$SU(2)$ Gluon Propagators and the Asymmetry $\langle \Delta_{A^2} \rangle$ in the Postconfinement Domain

V. G. Bornyakov^{*,†}, V. K. Mitrjushkin[†] and R. N. Rogalyov^{*}

^{*}*NRC "Kurchatov Institute" - IHEP, 142281 Protvino, Russia*

[†]*Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[‡]*School of Biomedicine, Far East Federal University, 690950 Vladivostok, Russia*

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We study numerically the chromoelectric-chromomagnetic asymmetry of the A^2 gluon condensate as well as the *transverse and longitudinal* gluon propagators at $T > T_c$ in the Landau-gauge $SU(2)$ lattice gauge theory. We show that the previously found point at which asymmetry changes sign is an artifact of the finite volume effects. We find that with increasing temperature the asymmetry decreases approaching zero value from above in agreement with perturbative result. Instead of the asymmetry we suggest the ratio of the transverse to longitudinal propagator taken at zero momentum as an indicator of the boundary of the postconfinement domain and find it at $T \simeq 1.7T_c$.

Keywords: Lattice gauge theory; finite temperature; gluon propagator; asymmetry.

1. Introduction

Studies of the gauge-boson condensate $\langle A^2 \rangle = g^2 \langle A_\mu^a(x) A_\mu^a(x) \rangle$ were initiated by the Ref. 1, where it was related to the confinement-deconfinement transition in $4D$ compact $U(1)$ gauge theory. In this theory the confinement and deconfinement phases are separated by the phase transition at zero temperature. It was found that the nonperturbative part of the condensate drops at critical coupling. This observation raised hopes that a similar quantity might be also of relevance for the finite-temperature transition in the $4D$ non-Abelian theories.

There are electric and magnetic condensates at nonzero temperatures:

$$\langle A_E^2 \rangle = g^2 \langle A_4^a(x) A_4^a(x) \rangle, \quad \langle A_M^2 \rangle = g^2 \langle A_i^a(x) A_i^a(x) \rangle. \quad (1)$$

The quantity of particular interest is the (color)electric-magnetic asymmetry

$$\langle \Delta_{A^2} \rangle \equiv \langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle \quad \text{or} \quad \mathcal{A} = \frac{\langle \Delta_{A^2}(x) \rangle}{T^2}, \quad (2)$$

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it peaks at the phase transition and monotonically decreases in the deconfinement phase.² Furthermore, it was found that the asymmetry crosses zero at $T \approx 2.2T_c$ and becomes negative at higher temperatures. The existence of this symmetric point was one of the main results of Ref. 2.

Here we improve the computation of the asymmetry by taking care of the finite-volume and Gribov-copy effects. We confirm that the asymmetry monotonically decreases in the deconfinement phase; however, it never crosses zero. Our result agrees with perturbative estimates.

There is some range of temperatures just above T_c , where the quark-gluon plasma (QGP) cannot be treated perturbatively. In the literature the respective range of temperatures is sometimes referred to as the postconfinement domain, and strongly interacting matter at these temperatures — as semi-QGP.^{3,4}

There is some evidence that an interplay of electric and magnetic degrees of freedom plays an important role in the dynamics of semi-QGP; the electric ones dominate.⁵ In this work we find the upper limit on the temperature range where electric fluctuations prevail. Our results mean that the asymmetry cannot give marker of the boundary of the postconfinement domain. We suggest a new quantity to indicate such a boundary - the ratio of the magnetic to electric propagator at zero momentum.

2. Definitions and simulation details

We consider $N_t \times N_s^3$ hypercubic lattice Λ with the standard Wilson action

$$S = \frac{4}{g^2} \sum_{x \in \Lambda} \sum_{\mu > \nu} \left[1 - \frac{1}{2} \text{Tr} \left(U_{x\mu} U_{x+\mu;\nu} U_{x+\nu;\mu}^\dagger U_{x\nu}^\dagger \right) \right],$$

where $U_{x\mu} \in SU(2)$ are link variables, g is the coupling. The temperature is defined as $T = 1/aN_t$, where a is lattice spacing. A definition of the vector potentials A_μ^a , propagators $D_L(p)$ and $D_T(p)$, and the Landau gauge in terms of lattice variables can be found in Ref. 6, 7. In terms of A_μ^a , the asymmetry has the form

$$\mathcal{A} = g^2 a^2 N_t^2 \sum_{b=1}^3 \left(\left\langle A_{x,4}^b A_{x,4}^b \right\rangle - \frac{1}{3} \sum_{i=1}^3 \left\langle A_{x,i}^b A_{x,i}^b \right\rangle \right). \quad (3)$$

One-loop perturbative estimate of the asymmetry at $T \gg T_c$ has the form⁸

$$\langle \Delta_{A^2} \rangle \simeq \frac{g^2 T^2}{4} \left(1 - \frac{g}{3\pi} \sqrt{\frac{2}{3}} \right). \quad (4)$$

In the deconfinement phase, we choose the sector where the Polyakov loop in the temperature direction is positive. There remain 8 sectors of the space of gauge configurations according to the signs of spatial Polyakov loops; they are referred to as flip sectors⁹ and transitions between them – as flip transformations (or non-periodic gauge transformations). We use them in gauge fixing considering that the gauge group is extended by the flip transformations.

We employ the gauge-fixing algorithm combining simulated annealing (SA) with finalizing overrelaxation and Z_2 flips. We generate $n_{copy} = 1 \div 3$ gauge copies per flip sector each time starting from a random gauge transformation of the Monte Carlo configuration, obtaining $N_{copy} = 8n_{copy}$ Gribov copies for each configuration in the Landau gauge. We take the copy with maximal value of the gauge-fixing functional as our best estimator of the global maximum and denote it as best (“bc”) copy. In order to demonstrate the Gribov copy effect we compare with the results obtained from the randomly chosen copy (“fc”, first copy) and with the ‘worst’ copy (“wc”), i.e. the copy with the lowest value of the gauge-fixing functional.

3. A^2 asymmetry in the deconfinement phase

A decade ago, it was found² that $\mathcal{A} > 0$ at $T < 2.21(5)T_c$ and $\mathcal{A} < 0$ at $T > 2.21(5)T_c$ indicating that at high temperatures magnetic fluctuations dominate, however, finite-volume effects were not checked though the lattice size at $T = 6 T_c$ was as small as $L \equiv aN_s = 0.44$ fm with the corresponding minimal momentum $p_{min} \simeq 2.8$ GeV. In this work we study the finite-volume and Gribov-copy effects on lattices up to $L = 3$ fm.

Fig. 1. Temperature dependence of the asymmetry at fixed $N_s = 32$. The values for the first copy agree with the results of earlier studies.²

Fig. 2. Temperature dependence of the asymmetry at fixed $L = aN_s = 2$ fm.

In Fig. 1 we show our results for $\mathcal{A}$ obtained on the same lattices $32^3 \times 8$ as were used long ago.² Lower data set shows our result for $\mathcal{A}_{fc}$, which is in a good agreement with $\mathcal{A}_{old}$.² Upper dots correspond to $\mathcal{A}_{bc}$, which differs dramatically from both $\mathcal{A}_{fc}$ and $\mathcal{A}_{old}$ and this difference grows with temperature.

The main qualitative difference is that $\mathcal{A}_{bc}$ does not cross zero within the range of temperatures under study. Since the difference between the two procedures consists in the use of flips, we attribute the observed difference to the flip effects.

In the case of bc data set the best fit is provided by the fit function

$$\mathcal{A}_{bc} \simeq b_0 + \frac{b_1}{\xi}, \quad \text{where} \quad b_0 = -0.03(1), \quad b_1 = 0.44(3), \quad (5)$$

and $\chi^2/N_{dof} = 2.22$. Even if $\mathcal{A}_{bc}$ at $N_s = 32$ becomes negative, this occurs at temperatures much greater than the upper limit of the range under our consideration.

Now we consider the temperature dependence of the asymmetry at the fixed lattice size $L = 2$ fm. We fit the data to the function motivated by formula (4)

$$\mathcal{A} \simeq \frac{zg^2(T)}{4} \left(1 - \frac{g(T)}{3\pi} \sqrt{\frac{2}{3}} \right), \quad (6)$$

where the running coupling is taken in the two-loop approximation,

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2} \left(\frac{11}{6} \ln \left(\frac{T^2}{\Lambda^2} \right) + \frac{17}{11} \ln \ln \left(\frac{T^2}{\Lambda^2} \right) \right), \quad (7)$$

z and Λ are the fit parameters; the results are shown in Table 1.

Table 1.

Gauge-fixing algorithm	z	Λ/T_c	χ^2/N_{dof}
bc	0.1284(14)	0.845(7)	1.50
fc	0.1045(13)	0.826(7)	1.12
wc	0.0749(17)	0.811(14)	1.93

Note: Parameters of the fit of the asymmetry to eq. (6) over the range $1.24 < \xi < 3.32$.

Thus we arrive at a good agreement with perturbation theory (PT) modulo the normalization factor of the propagator. To make quantitative comparison, we should use the same normalization condition; however, the $\overline{MS}$ scheme employed in perturbative studies,⁸ runs into difficulties beyond PT.

We infer that, contrary to the well-known conclusions,² the asymmetry never crosses zero in the deconfining phase and it cannot indicate the boundary between two regions of the deconfining phase, whose existence was widely discussed.^{3, 10–13}

4. Boundary of the postconfinement domain

The ratio of electric and magnetic screening masses characterizing relative strength of chromoelectric and chromomagnetic forces was computed¹⁴ from the long-distance behavior of the gluon propagators two decades ago^a. It was shown¹⁴ that the data at $T > 2T_c$ are well described by the fit formula

$$\frac{m_e^2(T)}{m_m^2(T)} = \frac{C}{g^2(T)}. \quad (8)$$

^aSee also analogous computation in $SU(3)$ gluodynamics.¹⁵

However, the analysis of finite-size effects was missing though, over the temperature range explored in our study, their lattice size decreased from 2 fm down to 0.8 fm.

The square of a screening mass can be interpreted as the inverse of the respective zero-momentum propagator,¹⁶ we also recollect that Linde in his pioneer work¹⁷ related the magnetic mass to the transverse gluon propagator at zero momentum. However, currently this interpretation is not well justified and we adopt the following reasoning. It is natural to expect that the existence of the postconfinement region is due to nonperturbative contributions from low-momentum modes of the gauge field, thus we consider $r = D_T(0)/D_L(0)$ as a possible indicator of the boundary of the postconfinement region. We assume that the temperature T_p such that

$$r(T_p) = 1 \quad (9)$$

determines the boundary of the postconfinement region. It should be emphasized that T_p indicates some interval of temperatures rather than an exact value of T .

Finite-size effects for $r(T)$ are shown in Fig. 3 for $T/T_c = 2.49$. Huge difference between bc and fc data at small volumes illustrates that taking the flip sectors into account substantially reduces the finite-size effects. For $L = 3$ fm finite-size correction is comparable to statistical error.

The bc ratio $r(T)$ for two fixed lattice sizes $L = 2$ fm and $L = 3$ fm versus temperature is plotted in Fig. 4, here we employ the fit function

$$r(T) \simeq R_0 + R_1 \ln \left(\frac{T}{T_c} - 1 \right) = R_1 \ln \left(\frac{T - T_c}{Q} \right). \quad (10)$$

The results of this fit are shown in Fig. 4 and Table 2. It is seen that this fit function works well. The slope is independent of the volume, whereas the intercept slowly decreases with an increase of the lattice size. We have every reason to consider $L = 3$ fm as a good approximation to the infinite-volume limit.

Fig. 3. Lattice size dependence of the ratio $r(T)$ for first copies (empty symbols) and best copies (filled symbols) at $T/T_c = 2.49$.

Fig. 4. Ratio $r(T)$ for best copies on lattices with fixed sizes $L = 2$ fm and $L = 3$ fm versus $\log(T/T_c - 1)$ with fits to eq. 10.

The parameter Q in (10) sets the scale over the postconfinement domain. We find that $Q \approx 230$ MeV, which comes close to the quantity $m = 201(8)$ MeV² that sets the temperature scale for the asymmetry in the confinement phase.

Table 2.

Lattice size	R_0	R_1	T_p/T_c	χ^2/N_{dof}
2 fm	1.21(1)	0.293(6)	1.488(13)	1.35
3 fm	1.115(15)	0.283(9)	1.667(27)	1.92

Note: Parameters of the fit of the ratio $r(T)$ for best copies on lattices with $L = 2$ fm and $L = 3$ fm to fit function (10).

Fit function (10) implies that $D_T(0)/D_L(0)$ goes to infinity in the infinite temperature limit. Results at higher temperatures are needed to confirm this prediction.

Our findings can be summarized as follows:

(I) In contrast to earlier conclusions,² $\mathcal{A} > 0$ at $T > T_c$ and its high- T behavior agrees with PT. (II) A good indicator of the boundary of the postconfinement domain is provided by $r = D_T(0)/D_L(0)$ rather than by $\mathcal{A}$. At $L = 3$ fm, $T_p \simeq 1.7T_c$. (III) At $1.2T_c < T < 3.3T_c$, the effect of flip sectors is substantial at $L \simeq 2$ fm and crucial at $L < 1$ fm. In the latter case, it dramatically changes the behavior of $\mathcal{A}$ and r . (IV) $r(T)$ can well be fitted by the linear function of $\ln(T - T_c)$ (10).

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