the effect of the larger exchange constant is negligible.

Similar results are apparent in Fig. 5 for hexagonal arrangements of cylinders embedded in a magnetic host. The gaps in spin-wave dispersion are greater when the cylinders have larger magnetization; for Fe cylinders embedded in YIG the spin-wave dispersion curves are almost flat (bottom left of Fig. 5), whereas there are no spin-wave gaps

apparent for YIG embedded in Fe (bottom right of Fig. 5).

We now show the detailed dispersion curves (Fig. 6) and spin-wave relaxation rates (Fig. 7) for a square lattice of Fe cylinders in Ni. Plotted are the lowest nine spin-wave modes ($\text{Re}(\Omega)$) in the entire first Brillouin zone as well as the corresponding inverse spin wave lifetimes ($\text{Im}(\Omega)$). Quality factors for these modes, corresponding to the ratio of the relaxation rate to the mode frequency, can exceed 100 for such spin waves, especially for the lowest-frequency modes. We note that these plots exhibit the correct square symmetry of the lattice. The linewidths of the lowest-frequency mode are found

Fig. 9. The spin wave relaxation rates (in units of GHz) corresponding to the lowest nine spin wave modes from Fig. 8. The figures include the entire Brillouin zone of the hexagonal lattice.

to be smallest at the zone boundary, although this result does not extend to higher-frequency spin waves of the lattice.

The detailed dispersion curves (Fig. 8) and spin-wave relaxation rates (Fig. 9) for a hexagonal lattice of Fe cylinders in Ni show similar features, although here the linewidths for the lowest spin-wave modes are small at the origin as well as at the zone boundary. The overall linewidths, however, tend to be larger than those in the square lattice.

4. Comparison with Alternate Effective Field

Some calculations of magnonic crystals dispersion curves used a different exchange field^{14,33} than the one derived in Sec. 2. The alternate form,

$$\mathbf{H}_{\text{ex}}(\mathbf{r}, t) = \frac{2}{\mu_0 M_s(\mathbf{r})} \left(\nabla \cdot \frac{A(\mathbf{r})}{M_s(\mathbf{r})} \nabla \right) \mathbf{M}(\mathbf{r}, t) \quad (26)$$

differs by the positioning of one factor of $M_s(\mathbf{r})^{-1}$ outside the gradient operators. A comparison of the band structures obtained for the two different exchange fields is shown in Fig. 10. An examination of the band structure for a homogeneous material composed of Fe or Ni (Fig. 2) indicates that the results for the correctly derived exchanged field (Eq. (8)) produce a band structure that is appreciably different from the homogeneous case, whereas the band structures produced by the alternate exchange field (Eq. (26)) are very similar to the homogeneous crystal.

In Fig. 11, we show the lowest spin-wave mode and corresponding relaxation rate obtained for Fe cylinders in Ni when using Eq. (26) as the exchange field. When looking at these contours, we might expect them to have the same symmetries as the real space lattice. For a square lattice, that would be symmetry under rotations of 90° and symmetry

Fig. 10. Magnonic band structure of Fe cylinders embedded in Ni calculated from the LLG equation using exchange field in Eq. (8) (left) and Eq. (26) (right). Frequencies are in units of THz.

Fig. 11. The lowest spin wave mode (in THz) and the corresponding relaxation rate (in units of GHz) for a square lattice magnonic crystal composed of Fe cylinders in Ni with a filling fraction of $f = 0.5$, and a lattice constant of $a = 10$ nm. These results were obtained using the exchange field in Eq. (26).

under reflections about either axis. However, the nature of the dipolar field breaks the point group symmetry. The size of the point group symmetry breaking from the dipolar field is very different for the two boundary conditions; the asymmetry in Fig. 11 is much larger than that in Figs. 7 and 9 (which use the correct boundary conditions).

5. Formation of Magnonic Band Gaps from Spatially-Periodic Electric Field

We consider a slab of YIG with magnetization in plane and a periodic electric field perpendicular to both the magnetization and the spin-wave propagation direction (Fig. 12). This system is described with the LLG equation, Eq. (1). For the above geometry the effective magnetic field $\mathbf{H}_{\text{eff}}(\mathbf{r}, t)$ of Eq. (2) acting on the magnetization consists of the following terms:

$$\mathbf{H}_{\text{eff}} = (H_0 - H_{\text{demag}})\hat{\mathbf{z}} + \mathbf{h} + \lambda_{\text{ex}}^2 \nabla^2 \mathbf{M} + \mathbf{H}_{\text{DM}}, \quad (27)$$

where λ_{ex} is the exchange length. H_0 is the magnitude of the external field (pointing along $\hat{\mathbf{z}}$), H_{demag} is the demagnetizing field,⁴³ $\mathbf{h}(\mathbf{r}, t)$ is the dynamic dipolar field obtained by satisfying Maxwell's

Fig. 12. Schematic diagram of the YIG slab with interdigitated electrodes to generate a periodically-varying electric field. The magnetization (M) is in-plane, and the size of the slab along the $\hat{y}$ and $\hat{z}$ directions is small enough that the transverse modes do not influence the spin-wave propagation.

equations, and $\lambda_{\text{ex}}^2 \nabla^2 \mathbf{M}$ is the effective exchange field obtained from the exchange energy.²⁶

The final term $\mathbf{H}_{\text{DM}}(\mathbf{r})$ is an effective magnetic field that appears as a result of the DMI in the presence of an electric field and the spin-orbit interaction. The form of the DMI field has previously been obtained for a uniform electric field applied perpendicularly to a YIG slab,⁵

$$\mathbf{H}_{\text{DM}}(\mathbf{r}) = \frac{2}{\mu_0} \mathbf{D} \times [(\hat{\mathbf{e}}_{ij} \cdot \nabla) \mathbf{M}(\mathbf{r})], \quad (28)$$

where

$$\mathbf{D} = \mu_0 \lambda_{\text{ex}}^2 \frac{e \mathbf{E} \times \hat{\mathbf{e}}_{ij}}{E_{\text{so}}} \quad (29)$$

is the DM vector. For a periodic electric field its form can be obtained as in Ref. 5, from the DMI energy, assuming a spatially dependent electric field:

$$\begin{aligned} \mu_0 \mathbf{H}_{\text{DM}}(\mathbf{r}) = & 2 \mathbf{D}(\mathbf{r}) \times [(\hat{\mathbf{e}}_{ij} \cdot \nabla) \mathbf{M}(\mathbf{r})] \\ & - \mathbf{M}(\mathbf{r}) \times [(\hat{\mathbf{e}}_{ij} \cdot \nabla) \mathbf{D}(\mathbf{r})]. \end{aligned} \quad (30)$$

We now simplify the matrix expressions from Sec. 2 for the 1D case to yield the following expressions:

$$\begin{bmatrix} \frac{i\omega}{|\gamma|\mu_0 H_0} - \mathbf{B}^{xx} & \frac{i\omega\alpha}{|\gamma|\mu_0 H_0} - \mathbf{B}^{xy} \\ -\frac{i\omega\alpha}{|\gamma|\mu_0 H_0} - \mathbf{B}^{yx} & \frac{i\omega}{|\gamma|\mu_0 H_0} - \mathbf{B}^{yy} \end{bmatrix} \begin{bmatrix} m_{x,k}(G_{-N}) \\ \vdots \\ m_{x,k}(G_N) \\ m_{y,k}(G_{-N}) \\ \vdots \\ m_{y,k}(G_N) \end{bmatrix} = 0, \quad (31)$$

where

$$\begin{aligned} B_{mn}^{xx} &= B_{mn}^{yy} = -i \frac{\mathbf{M}_0 \cdot \tilde{\mathbf{D}}(G_m - G_n)}{H_0} \\ &\quad \times (G_n + G_m + 2k), \\ B_{mn}^{xy} &= \left(1 + \frac{M_s}{H_0} \lambda_{\text{ex}}^2 (k + G_n)^2\right) \delta_{mn}, \\ B_{mn}^{yx} &= \left(-1 - \frac{M_s}{H_0} \lambda_{\text{ex}}^2 (k + G_n)^2 - \frac{M_s}{H_0}\right) \delta_{mn}. \end{aligned} \quad (32)$$

Here $G_m = 2\pi m/a$ is a reciprocal lattice vector of the system in the x -direction, $\tilde{\mathbf{D}}(G)$ is the Fourier transform of $\mathbf{D}(\mathbf{r})$, and n and m are integers.

The boundary conditions for spin waves in the transverse directions are obtained from the general exchange boundary conditions⁴⁴:

$$2 \frac{k_y}{\beta} \cos(k_y y_0) - \left(1 - \frac{k_y^2}{\beta^2}\right) \sin(k_y y_0) = 0, \quad (33)$$

$$2 \frac{k_z}{\beta} \cos(k_z z_0) + \left(1 - \frac{k_z^2}{\beta^2}\right) \sin(k_z z_0) = 0, \quad (34)$$

where y_0 and z_0 are the widths of the slab in the y and z directions, and β is the pinning parameter of YIG. In order to ensure that these transverse modes do not interfere with the band gaps produced by the electric field, we set the widths to be small enough so that if there is a transverse component, the lowest spin-wave band is shifted to above the frequency of interest. Using Eqs. (33) and (34) with⁴⁵ $\beta = 10^7 \text{ m}^{-1}$, a width of 50 nm or less will shift the dispersions by more than 2 GHz for either transverse mode, putting the lowest mode above the frequency where the band gap will open.

The frequencies and linewidths are obtained from the complex eigenvalues of Eq. (31). We consider an external field $\mu_0 H_0 = 0.27 \text{ T}$, saturation magnetization $M_s = 1.4 \times 10^5 \text{ A/m}$, exchange length $\lambda_{\text{ex}} = 16.1 \text{ nm}$, and damping parameter $\alpha = 0.0006$. From the dispersion relations (Fig. 13), we see that the YIG slab has no band gaps when there is no electric field. However, when a periodic electric field is applied, the band levels begin to split, opening multiple band gaps with the first few modes with a quality factor of the first mode on the order of 100. An additional effect is that the wave vector of the dispersion curves is phase shifted due to the electric field breaking the rotational symmetry of the system. By looking at the density of states of the spin waves (Fig. 16), we see that the appearance of the band gaps relies on the electric field having a periodic variation. A uniform electric field only shifts the band levels to a lower frequency.

Both the width and location of the band gaps can be controlled by adjusting various parameters of the system (Fig. 15). Since the electric field introduces the band gaps into the system, increasing its magnitude has the expected effect of increasing the width of the band gap. Additionally, due to the

Fig. 13. Spin-wave frequencies (dispersion relations) of YIG with $a = 200$ nm, $E_A = 0$, and (a) $E_B = 0$ and (b) $E_B = 8 \cdot 10^7$ V/m.

Fig. 14. Spin-wave linewidths of YIG with $a = 200$ nm, $E_A = 0$, and (a) $E_B = 0$ and (b) $E_B = 8 \cdot 10^7$ V/m.

overall effect of the electric field lowering the spin-wave frequency (Fig. 16), increasing the strength of the electric field will also lower the position of the band gap. The lattice constant has a comparatively

smaller effect on the band gap width, but can have a significant impact on its location, with the frequency rapidly increasing when the lattice constant is below 100 nm.

Fig. 15. Variation of band gap width and center between the first two modes for (a) $a = 200$ nm and (b) $E = 8 \cdot 10^7$ V/m.

Fig. 16. Density of States for zero electric field (bottom), uniform electric field (middle), and periodic electric field (top).

6. Conclusion

Spin-wave dispersion curves and relaxation rates have been calculated for hexagonal and square 2D lattices of magnetic cylinders embedded in another magnetic material. The correct form of the exchange field at the boundary between these two magnetic materials has been found, and the difference from another form used in the literature has been shown to be significant. Full-zone magnonic gaps are obtained for lattice materials that differ substantially in their saturation magnetization, such as Fe and YIG. Quality factors for spin waves can exceed 100, especially for the lowest-frequency spin mode. These results should assist in the design of magnonic crystals that can focus or redirect spin waves due to their effective band structure.

We have also calculated the band structures for a slab of YIG in the presence of a periodically varying electric field. This field opens band gaps in the dispersion relations whose frequency and width can be adjusted by modifying the strength of the electric field and the length scale of the periodicity. This principle could be used in the development of a magnon transistor and is more efficient than other proposed methods of controlling the spin-wave band structure^{20,25,46} since power is required only when switching the electric field on or off. The band gaps obtained by this method are also much larger than

those obtained via other methods. For spin waves with a similar frequency as described here, band gaps of tens of MHz were reported,^{20,46} whereas we obtained gap widths on the order of several hundred MHz.

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References

1. A. A. Serga, A. V. Chumak and B. Hillebrands, *J. Phys. D* **43**, 264002 (2010).
2. S. O. Demokritov, V. E. Demidov, O. Dzyapko, G. A. Melkov, A. A. Serga, B. Hillebrands and A. N. Slavin, *Nature* **443**, 430 (2006).
3. K. Ando, S. Takahashi, J. Ieda, H. Kurebayashi, T. Trypiniotis, C. H. W. Barnes, S. Maekawa and E. Saitoh, *Nat. Mater.* **10**, 655 (2011).
4. T. Liu and G. Vignale, *Phys. Rev. Lett.* **106**, 247203 (2011).
5. T. Liu and G. Vignale, *J. Appl. Phys.* **111**, 083907 (2012).
6. X. Zhang, T. Liu, M. E. Flatté and H. X. Tang, *Phys. Rev. Lett.* **113**, 037202 (2014).
7. M. P. Kostylev, A. A. Serga, T. Schneider, B. Leven and B. Hillebrands, *Appl. Phys. Lett.* **87**, 153501 (2005).
8. A. Khitun, M. Bao and K. L. Wang, *J. Phys. D* **43**, 264005 (2010).
9. D. L. Smith and C. Mailhot, *Rev. Mod. Phys.* **62**, 173 (1990).
10. E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987).
11. S. John, *Phys. Rev. Lett.* **58**, 2486 (1987).
12. E. Istrate and E. H. Sargent, *Rev. Mod. Phys.* **78**, 455 (2006).
13. B. Hillebrands, *Phys. Rev. B* **41**, 530 (1990).
14. J. O. Vasseur, L. Dobrzynski, B. Djafari-Rouhani and H. Puzkarski, *Phys. Rev. B* **54**, 1043 (1996).
15. S. A. Nikitov, Ph. Tailhades and C. S. Tsai, *J. Magn. Magn. Mater.* **236**, 320 (2001).
16. A. V. Chumak, A. A. Serga, B. Hillebrands and M. P. Kostylev, *Appl. Phys. Lett.* **93**, 022508 (2008).
17. M. E. Dokukin, K. Togo and M. Inoue, *J. Magn. Soc. Jpn.* **32**, 103 (2008).
18. A. V. Chumak, A. A. Serga, S. Wolff, B. Hillebrands and M. P. Kostylev, *Appl. Phys. Lett.* **94**, 172511 (2009).

19. A. Chumak *et al.*, *Appl. Phys. Lett.* **95**, 262508 (2009).
20. A. V. Chumak, T. Neumann, A. A. Serga, B. Hillebrands and M. P. Kostylev, *J. Phys. D* **42**, 205005 (2009).
21. K.-S. Lee, D.-S. Han and S.-K. Kim, *Phys. Rev. Lett.* **102**, 127202 (2009).
22. M. Mruczkiewicz, M. Krawczyk, R. V. Mikhaylovskiy and V. V. Kruglyak, *Phys. Rev. B* **86**, 024425 (2012).
23. T. Schneider, A. A. Serga, B. Leven, B. Hillebrands, R. L. Stamps and M. P. Kostylev, *Appl. Phys. Lett.* **92**, 022505 (2008).
24. K. Di, H. S. Lim, V. L. Zhang, S. C. Ng, M. H. Kuok, H. T. Nguyen and M. G. Cottam, *J. Appl. Phys.* **115**, 053904 (2014).
25. A. V. Chumak, A. A. Serga and B. Hillebrands, *Nat. Commun.* **5**, 4700 (2014).
26. T. L. Gilbert, *IEEE Trans. Magn.* **40**, 3443 (2004).
27. A. Khitun and K. L. Wang, *Superlattices and Microstruct.* **38**, 184 (2005).
28. A. Khitun, D. E. Nikonov, M. Bao, K. Galatsis and K. L. Wang, *Nanotechnology* **18**, 465202 (2007).
29. M. Krawczyk and H. Puzskarski, *Acta Phys. Polon. A* **93**, 805 (1998).
30. H. Puzskarski and M. Krawczyk, *Solid State Phenom.* **94**, 125 (2003).
31. M. Krawczyk and H. Puzskarski, *Phys. Rev. B* **77**, 054437 (2008).
32. Y. Cao, G. Yun, X. Liang and N. Bai, *J. Phys. D* **43**, 305005 (2010).
33. R. P. Tiwari and D. Stroud, *Phys. Rev. B* **81**, 220403(R) (2010).
34. J. W. Kłos, M. L. Sokolovskyy, S. Mamica and M. Krawczyk, *J. Appl. Phys.* **111**, 123910 (2012).
35. C. Kittel, *Rev. Mod. Phys.* **21**, 541 (1949).
36. G. T. Rado and J. R. Weertman, *J. Phys. Chem. Solids* **11**, 315 (1959).
37. F. Hoffmann, A. Stankoff and H. Pascard, *J. Appl. Phys.* **41**, 1022 (1970).
38. F. Hoffmann, *Phys. Stat. Sol.* **41**, 807 (1970).
39. R. Skomski and D. J. Sellmyer, *Handbook of Advanced Magnetic Materials: Vol 1. Nanostructural Effects* (Springer, New York, 2006).
40. M. Oogane, T. Wakitani, S. Yakata, R. Yilgin, Y. Ando, A. Sakuma and T. Miyazaki, *Jpn. J. Appl. Phys.* **45**, 3889 (2006).
41. J. C. Slonczewski, A. P. Malozemoff and E. A. Giess, *Appl. Phys. Lett.* **24**, 396 (1974).
42. Z. Zhang, P. C. Hammel and P. E. Wigen, *Appl. Phys. Lett.* **68**, 2005 (1996).
43. J. Kaczér and L. Murtinová, *Phys. Stat. Sol. (a)* **23**, 79 (1974).
44. R. F. Soohoo, *Phys. Rev.* **131**, 594 (1963).
45. P. E. Wigen, T. S. Stakelon, H. Puzskarski and P. Yen, Determination of complex magnetic surface energies from SWR spectra, in *Magnetism and Magnetic Materials* (North Holland, Amsterdam, 1976).
46. A. A. Nikitin, A. B. Ustinov, A. A. Semenov, A. V. Chumak, A. A. Serga, V. I. Vasyuchka, E. Lähderanta, B. A. Kalinikos and B. Hillebrands, *Appl. Phys. Lett.* **106**, 102405 (2015).