The QCD Phase-Diagram Obtained from the NJL and the Extended-NJL Models for Quark and Hadron Phases

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Published 15 August 2017

We analyse the hadron/quark phase transition described by the Nambu-Jona-Lasinio (NJL) model [quark phase] and the extended Nambu-Jona-Lasinio model (eNJL) [hadron phase]. While the original formulation of the NJL model is not capable of describing hadronic properties due to its lack of confinement, it can be extended with a scalar-vector interaction so it exhibits this property, the so-called eNJL model. As part of this analysis, we obtain the equations of state within the SU(2) versions of both models for the hadron and the quark phases and determine the binodal surface.

Keywords: Phase-Transition, NJL, Binodal.

PACS numbers: 26.60.-c, 26.60.Kp, 21.65.Qr, 25.75.Nq

1. Motivation

The study of the QCD phase-diagram is relevant to the understanding of many aspects of the physics in our universe. Two of the aspects of interest are related to relativistic heavy ion collisions (low densities, high temperatures) and stars (high densities, low temperatures). However, the direct solution of QCD is not possible at finite chemical potential.

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Given the importance of the study of the phases and the phase separation boundaries, we propose an investigation of the hadron/quark phase transition within a two phase model.

2. Quark Phase Matter

The quark phase is described by a SU(2) NJL model lagrangian, including a vector-isoscalar term, given by ¹

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_0)\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_V(\bar{\psi}\gamma^\mu\psi)^2. \quad (1)$$

Here ψ represents the quark field, m_0 the quark bare mass, and G_S and G_V are coupling constants that are chosen by fitting the pion mass $m_\pi = 135.0$ MeV and decay constant $f_\pi = 92.4$ MeV. As the theory is non-renormalizable, a momentum cutoff Λ is employed, which acts as a new parameter.

Table 1. Parameters sets for the lagrangian density (1). (B_i from Ref. ², B_R Set 2 of 1).

Model	Λ	G_S (fm ²)	G_V (fm ²)	m_0 (MeV)	m (MeV)
B ₁	650	0.19721	–	0	313
B ₂	600	0.26498	–	0	400
B ₃	570	0.34034	–	0	500
B _R	587.9	0.27449	$G_V \propto G_S$	5.6	400

3. Hadron Phase Matter

Even though the original NJL model is unable to describe the saturation properties of the nuclear matter, this can be fixed by the inclusion of extra channels which combine the scalar and vector terms ³. An extended NJL model (eNJL) ⁴ which includes such channel is given by the lagrangian density

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu \partial_\mu - m_0)\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_v(\bar{\psi}\gamma^\mu\psi)^2 \\ & - G_\rho[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2] - G_{sv}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2](\bar{\psi}\gamma^\mu\psi)^2 \\ & - G_{v\rho}(\bar{\psi}\gamma^\mu\psi)^2[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2] \\ & - G_{s\rho}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2][(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2], \end{aligned} \quad (2)$$

where ψ represents the nucleon field and the constants G_i represent the coupling constants for the different channels.

4. Binodals

The QCD phase-diagram is characterized by potentially multiple phases, whose phase separation boundaries are referred as *binodals*. Over those boundaries, the

phases from the regions of either side of the boundary can coexist. The binodals may be determined using Gibbs' conditions ⁷:

$$\mu_B^Q = \mu_B^H; \quad T^Q = T^H; \quad P^Q = P^H. \quad (3)$$

with

$$\mu_B^H = \frac{\mu_p + \mu_n}{2}; \quad \mu_B^Q = \frac{3}{2}(\mu_u + \mu_d) = 3\mu_q,$$

where the indexes H and Q refer to the hadronic and quark phases. The phase coexistence may then be obtained by simply plotting $P^i \times \mu_B^i$, $i = Q, H$, and looking for the intersection of both lines ^{5,6}.

Fig. 1. The crossing point at a $P^i \times \mu_B^i$ plot indicate the phase-coexistence.

5. Results

We determine the equations of state for both models at $T = 0$ and proton fraction $y_p = 0.5$. The crossing of the lines, as in Figure 1 (right), indicates a phase transition.

In Table 3 we show the chemical potential μ_B^i at the phase-transition for the combinations of parameterizations of Tables 1 and 2. Combinations for which there is no transition are omitted.

Table 2. Parameters sets for the lagrangian density (2) ⁴. The parameters G_i are in fm^2 , except for G_{sv} , G_{vp} and G_{sp} which are in fm^6 .

Model	G_s	G_v	G_{sv}	G_ρ	G_{vp}	G_{sp}	Λ (MeV)	m (MeV)
eNJL1	4.855	4.65	-6.583	0.5876	0	0	388.189	0
eNJL1 ^m	1.3833	1.781	-2.943	0.7	0	0	478.248	450
eNJL1 _{$\omega\rho$1}	4.855	4.65	-6.583	0.5976	-1	0	388.189	0
eNJL2	3.8	3.8	-4.228	0.6313	0	0	422.384	0
eNJL2 ^m	1.078	1.955	-2.74	0.75	0	0	502.466	450
eNJL2 _{$\omega\rho$1}	3.8	3.8	-4.228	0.6413	-1	0	422.384	0
eNJL3	1.93	3.0	-1.8	0.65	0	0	534.815	0

Table 3. Chemical potential μ_B^i at crossing point por different parameterization combinations. For B_R we have transition only for $G_V = 0$.

Q	H	μ_B^i (MeV)	P^i ($\frac{\text{MeV}}{\text{fm}^3}$)	Q	H	μ_B^i (MeV)	P^i ($\frac{\text{MeV}}{\text{fm}^3}$)
B ₁	eNJL1	1244	122	B ₃	eNJL1	1615	330
B ₁	eNJL1 $_{\omega\rho 1}$	1244	122	B ₃	eNJL1 $_{\omega\rho 1}$	1615	331
B ₁	eNJL2	1373	216	B ₃	eNJL2	1700	456
B ₁	eNJL2 ^m	1279	145	B ₃	eNJL2 $_{\omega\rho 1}$	1700	456
B ₁	eNJL2 $_{\omega\rho 1}$	1373	216	B ₃	eNJL3	1744	530
B ₁	eNJL3	1313	169	B _R	eNJL1	1475	244
B ₂	eNJL1	1460	235	B _R	eNJL1 $_{\omega\rho 1}$	1475	244
B ₂	eNJL1 $_{\omega\rho 1}$	1460	235	B _R	eNJL2	1570	354
B ₂	eNJL2	1557	343	B _R	eNJL2 ^m	1730	587
B ₂	eNJL2 ^m	1675	506	B _R	eNJL2 $_{\omega\rho 1}$	1570	354
B ₂	eNJL2 $_{\omega\rho 1}$	1557	343	B _R	eNJL3	1587	376
B ₂	eNJL3	1571	361				

6. Conclusions

From the leftmost plot at Figure 1, we can see that not all combinations of parameterizations produce a system in which a phase-transition is favored. Another manifestation of the dependence of the results on the choice of parameterizations is the range of chemical potential for which the transition takes place: it spans from 1244 MeV to 1744 MeV, a 40% difference in relation to the lowest value. This indicates that the choice of parameterizations to produce a complete phase-diagram should be made with care.

As a next step on this analysis, we shall turn our attention to expand our results for finite temperature and obtain the binodal sections.

Acknowledgments

D. P. M. acknowledges partial support by CNPq. C. A. G. thanks D. P. M. for the warm welcome and supervision of post-doctorate studies at Universidade Federal de Santa Catarina (UFSC).

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