Neutral Pion Pole Mass Calculation in a Strong Magnetic Field: Lattice QCD Versus NJL Model

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Within the framework of cold magnetized SU(2) Nambu-Jona-Lasinio model we evaluate the π_0 and σ pole mass, as well as, f_{π_0} , $g_{\pi_0 qq}$ at zero baryon density. We employ a magnetic field dependent coupling, $G(eB)$, fitted to reproduce lattice QCD results for the quark condensates. In particular, we find that the π_0 meson mass systematically decreases when the magnetic field increases, in good agreement with recent lattice calculations.

Keywords: Neutral Meson Mass, RPA Approximation, NJL Model in a Magnetic Field.

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1. Introduction

In this work the π_0 neutral meson pole mass in a strong magnetic field is calculated within the random phase approximation (RPA) ¹ at zero temperature and baryon

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density. The model coupling constant, $G(eB)^{2,3}$, is taken to be magnetic field dependent and it has been fitted in order to reproduce lattice QCD results for the quark condensates⁴. We employ the magnetic field independent regularization scheme (MFIR)⁵.

2. Meson Properties Under Strong Magnetic Field

The SU(2) NJL model lagrangian, in the presence of a magnetic field, is described by

$$\mathcal{L} = \bar{\psi}_f (i\mathcal{D} - \tilde{m}) \psi_f + G [(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \vec{\tau} \psi_f)^2] - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \quad (1)$$

where a sum over repeated f is implied. The electromagnetic gauge field is represented by A^μ , $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\vec{\tau}$ is the isospin matrix, the coupling constant by G while $Q = \text{diag}(q_u = 2e/3, q_d = -e/3)$ represents the charge matrix, $D^\mu = (i\partial^\mu - QA^\mu)$ is the covariant derivative, $\psi^T = (\psi_u, \psi_d)$ is the quark fermion field and $\tilde{m} = \text{diag}(m_u, m_d)$ represents the bare quark mass matrix. It has been shown by Avancini et al.¹ that within the RPA approximation the π_0 mass can be written as:

$$m_{\pi_0}^2(B) = -\frac{m}{M(B)} \frac{1}{4iGN_c N_f I(m_{\pi_0}^2, B)}, \quad (2)$$

where

$$I(m_{\pi_0}^2, B) = \frac{1}{4(2\pi)^3} \sum_{f=u,d} \beta_f \sum_{n=0}^{\infty} g_n I_{f,n}(k_{\parallel}^2 = m_{\pi_0}^2), \quad (3)$$

where $M(B)$ is the effective mass, $\beta_f = |q_f|B$, $f = (u, d)$, $N_c = 3$, $g_n = 2 - \delta_{n0}$, $p_{\parallel} = p_0 - p_3$, and

$$I_{f,n}(k_{\parallel}^2) = \int d^2 p_{\parallel} \frac{1}{[p_{\parallel}^2 - M^2 - 2\beta_f n][(p + k)_{\parallel}^2 - M^2 - 2\beta_f n]}. \quad (4)$$

It can be shown¹ that the σ -meson mass is given by the relation $m_{\sigma}^2(B) = 4M^2(B) + m_{\pi_0}^2(B)$ and that the Gell-Mann-Oakes-Renner (GOR) relation in a magnetic medium is recovered¹. The detailed calculation of f_{π_0} pion decay constant and $g_{\pi_0 qq}$ can be found elsewhere¹. We use the four different sets of parameters displayed in Table 1.

Table 1. Parameter sets for the NJL model at $T = B = 0$. The correct $eB \rightarrow 0$ limit of our ansatz requires that $G_{II} = G(eB = 0)$.

	m_{π_0} (MeV)	m_0 (MeV)	G (GeV ⁻²)	Λ (MeV)
Set I	135.62	5.0	$G_I = 4.67$	664.3
Set II	143.31	5.5	$G_{II} = 4.50$	650.0
Set III	417	48.41	$G_{III} = G_I$	664.3
Set IV	417	50.16	$G_{IV} = G_{II}$	650.0

Fig. 1. Condensates average and difference as functions of eB for the NJL model with G_{II} , $G(eB)$ compared to lattice QCD calculations ⁴.

3. Field Dependent Coupling $G(eB)$

Next, we present our ansatz for $G(eB)$ which reproduces the lattice results of Bali et al. ⁴. Lattice simulations for the average $(\Sigma_u + \Sigma_d)/2$ can be obtained by using

$$G(eB) = \alpha + \beta e^{-\gamma (eB)^2}, \quad (5)$$

where $\alpha = 1.44373 \text{ GeV}^{-2}$, $\beta = 3.06 \text{ GeV}^{-2}$ and $\gamma = 1.31 \text{ GeV}^{-4}$. In Fig.1 we show NJL results for the condensate average and difference using $G(eB)$ and G , and compare with lattice results.

4. Numerical Results and Conclusions

The properties of magnetized neutral mesons have been investigated using a fixed and a B -dependent coupling constant so that model predictions and LQCD results related to inverse magnetic catalysis agree ^{2,3}. In Fig. 2 our results for the m_{π_0} and $g_{\pi_0 qq}$ are presented for both fixed G and $G(eB)$. We have found that the use

Fig. 2. Normalized π_0 meson mass and meson-quark coupling as a function of eB in the NJL model with different coupling schemes.

Fig. 3. Normalized neutral pion mass $m_{\pi_0}(eB)/m_{\pi_0}(0)$ in the NJL model with different coupling schemes and a large current quark mass compared to recent lattice results ⁶.

of $G(eB)$ and G_I (and G_{II}) indicate an opposite behavior for $g_{\pi_0 qq}$ with B . As a major result, we show that the π_0 remains a soft mode even at rather high field strengths ($\approx 1.5 \text{ GeV}^2$) since its mass decreases by about 30%. Our results in Fig.7 show a remarkable agreement with recent LQCD predictions ⁷.

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