A Heavy-Light Quark Potential

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Published 15 August 2017

In many studies of meson-baryon interactions with short one gluon exchange potential (OGEP), usually a full non-relativistic reduction, at the quark level Hamiltonian, is performed. In systems like $\eta_c N$, light and heavy quarks are present, which in principle would require only a partial non-relativistic reduction. We shall start from a JKK relativistic quark Hamiltonian and apply a partial non-relativistic reduction in order to obtain a OGEP between heavy and light quarks (heavy-light quark potential).

Keywords: Quark Model, Relativistic Approximation, Meson-Nucleon Scattering.

PACS numbers: 12.39.Pn, 12.40.-y, 13.90.+i

1. $\eta_c N$ Interaction

Due to difficulties in dealing with non-perturbative aspects of QCD such as color confinement and the necessity to attend to the internal structure of hadrons to properly deal with several systems, effective quark models were developed. One such approach is the Fock-Tani formalism, in which a change of representations maps the physical Hilbert space into an ideal Hilbert space, where the bound-states are represented by “ideal particles” whose elementary creation and annihilation operators obey canonical commutation relations. The information about the hadrons internal structure is transferred to an effective Hamiltonian^{1–3}. This formalism was shown to be equivalent, in lower orders, to the Resonating Group Method (RGM).

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Fig. 1. One Gluon Exchange Potential scattering diagrams.

In particular the $\eta_c N$ system is formed by a mesons η_c ($c\bar{c}$) and the nucleon N (qqq), which in terms of the RGM scattering diagrams, with a One Gluon Exchange Potential (OGEP), can be seen in Fig. 1.

In the $\eta_c N$ interaction, quark interchange is associated with a one gluon exchange between constituent quarks. Traditionally, OGEP is obtained from a relativistic two-body quark interaction, taking an expansion in powers of the momentum of the constituent spinors, obtaining a non-relativistic approximation in the lowest order of momentum ⁴. In the $\eta_c N$ system the η_c -meson has a pair of heavy quarks which will interact with the quarks from the nucleon. In this case the corresponding expansion should consider the interaction of a light relativistic quark with a heavy non-relativistic quark. The Cornell JKJ ⁵ type relativistic quark interaction Hamiltonian is

$$H_I = \frac{1}{2} \sum_{a=1}^8 \mathcal{F}^a \mathcal{F}^a \int d\vec{x} d\vec{y} J^\mu(\vec{x}) K_{\mu\nu}(\vec{x} - \vec{y}) J^\nu(\vec{y}), \quad (1)$$

where

$$J^\mu(\vec{x}) = \bar{q}(\vec{x}) \gamma^\mu q(\vec{x}),$$

are the quark currents, $\mathcal{F}^a = \lambda^a/2$ are the generators of $SU(3)$ and the gluon potential $K_{\mu\nu}$, in the momentum space, is

$$K_{00}(\vec{q}) = -\frac{D_0}{q^2}, \quad K_{ij}(\vec{q}) = \frac{D_0}{q^2} \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right); \quad (2)$$

$K_{0j}(\vec{q}) = K_{i0}(\vec{q}) = 0$ and $D_0 = 4\pi\alpha_s$. After using Eq. (2), the basic structure of Eq. (1) reduces to

$$J^\mu K_{\mu\nu}(\vec{q}) J^\nu = -\frac{D_0}{q^2} J^0 J^0 + \frac{D_0}{q^2} \vec{J} \cdot \vec{J} - \frac{D_0}{q^2} (\vec{J} \cdot \vec{q})(\vec{J} \cdot \vec{q}). \quad (3)$$

The spinor can be written as

$$u_s(\vec{p}) = \begin{pmatrix} f(\vec{p}) \chi_s \\ g(\vec{p}) \vec{\sigma} \cdot \vec{p} \chi_s \end{pmatrix}, \quad v_s(\vec{p}) = \begin{pmatrix} g(\vec{p}) \vec{\sigma} \cdot \vec{p} \chi_s^c \\ f(\vec{p}) \chi_s^c \end{pmatrix}, \quad (4)$$

where

$$f(\vec{p}) = \sqrt{\frac{E_p + m}{2E_p}} \quad , \quad g(\vec{p}) = \frac{1}{E_p + m} \sqrt{\frac{E_p + m}{2E_p}} \quad (5)$$

and χ_i, χ_i^c are the Pauli spinors and where $E_p = \sqrt{p^2 + m^2}$.

2. Heavy-Light Potential

The calculation will be restricted to the spin-spin interaction, which implies that the only relevant term is the second term in Eq. (3). Considering momentum conservation and defining $J_{s_1 s_2}^i(\vec{p} + \vec{q}, \vec{p})$ the "heavy current" and $J_{s_3 s_4}^j(\vec{p}' - \vec{q}, \vec{p}')$ as the "light current", with $f_i = f(\vec{p}_i)$ and $g_j = g(\vec{p}_j)$, one has the relevant part of J^i is

$$\begin{aligned} J_{s_1 s_2}^i(\vec{p} + \vec{q}, \vec{p}) &= i(f_1 g_2 - f_2 g_1)(\vec{\sigma} \times \vec{p})^i + f_2 g_1 [\delta_{12} q^i + i(\vec{\sigma} \times \vec{q})^i] \\ J_{s_3 s_4}^j(\vec{p}' - \vec{q}, \vec{p}') &= i(f_3 g_4 - f_4 g_3)(\vec{\sigma}' \times \vec{p}')^j - f_4 g_3 [\delta_{34} q^j + i(\vec{\sigma}' \times \vec{q}')^j]. \end{aligned} \quad (6)$$

The heavy current contribution of order $J_{s_1 s_2}^i \sim p$, which implies that $f_1 g_2 - f_2 g_1 \approx 0$ and $f_2 g_1 \approx 1/2m_p$, resulting in

$$J_{s_1 s_2}^i(\vec{p} + \vec{q}, \vec{p}) = \frac{i}{2m_p} (\vec{\sigma} \times \vec{q})^i. \quad (7)$$

The light current is $J_{s_3 s_4}^j \sim p^3$ and is obtained by similar procedure:

$$\begin{aligned} J_{s_3 s_4}^j(\vec{p}' - \vec{q}, \vec{p}') &= i \left[\frac{q^2}{8m_l^3} - \frac{\vec{p}' \cdot \vec{q}}{4m_l^3} \right] (\vec{\sigma}' \times \vec{p}')^j \\ &\quad - i \left[\frac{1}{2m_l} - \frac{p'^2}{4m_l^3} + \frac{3\vec{p}' \cdot \vec{q}}{8m_l^3} - \frac{3q^2}{16m_l^3} \right] (\vec{\sigma}' \times \vec{q})^j. \end{aligned} \quad (8)$$

The spin-spin contribution [second term in Eq. (3)] for the Qq system is

$$\begin{aligned} \vec{J}_{s_1 s_2} \cdot \vec{J}_{s_3 s_4} &= -C_1 [(\vec{\sigma} \cdot \vec{\sigma}')(\vec{q} \cdot \vec{p}') - (\vec{\sigma} \cdot \vec{p}')(\vec{\sigma}' \cdot \vec{q})] \\ &\quad + C_2 [(\vec{\sigma} \cdot \vec{\sigma}')q^2 - (\vec{\sigma} \cdot \vec{q})(\vec{\sigma}' \cdot \vec{q})], \end{aligned} \quad (9)$$

and for the $\bar{Q}q$ system is

$$\begin{aligned} \vec{J}_{s_1 s_2} \cdot \vec{J}_{s_3 s_4} &= -C_1 [(\vec{\tau} \cdot \vec{\sigma}')(\vec{q} \cdot \vec{p}') - (\vec{\tau} \cdot \vec{p}')(\vec{\sigma}' \cdot \vec{q})] \\ &\quad + C_2 [(\vec{\tau} \cdot \vec{\sigma}')q^2 - (\vec{\tau} \cdot \vec{q})(\vec{\sigma}' \cdot \vec{q})], \end{aligned} \quad (10)$$

where

$$\begin{aligned} C_1 &= \frac{q^2}{16m_p m_l^3} - \frac{\vec{p}' \cdot \vec{q}}{8m_p m_l^3}; \\ C_2 &= \frac{1}{4m_p m_l} - \frac{p'^2}{8m_p m_l^3} + \frac{3\vec{p}' \cdot \vec{q}}{16m_p m_l^3} - \frac{3q^2}{32m_p m_l^3}; \\ \vec{\sigma}_{ij} &= \chi_{s_i}^\dagger \vec{\sigma} \chi_{s_j} \quad ; \quad \vec{\tau}_{ij} = \chi_{s_i}^{c\dagger} \vec{\sigma} \chi_{s_j}^c, \end{aligned} \quad (11)$$

where the prime in $\vec{\sigma}$ and $\vec{\tau}$ indicates they are related to heavy currents and m_p (m_l) is the mass of the heavy (light) quark.

The next step is to isolate the spin-spin interaction terms from the potential through a Fourier transform, resulting in

$$V_{ss} = [a_1 + a_2 q^2 + a_3 (\vec{p}' \cdot \vec{q}) + a_4 p'^2] \frac{\vec{\sigma}}{2} \cdot \frac{\vec{\sigma}'}{2}, \quad (12)$$

where

$$a_1 = -\frac{8\pi\alpha_s}{3m_p m_l} \quad ; \quad a_2 = \frac{3\pi\alpha_s}{2m_p m_l^3} \quad ; \quad a_3 = -\frac{4\pi\alpha_s}{m_p m_l^3} \quad ; \quad a_4 = \frac{2\pi\alpha_s}{m_p m_l^3}. \quad (13)$$

3. Perspectives

The resulting heavy-light potential can be used to define a semi-relativistic effective meson-baryon potential which opens a perspective for future calculations like charmonium-nucleon cross sections and phase-shifts. One can also add to the potential a color confinement and a Coulombian color terms besides adopting a running coupling constant to have a more realistic model.

Acknowledgments

This work was funded by Capes and Universidade Federal do Rio Grande do Sul (UFRGS).

References

1. D. Hadjimichef, G. Krein, S Szpigel, and J. S. da Veiga, *Ann. of Phys.* **268** n.1, 105 (1998).
2. D. Hadjimichef, J. Haidenbauer, and G. Krein, *Phys. Rev.* **C63**, 035204 (2001).
3. D. Hadjimichef, J. Haidenbauer, and G. Krein, *Phys. Rev.* **C66**, 055214 (2002).
4. V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii; *Quantum Electrodynamics*, Course of Theoretical Physics, Vol. 4, 2nd edn. (Elsevier, Oxford, 1982).
5. E. S. Ackleh, T. Barnes, and E. S. Swanson, *Phys. Rev.* **D54**, 6811 (1996).