Bound State Correction in CP Violation

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We study hadronic weak decays using an approach which consists in a mapping technique, the Fock-Tani formalism, in order to obtain an effective weak Hamiltonian starting from a contact microscopic interaction. An additional effect is manifest in this formalism associated to the extended nature of mesons: bound-state corrections which can be related to CP violation parameters.

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1. Introduction

The Fock-Tani formalism is a field theory appropriated for the simultaneous treatment of composite particles and their constituents. This technique was largely used in atomic physics (Ref. ¹) and hadron physics to describe hadron-hadron scattering interactions (Ref. ²⁻³) and meson decay (Ref. ⁴).

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In the Fock-Tani formalism one starts with the Fock representation of the system using field operators of elementary constituents which satisfy canonical (anti-) commutation relations. Composite particle field operators are linear combinations of the elementary-particle operators and do not generally satisfy canonical (anti-) commutation relations. “Ideal” field operators acting on an enlarged Fock space are then introduced in close correspondence with the composite ones. Next, a given unitary transformation, which transforms the single composite states into single ideal states, is introduced. Application of the unitary operator on the microscopic Hamiltonian (Ref. ⁵), or on other hermitian operators expressed in terms of the elementary constituent field operators, gives equivalent operators which contain the ideal field operators. The effective Hamiltonian in the new representation has a clear physical interpretation in terms of the processes it describes. Since all field operators in the new representation satisfy canonical (anti)commutation relations, the standard methods of quantum field theory can then be readily applied.

CP violation in a neutral meson decay into charged mesons $M^0 \rightarrow M_1 M_2$ can be evaluated by

$$\mathcal{A}_{CP}(M^0 \rightarrow M_1 M_2) = \frac{\Gamma(\bar{M}^0 \rightarrow M_1^+ M_2^-) - \Gamma(M^0 \rightarrow M_1^- M_2^+)}{\Gamma(\bar{M}^0 \rightarrow M_1^+ M_2^-) + \Gamma(M^0 \rightarrow M_1^- M_2^+)} . \quad (1)$$

Fock-Tani bound state corrections introduce modifications in Γ , which in principle could alter c_P values.

2. The Meson in the Fock-Tani Formalism

In the Fock-Tani formalism we can write the meson creation operators in the following form $M_\alpha^\dagger = \Phi_\alpha^{\mu\nu} q_\mu^\dagger \bar{q}_\nu^\dagger$, where $\Phi_\alpha^{\mu\nu}$ is the bound-state wave-function for two-quarks. The quark and antiquark operators obey the following anti-commutation relations

$$\{q_\mu, q_\nu\} = \{\bar{q}_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu^\dagger\} = 0 \quad (2)$$

and

$$\{q_\mu, q_\nu^\dagger\} = \{\bar{q}_\mu, \bar{q}_\nu^\dagger\} = \delta_{\mu\nu} . \quad (3)$$

The composite meson operators satisfy non-canonical commutation relations

$$[M_\alpha, M_\beta] = 0 , \quad (4)$$

and

$$[M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \Delta_{\alpha\beta} , \quad (5)$$

where

$$\Delta_{\alpha\beta} = \Phi_\alpha^{*\mu\gamma} \Phi_\beta^{\gamma\rho} q_\rho^\dagger q_\mu + \Phi_\alpha^{*\mu\gamma} \Phi_\beta^{\gamma\rho} \bar{q}_\rho^\dagger \bar{q}_\mu . \quad (6)$$

The idea of the Fock-Tani formalism is to make a representation change, where the composite particles operators are described by operators that satisfy canonical commutation relations, i. e., which obey canonical relations $[m_\alpha, m_\beta] = 0$ and

$[m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta}$, where $m_\alpha^\dagger$ is the operators of the “ideal particles” creation. This way one can transform the composite state $|\alpha\rangle$ into an ideal state $|\alpha\rangle$. The transformations are given of the following form

$$|\alpha\rangle \rightarrow |\alpha\rangle = U^{-1} |\alpha\rangle, \quad (7)$$

and

$$O \rightarrow O_{FT} = U^{-1} O U, \quad (8)$$

where the U operator must be unitary so that

$$\langle\alpha|\alpha\rangle = \langle\alpha|\alpha\rangle, \quad (9)$$

and

$$\langle\alpha|O|\alpha\rangle = \langle\alpha|O_{FT}|\alpha\rangle. \quad (10)$$

In the meson case for example we have

$$U^{-1} M_\alpha^\dagger |0\rangle = m_\alpha^\dagger |0\rangle \equiv |\alpha\rangle, \quad (11)$$

where

$$U = \exp(tF), \quad (12)$$

and F is the generator of the meson transformation given by

$$F = m_\alpha^\dagger \tilde{M}_\alpha - \tilde{M}_\alpha^\dagger m_\alpha, \quad (13)$$

with

$$\tilde{M}_\alpha(t) = \sum_{i=0}^{\infty} \tilde{M}_\alpha^{(i)}(t). \quad (14)$$

3. The Microscopic Model

The Hamiltonian in this model is used to weak interaction with fermions current-current (Ref. ⁵), that is approach

$$H_W = \frac{G_F}{\sqrt{2}} \int d^3x \left\{ J_i^{(+)}(\vec{x}), J^{(-)i}(\vec{x}) \right\}, \quad (15)$$

where G_F is the Fermi coupling and

$$J_i^{(+)}(\vec{x}) = \frac{1}{2} \bar{\psi}_i(\vec{x}) \gamma_\mu (1 - \gamma_5) (V_{CKM})_{ij} \psi_j(\vec{x}), \quad (16)$$

$$J_i^{(-)}(\vec{x}) = \left(J_i^{(+)}(\vec{x}) \right)^\dagger. \quad (17)$$

In the Eq. (17), V_{CKM} is the Cabbibo-Kobayashi-Maskawa (CKM) matrix, $\Psi(\vec{x})$ is the Dirac quark field and the fermions are defined as $(\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3) \rightarrow (u, c, t)$ and $(\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3) \rightarrow (d, s, b)$ where $\mathcal{U}$ and $\mathcal{D}$ represent $\bar{\Psi}(\vec{x})$ and $\Psi(\vec{x})$.

Note that the operators $J_i^{(+)}(\vec{x}) J^{(-)i}(\vec{x})$ and your hermitian complex leads to the decay $(q\bar{q})_A \rightarrow (q\bar{q})_B + (q\bar{q})_C$ through the $b^\dagger d^\dagger b^\dagger b$ and $b^\dagger d^\dagger d^\dagger d$ terms. Introducing

the following notation $b \rightarrow q$, $d \rightarrow \bar{q}$, $\mu = (\vec{p}_1, s_1)$, $\nu = (\vec{p}_2, s_2)$, $\sigma = (\vec{p}_3, s_3)$ e $\rho = (\vec{p}_4, s_4)$, after the expansion one obtains

$$H_W = V_{qq}(\mu\nu; \sigma\rho) q_\mu^\dagger \bar{q}_\nu^\dagger q_\rho^\dagger q_\sigma - V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho) q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho^\dagger \bar{q}_\sigma, \quad (18)$$

where the sum (integration) is applied over repeated indexes and

$$\begin{aligned} V_{qq}(\mu\nu; \sigma\rho) &\equiv \frac{G_F \sqrt{2}}{(2\pi)^3} \int d\vec{p}_\mu d\vec{p}_\nu d\vec{p}_\rho d\vec{p}_\sigma \delta(-\vec{p}_\mu - \vec{p}_\nu - \vec{p}_\rho + \vec{p}_\sigma) \\ &\times \sum_{s_\mu, s_\nu, s_\sigma, s_\rho} \sum_{f_\mu, f_\nu, f_\sigma, f_\rho} \sum_{C_\mu, C_\nu, C_\sigma, C_\rho} \delta_{C_\mu C_\nu} \delta_{C_\rho C_\sigma} \\ &\times \left[\chi_{s_\rho}^* \left[\frac{\vec{p}_\rho \cdot (\vec{\sigma} \times \vec{\sigma})}{2m_\rho} \right] \chi_{s_\sigma} \chi_{s_\mu}^* \left\{ \frac{[\vec{p}_\mu \cdot (\vec{\sigma} \times \vec{\sigma})] (\vec{\sigma} \cdot \vec{p}_\nu)}{4m_\mu m_\nu} \right\} \chi_{s_\nu}^c (V_{CKM})_{\rho\sigma}^\dagger (V_{CKM})_{\mu\nu} \right. \\ &\left. + \chi_{s_\mu}^* \left\{ \frac{[\vec{p}_\mu \cdot (\vec{\sigma} \times \vec{\sigma})] (\vec{\sigma} \cdot \vec{p}_\nu)}{4m_\mu m_\nu} \right\} \chi_{s_\nu}^c \chi_{s_\rho}^* \left[\frac{\vec{p}_\rho \cdot (\vec{\sigma} \times \vec{\sigma})}{2m_\rho} \right] \chi_{s_\sigma} (V_{CKM})_{\mu\nu}^\dagger (V_{CKM})_{\rho\sigma} \right]; \end{aligned} \quad (19)$$

$$\begin{aligned} V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho) &\equiv \frac{G_F \sqrt{2}}{(2\pi)^3} \int d\vec{p}_\mu d\vec{p}_\nu d\vec{p}_\rho d\vec{p}_\sigma \delta(-\vec{p}_\mu - \vec{p}_\nu - \vec{p}_\rho + \vec{p}_\sigma) \\ &\times \sum_{s_\mu, s_\nu, s_\sigma, s_\rho} \sum_{f_\mu, f_\nu, f_\sigma, f_\rho} \sum_{C_\mu, C_\nu, C_\sigma, C_\rho} \delta_{C_\mu C_\nu} \delta_{C_\rho C_\sigma} \\ &\times \left[\chi_{s_\sigma}^{c*} \left[\frac{(\vec{\sigma} \times \vec{\sigma}) \cdot \vec{p}_\rho}{2m_\rho} \right] \chi_{s_\rho}^c \chi_{s_\mu}^* \left\{ \frac{[\vec{p}_\mu \cdot (\vec{\sigma} \times \vec{\sigma})] (\vec{\sigma} \cdot \vec{p}_\nu)}{4m_\mu m_\nu} \right\} \chi_{s_\nu}^c (V_{CKM})_{\sigma\rho}^\dagger (V_{CKM})_{\mu\nu} \right. \\ &\left. + \chi_{s_\mu}^* \left\{ \frac{[\vec{p}_\mu \cdot (\vec{\sigma} \times \vec{\sigma})] (\vec{\sigma} \cdot \vec{p}_\nu)}{4m_\mu m_\nu} \right\} \chi_{s_\nu}^c \chi_{s_\sigma}^{c*} \left[\frac{(\vec{\sigma} \times \vec{\sigma}) \cdot \vec{p}_\rho}{2m_\rho} \right] \chi_{s_\rho}^c (V_{CKM})_{\mu\nu}^\dagger (V_{CKM})_{\sigma\rho} \right]. \end{aligned} \quad (20)$$

Applying the Fock-Tani transformation to H_W one obtains the effective Hamiltonian

$$H_{FT} = U^{-1} H_W U = H_0 + \delta H_1. \quad (21)$$

For the calculate of the decay amplitude h_{fi} we consider the transition $m_\gamma \rightarrow m_\alpha + m_\beta$, with h_{fi} given by

$$\langle f | H_{FT} | i \rangle = \delta(P_\gamma - P_\alpha - P_\beta) h_{fi}, \quad (22)$$

where $|i\rangle = m_\gamma^\dagger |0\rangle$ and $|f\rangle = m_\alpha^\dagger m_\beta^\dagger |0\rangle$. After developing the calculations, the matrix element showed in the Eq. (22), finally we find

$$\begin{aligned} \langle f | H_{FT} | i \rangle &= V_{qq}(\mu\nu; \sigma\rho) \left[\Phi_\alpha^{*\mu\nu} \Phi_\beta^{*\rho\varepsilon} + \Phi_\beta^{*\mu\nu} \Phi_\alpha^{*\rho\varepsilon} - \Phi_\alpha^{*\mu\varepsilon} \Phi_\beta^{*\rho\nu} - \Phi_\beta^{*\mu\varepsilon} \Phi_\alpha^{*\rho\nu} \right] \Phi_\gamma^{\sigma\varepsilon} \\ &- V_{q\bar{q}}(\mu\nu; \sigma\rho) \left[\Phi_\alpha^{*\mu\tau} \Phi_\beta^{*\eta\nu} + \Phi_\beta^{*\mu\tau} \Phi_\alpha^{*\eta\nu} \right] \Delta(\eta\tau; \rho\varepsilon) \Phi_\gamma^{\sigma\varepsilon} \end{aligned}$$

$$\begin{aligned}
& + V_{\bar{q}q}(\mu\nu; \sigma\rho) \left[\Phi_{\alpha}^{*\mu\nu} \Phi_{\beta}^{*\lambda\rho} + \Phi_{\beta}^{*\mu\nu} \Phi_{\alpha}^{*\lambda\rho} - \Phi_{\alpha}^{*\mu\rho} \Phi_{\beta}^{*\lambda\nu} - \Phi_{\beta}^{*\mu\rho} \Phi_{\alpha}^{*\lambda\nu} \right] \Phi_{\gamma}^{\lambda\sigma} \\
& - V_{\bar{q}q}(\mu\nu; \sigma\rho) \left[\Phi_{\alpha}^{*\mu\tau} \Phi_{\beta}^{*\eta\nu} + \Phi_{\beta}^{*\mu\tau} \Phi_{\alpha}^{*\eta\nu} \right] \Delta(\eta\tau; \lambda\rho) \Phi_{\gamma}^{\lambda\sigma}.
\end{aligned} \quad (23)$$

In Eq. (23), the terms dependents of the bound state kernel Δ are the bound state correction, where the kernel represent the intermediate state of transition of the particle from the initial state to the final state. For the kernel is made a sum over mesons with the quantum numbers of the final state.

The meson wavefunction is defined as

$$\Phi_{\alpha}^{\mu\nu} = \chi_{S_{\alpha}}^{s_1 s_2} f_{f_{\alpha}}^{f_1 f_2} C^{c_1 c_2} \Phi_{nl}^{\vec{P}_{\alpha} - \vec{p}_1 - \vec{p}_2}, \quad (24)$$

where χ denotes the spin; f is the flavor and C are color coefficients. The spatial part is given by the SHO (simple harmonic oscillation) wavefunction.

4. Conclusions

The model for the weak interaction and CP violation with correct bound state is obtained through the Fock-Tani formalism.

The next step will be to make a mapping of the spectroscopy sectors (Ref. ⁶) as an application of the model.

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