Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential

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In this work we study the chiral transition in quark matter at finite temperature and in the presence of a chiral imbalance in the Nambu–Jona-Lasinio model. We use the Medium Separation Scheme (MSS) to rewrite divergent momentum integrals that appear in the model in such way that we can conciliate results obtained with the model and recent lattice results for the chiral transition.

Keywords: Chiral Imbalance, Chiral Transition, Medium Separation.

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1. Introduction

The fact that a chiral imbalance of right- and left-handed quarks in quark matter can influence the phase diagram of quantum chromodynamics (QCD) has motivated many investigations recently. There are different reasons for this interest, e.g the nontrivial nature of the vacuum of QCD that allows for the existence of topological field configurations like instantons and sphalerons; the latter, in particular, can generate an asymmetry between the number of left- and right-handed quarks due to the Adler-Bell-Jackiw anomaly. This chiral imbalance in quark matter is expected to occur in event-by-event C – and CP –violating processes in heavy-ion collisions^{1,2}.

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Another interesting scenario where a chiral imbalance plays an important role is in the chiral magnetic effect (CME): in this case, a charge separation can be produced due the magnetic field created in off-central collisions that, in the presence of a chiral imbalance, gives rise to an electric current along the magnetic field—Ref. ³ provides a review with is a large list of references. The CME can be found in different physical systems and actually has been observed experimentally in semi-metals—for a recent review, see Ref. ⁴.

Effects of a chiral imbalance on the QCD phase diagram can be studied in the grand canonical ensemble introducing a chiral chemical potential μ_5 via a term $\mu_5 \bar{\psi} \gamma_0 \gamma_5 \psi$ in the QCD Lagrangian ². This allows, at least in principle, mapping the QCD phase diagram in the $T - \mu_5$ plane into the real phase diagram in the $T - \mu$ plane ^{5,6}, where μ is the baryon chemical potential. That is, such a mapping can be helpful in finding the expected critical end point (CEP) of the chiral transition (see Refs. ^{7,8} for opposite views). An additional interesting feature of QCD at finite μ_5 is that that it is free from the sign problem that afflicts QCD at finite μ and, therefore, it is amenable to Monte Carlo sampling in lattice simulations.

Recent lattice QCD results ^{9,10} at finite T and μ_5 are, to some extent surprisingly, in contradiction with *all* corresponding results obtained with effective models of QCD. Results based on Nambu–Jona-Lasinio (NJL)-type of models ^{5,11–15} and quark linear sigma models ^{5,16} predict that T_c decreases with μ_5 , while the lattice results of Refs. ^{9,10} find the opposite behavior. The contradiction came as a surprise since the models have been very helpful in giving insight in different pieces of chiral physics and seem to fail in this particular chiral problem. On the other hand, the behavior found for T_c in lattice simulations seems to be expected on general physical grounds, as the addition of left- and right-handed quarks to a system favors quark-antiquark pairing, that is, μ_5 increases the quark condensate ¹⁷. Universality arguments for QCD in the limit of a large number of colors N_c ¹⁸ lend further support to the lattice results. Different approaches have appeared in the recent literature trying to understand this discrepancy, e.g. studies based using phenomenological quark-gluon interactions in the framework of the Dyson-Schwinger equations (DSE) for the quark propagator ^{7,8} and nonlocal finite-range NJL models ^{19,20} find a T_c increasing with μ_5 . A qualitative agreement with the lattice results for T_c was also found in Ref. ²¹ using the quark linear sigma model with a nonstandard renormalization scheme.

The motivation for this work is try to understand why the local version of the NJL model do not reproduce the behavior for T_c as a function of μ_5 founded in lattice simulations. In a recent work ²², we propose that this discrepancy can be solved using a strategy based in a proper separation of the medium effects from the divergent integrals (Medium Separation scheme (MSS)), so that all divergent integrals are the same as those that appear in the vacuum ($T = 0$ and $\mu_5 = 0$). The improper separation of the medium effects can introduce unphysical effects ^{23–25}. Our procedure consist in rewrite all divergent integras in terms of integrals that appear in the vacuum, once the divergent integrals are fixed to reproduce physical

results, our results predicts that T_c is an increasing function of μ_5 . We apply MSS in the NJL model, but it can be applied in other effective models for QCD like the Polyakov–Nambu–Jona-Lasinio (PNJL) model^{11,15}.

This paper is organized as follows. In Sec. 2 we describe the model and the regularization scheme used to implement the separation of the medium effects. In Sec. 3 we compare our numerical results obtained in the context of this MSS with the traditional cutoff scheme (traditional regularization scheme (TRS)). Our final remarks and conclusions are presented in Sec. 4.

2. Chiral Imbalanced NJL Model

The version of the NJL model including chiral chemical potential is given by the Lagrangian density:

$$\mathcal{L} = \bar{\psi} (i\partial - m_c + \mu_5 \gamma^0 \gamma^5) \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5 \vec{\tau}\psi)^2 \right], \quad (1)$$

where m_c is the current quark mass ($m_c = 0$ in the chiral limit), G is the coupling and ψ represents a quark field. There are implicit sums over flavors, $N_f = 2$ and over color, $N_c = 3$. The thermodynamic potential of the model in mean field approximation can be written as

$$\Omega(M, T, \mu_5) = \frac{(M - m_c)^2}{4G} - N_f N_c \sum_{s=\pm 1} \int \frac{d^3 k}{(2\pi)^3} \omega_s(k) - 2N_f N_c T \sum_{s=\pm 1} \int \frac{d^3 k}{(2\pi)^3} \ln \left[1 + e^{-\omega_s(k)/T} \right], \quad (2)$$

where the dynamical quark mass $M \equiv M(T, \mu_5)$ is given by the gap equation

$$\partial \Omega(M, T, \mu_5) / \partial M = 0,$$

and

$$\omega_s(k) = \sqrt{(|\mathbf{k}| + s\mu_5)^2 + M^2}.$$

The momentum integral in the first line of Eq. (2) is UV divergent and requires regularization.

The MSS scheme of separating medium-dependent quantities from the integrands of divergent integrals proceeds as follows. Initially, we note that in the gap equation, which comes from the derivation of $\Omega(M, T, \mu_5)$ with respect to M , there appears a quadratic divergence

$$\frac{\partial}{\partial M^2} \left[\int \frac{d^3 k}{(2\pi)^3} \omega_s(k) \right] = \int_{-\infty}^{+\infty} \frac{dk_4}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k_4^2 + \omega_s^2(k)}. \quad (3)$$

Next, the integrand of the last integral is rewritten using identity^{26,23}

$$\begin{aligned} \frac{1}{k_4^2 + \omega_s^2(k)} &= \frac{1}{k_4^2 + k^2 + M_0^2} - \frac{A_s(k)}{(k_4^2 + k^2 + M_0^2)^2} + \frac{A_s^2(k)}{(k_4^2 + k^2 + M_0^2)^3} \\ &\quad - \frac{A_s^3(k)}{(k_4^2 + k^2 + M_0^2)^3 [k_4^2 + \omega_s^2(k)]}, \end{aligned} \quad (4)$$

where

$$A_s(k) = \mu_5^2 + 2sk\mu_5 + M^2 - M_0^2,$$

and M_0 is the quark mass in the vacuum, solution of the gap equation at $T = 0$ and $\mu_5 = 0$. Finally, the gap equation can then be written as

$$\begin{aligned} \frac{M - m_c}{4N_f N_c G M} = & I_{\text{quad}}(\Lambda, M_0) + (2\mu_5^2 - M^2 + M_0^2) I_{\text{log}}(\Lambda, M_0) - \frac{2\mu_5^2 + M^2 - M_0^2}{8\pi^2} \\ & + \frac{M^2 - 2\mu_5^2}{8\pi^2} \ln\left(\frac{M^2}{M_0^2}\right) - \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_s(k)} \frac{1}{e^{\omega_s(k)/T} + 1}, \end{aligned} \quad (5)$$

where

$$I_{\text{quad}}(\Lambda, M_0) = \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{1}{k_4^2 + k^2 + M_0^2}, \quad I_{\text{log}}(\Lambda, M_0) = -I'_{\text{quad}}(\Lambda, M_0), \quad (6)$$

the prime in I'_{quad} means derivative with respect to M_0^2 . Note that where M_0 is the vacuum quark mass and Λ denotes the regularization parameter used in the divergent integrals. The values of I_{quad} and I_{log} are fixed by fitting vacuum properties: quark condensate $\langle \bar{q}q \rangle$, the leptonic decay constant f_π and the pion mass m_π . We also note to arrive to the result in Eq. (5), all finite integrals are integrated without a regulator; the same is done with the last integral in this equation that depends explicitly in temperature and μ_5 .

The main point here is that the divergent integrals do not present explicit or implicit dependence on T or μ_5 in their integrands, they are the same that appear in the vacuum of the model. We can regularize the divergent integrals with different schemes, as, e.g., by a three-dimensional momentum cutoff Λ , and fix Λ by fitting a vacuum physical quantity.

In addition, other works that have used TRS in different effective models of QCD ^{5,6,11–14,16} they also find a CEP on the phase diagram (μ_5, T_c) . The recent lattice results ^{9,10} show explicitly that besides T_c increasing with μ_5 they also obtain a transition that is only a crossover.

3. Numerical Results

Our results for the dynamical quark mass M as a function of μ_5 at $T = 0$ are presented in Fig. 1. We can see that TRS scheme seems to indicate that the chiral chemical potential initially strengthens dynamical chiral symmetry breaking (DCSB), but this behavior changes at around $\mu_5 \simeq 0.6\Lambda$, beyond which it starts to disfavor DCSB. However, in the MSS scheme, DCSB is always strengthened by the chiral chemical potential. This change of behavior is due the separation of the medium effects from vacuum effects. This separation will also have an impact on how the critical temperature changes with μ_5 .

We use a three-dimensional cutoff to evaluate the vacuum divergent integrals. The values of the parameters are: $m_c = 5.37$ MeV, $G = 4.75$ GeV⁻² and $\Lambda =$

Fig. 1. The zero temperature quark mass M , normalized by its respective vacuum value M_0 , as a function of chiral chemical potential μ_5 . The results obtained using the TRS regularization are given by the dashed (chiral limit) and dotted lines. In the MSS regularization the results are given by the solid (chiral limit, $m_c = 0$) and dash-dotted lines.

0.660 GeV; they are fixed using as input $\langle \bar{q}q \rangle^{1/3} = -0.250$ GeV, $m_\pi = 0.140$ GeV and $f_\pi = 92.3$ MeV. The constituent quark mass in the vacuum is $M_0 = 0.302$ GeV.

Table 1. Critical T_c and pseudo-critical T_{pc} temperatures at $\mu_5 = 0$.

Reg. scheme	T_c [GeV]	T_{pc} [GeV]
TRS	0.165	0.177
MSS	0.169	0.183

In Fig. 2 we show the results for the critical temperature T_c as a function of μ_5 using TRS and MSS. The values of T_0 , for the critical (T_c) and pseudo-critical (T_{pc}) temperatures for chiral symmetry restoration evaluated at $\mu_5 = 0$ are given in Tab. 1.

In the results of TRS scheme there appears a critical end point (CEP) that separates a crossover line from a first-order transition. In the chiral limit ($m_c = 0$), the transition is instead a tri-critical point (TP), which separates a line of second-order phase transition from one of first-order. In the MSS scheme, the transition is a crossover and in the chiral limit, the transition is second order throughout. Other very important result is that in MSS scheme one sees that T_c always increases with μ_5 . The MSS results are in qualitative accordance with the recent results from the lattice ^{9,10} and also with other non-perturbative treatments ^{7,8}. The absence of the

Fig. 2. The critical temperature T_c , normalized by $T_0 = T_c(\mu_5 = 0)$, as a function of μ_5 . The black dot indicates a tricritical point in the chiral limit, while the square indicates the critical end point, both in the TRS regularization case (see text for a detailed explanation).

TP (in the chiral limit) or the CEP in the MSS regularization is also seen in the results obtained from the earlier lattice results²⁷, and also with the more recent ones, where no CEP (or TP) has been found. One should mention that the lattice results in Refs.^{9,10} should be taken with some caution, as they were obtained for a very large pion mass, $m_\pi = 418$ MeV.

4. Conclusions

We have shown a way to conciliate results obtained with the NJL model and recent lattice results for the critical temperature of chirally imbalanced quark matter. We pointed out that the discrepancy between the model and lattice results is strongly connected on how the UV momentum integrals are treated in these NJL-type of models.

Using the MSS, we implement a proper separation of medium effects from vacuum effects from the integrand of the divergent integrals that require regularization; finite integrals are performed without any cutoff, in the same way as the integrations over the terms depending explicitly on T are performed. Our results for the critical temperature dependence with the chiral chemical potential are in qualitative agreement with physical expectations, in that μ_5 is a catalyst of DCSB¹⁷ and, therefore, an increasing critical temperature as a function of μ_5 should be expected. Another result that we also have is a qualitative agreement with lattice results regarding the absence of a CEP. It is important to mention that recent studies^{7,8} based on a renormalizable, non-perturbative scheme based on the DSE of QCD also do not find a critical end point in the (T, μ_5) phase diagram— see also discussions

of Zhu-Fang Cui et al.¹⁵. It would be important to examine such result within similar approaches using DSE in QCD, particularly in models that incorporate different confining mechanisms, like those inspired in Coulomb gauge²⁸ and soliton models²⁹. The regularization scheme used in this work can be relevant for other problems that perform a mixing of medium and vacuum terms. We believe that our approach can improve the predictability of these effective models of QCD, which can help to study the QCD phase diagram.

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