Study of Deuteron System in a Two-Body Potential Model

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In this work, we study the Hamiltonian of Deuteron as a two-body system and solve the equation of the system in the non-relativistic limit. We obtain the ground state wave function as well as the corresponding energy eigenvalue of the deuteron system. The considered potentials are a combination of the confinement, Coulomb-like (as the one arising from one gluon exchange) and Goldstone boson exchange interaction.

Keywords: Potential model; exchange interactions; Deuteron system.

1. Introduction

One of the central goals of nuclear physics is to determine the structure and the dynamics of the nuclei [1-4]. Nucleons can form bound states (nucleus) due to the strong interactions. One way to study the nuclear two-body interactions is to make use of two-nucleon system such as deuteron. Deuteron is still fundamental to understand some basic properties of NN interaction. Deuteron is the only bound state of two nucleons with isospin $T=0$, spin-parity $J^\pi = 1^+$. In this work we study deuteron as a two-body system containing two nucleons in the non-relativistic limit. In Sec. 2 we introduce our potential model. In Sec. 3 we present our method to solve the Schrödinger equation of the deuteron system. Finally, the discussion and conclusion are presented in Sec. 4 and Sec. 5 respectively.

2. The Model

The interaction Hamiltonian (V_{nn}) for the nucleon-nucleon interaction leading to form the deuteron is given by

$$V_{nn} \ x = V_{Coul} \ x + V_{conf} \ x + V_{ope}(x) \quad (1)$$

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V_{Coul} , V_{conf} and V_{ope} are the coulomb-like, linear confining, and one pion exchange potentials respectively. The variable x is the relative nucleon-nucleon coordinate.

The linear confining and Coulombic-like interaction are as follows [5]

$$V_{Coul}(x) = -\frac{\lambda}{x} (1 - e^{-\mu x}), \quad V_{conf}(x) = \alpha x \quad (2)$$

Where α , λ and μ are constant. In the actual situation, one pion exchange interaction (as a Goldstone boson exchange interaction) contains spin and isospin parts. One can obtain spin-spin and tensor potentials as follows [1]

$$V_{ope}(x) = (V_{ss}(x) \delta_1 \cdot \delta_2 + V_T(x) S_{12}) \tau_1 \cdot \tau_2 \quad (3)$$

Where $S_{12} = [3 \delta_1 \cdot x \delta_2 \cdot x - \delta_1 \cdot \delta_2]$ and δ_1 and δ_2 are the Pauli matrix. In our model the radial part of V_{ss} and V_T is considered as

$$V_{ss} = V_T = \frac{D}{x^3} \quad (4)$$

D is a constant. This potential (V_{ope}) successfully explains NN-phase shifts at low energy as well as the quadrupole deformation of the deuteron system [1]. If the spins of the particles (proton and neutron) be parallel to x then $S_{12} = 2$ (the tensor force is attractive) and if be perpendicular then $S_{12} = -1$ (the tensor force is repulsive).

Since the deuteron ground state is in the $S=1$, $T=0$ state, therefore we can calculate the V_{ope} easily.

3. Solution to the Schrödinger Equation

We consider the V_{ope} exchange as perturbation and therefore, Since the potential (1) is assumed to depend on x only, one can factor out the angular part of the two-body wave function. We consider one pion exchange interaction (V_{ope}) as a perturbation. The remaining radial part of the wave function is then determined by the Schrödinger equation:

$$\left[\frac{d^2}{dx^2} + \frac{2}{x} \frac{d}{dx} - \frac{\gamma(\gamma+1)}{x^2} \right] \psi_\gamma(x) = -2m [E_\gamma - V_{conf}(x) - V_{Coul}(x)] \psi_\gamma(x), \quad (5)$$

where $\psi_\gamma(x)$, E_γ and γ are the wave function, energy eigenvalues and angular quantum number respectively. The two body problem is reduced to a problem of a single particle of reduced mass m moving in the central potential $V_{conf} + V_{Coul}$. The reduced mass of two-body system is

$$m = (m_1 m_2) / (m_1 + m_2). \quad (6)$$

The transformation [2-5]

$$\psi_{\gamma}(x) = x^{-1} \varphi_{\gamma}(x) \quad (7)$$

reduces Eq. (5) to the form of

$$\varphi_{\gamma}''(x) + [-2mV_{conf}(x) - 2mV_{Coul}(x) - \frac{1}{x^2}] \varphi_{\gamma}(x) = -2mE_{\gamma} \varphi_{\gamma}(x). \quad (8)$$

The wave function $\varphi_{\gamma}(x)$ is a solution of the reduced Schrödinger equation for each of two identical particles with the mass m and interacting potential (2). We Use the Maclaurin expansion of Yukawa potential in Eq. (2) and neglect all terms with powers higher than 3:

$$V(x) = V_{Coul}(x) + V_{Coul}(x) = \alpha x - \frac{\lambda}{x} [1 - e^{-\mu x}] = \alpha x - \frac{\lambda}{x} [1 + \mu x - \mu^2 x^2 + \frac{\mu^3 x^3}{6}]. \quad (9)$$

Now we use a variational method to solve Eq. (8). For variational solution [6-7] we rewrite Eq. (8) as follows

$$\varphi_{\gamma}''(x') + 2m[-V(x') - \frac{1}{mx'^2}] \varphi_{\gamma}(x') = -2mE_{\gamma} \varphi_{\gamma}(x'). \quad (10)$$

We use a new variable $x' = \sqrt{m}x$, and introduce a simple variational ansatz for $\varphi_{\gamma}(x')$ as

$$\varphi(x') = A \sqrt{p^3} x' e^{-p^2 x'^2}, \quad (11)$$

where p is the variational parameter, and the numerical factor A is chosen so that $\int \varphi^2(x') dx' = 1$. The trial two-body Hamiltonian admits explicit solutions for the wave function and the energy

$$E_0 = \min_p E(p), \quad (12)$$

where

$$E(p) = \langle \varphi | H | \varphi \rangle, \quad (13)$$

where

$$H = V_{Coul} + V_{conf}. \quad (14)$$

Now by using the condition

$$\left. \frac{dE}{dp} \right|_{p=p_0} = 0, \quad (15)$$

the value of p_0 is found.

4. Discussion

Using the obtained results in the previous section, we can study the static properties of deuteron. The first order energy correction from the non-confining potential $\langle H_{ope} \rangle$ can be obtained using the unperturbed wave function (7) as follows

$$\langle H_{ope} \rangle = \frac{\int \psi_0 H_{ope} \psi_0 x^2 dx}{\int \psi_0 \psi_0 x^2 dx} \quad (16)$$

Deuteron mass is obtained by nucleon masses and energy of perturbed system [7-10]:

$$M_D = 2m_N + E_0 + \langle H_{ope} \rangle \quad (17)$$

Within the Deuteron the mass of the nucleons may get modified due to its binding interactions with the other nucleon. The effective nucleon mass is defined as

$$m_N^{eff} = m_N \left(1 + \frac{E + \langle H_{ope} \rangle}{2m_N} \right) \quad (18)$$

Such that the mass of Deuteron is

$$M_D = 2m_N^{eff} \quad (19)$$

5. Conclusion

In this study, we present a simple method to solve two-body equation of the deuteron system in the non-relativistic limit. Hamiltonian of the system breaks to three parts: confining, Coulombic and one pion exchange interactions. A variational method is used to solve the Schrödinger equation. We consider the pion exchange interaction as a perturbation. Using the obtained results one can calculate the static properties of deuteron such as mass and magnetic moment. The method presented in our model can also be used to study two or three-body systems in the atomic and sub-atomic scales.

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