Bayesian Inference Applied to Pulsar's Models

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The goal of this work is to apply Bayesian statistics to the problem of pulsars in order to compute the Bayes factor and investigate which one among different EoS could better fit known pulsar data, regarding the rate of decrease of the angular velocity versus the angular velocity itself. We also find the posterior distribution and the best fit for some relevant parameters of the pulsar like the mass and the magnetic field.

Keywords: Pulsar, Equation of State, Bayesian Statistics.

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1. Introduction

There are strong evidences that pulsars are rotating magnetized neutron stars emitting electromagnetic radiation in a wide range of wavelengths. The decay of a pulsar's rotation angular frequency is an observational evidence that the rotation energy decreases with time. However, it is well known that the canonical model for magnetic dipole radiation cannot precisely explain the energy loss. There is a variety of models trying to explain the phenomenon, such as the energy loss due to the emission of gravitational radiation ¹, quantum vacuum friction ² and interaction of the magnetic field with the plasma surrounding the star, and/or also variations in magnetic field, angle inclination ^{3–5}, moment of inertia, etc. Moreover, the equations that describe the phenomenon are functions of the mass-radius relation of the neutron star and so far it is not known which equation of state (EoS) for nuclear matter should be used to describe the star's interior.

In the canonical model, pulsars are described as spheres which have a rotating magnetic dipole and with the magnetic dipole axis misaligned with respect to its axis of rotation by which the star radiates its rotational energy as electromagnetic energy. In such model, it does not consider the variation of the moment of inertia ($\dot{I} = 0$).

2. Equations of State for Neutron Stars

We selected up-to-date nuclear equations of state (EoS) for this study. In particular, we adopt the parametrizations GM1, TM1, and NL3 (see Refs. ^{6,7}). Following the Hartle's formalism, we compute rotating configurations, accurate up to second-order

Fig. 1. Mass-Radius relations for different equations of state for neutron stars.

in Ω , with the same central density as the seed static, non-rotating configurations. The mass-radius relations for non-rotating configurations in the case of local charge neutrality are shown in Figure 1.

3. Bayes' Theorem Probing EoS

In the following we consider Bayes' theorem probing EoS⁸. We are considering the simple canonical model for pulsars, for which⁶

$$\dot{\Omega} = -\frac{5\Omega^3 B^2 R^4}{12c^3 M}, \quad (1)$$

where Ω is the angular frequency of arrival of the pulses, $\dot{\Omega}$ is its time derivative, M is the mass of the pulsar, R is its radius and B is the magnetic field at the pole of the star.

For each EoS Figure 1 shows a different function $R = R(M)$. Therefore, since Ω and $\dot{\Omega}$ are observational data it is possible to estimate the distributions of M and B for pulsars.

We denote the data by $D = \{d_1, d_2, \dots, d_N\}$ for N pulsars. We consider

$$d_i = \dot{\Omega}_i + e_i, \quad i = 1, 2, \dots, N, \quad (2)$$

where e_i describes spurious information that we name "noise".

The Bayes' theorem states that given the hypothesis H_k that the true EoS is k , then the posterior probability density function (PDF) for the parameter θ is

$$P(\theta|H_k, D) = \frac{P(D|H_k, \theta)P(\theta|H_k)}{P(D|H_k)}, \quad (3)$$

where, $P(D|H_k, \theta)$ is the likelihood function, $P(\theta|H_k)$ is the prior's distribution and $P(D|H_k)$ is a normalization constant.

It is reasonable to assume a Gaussian likelihood function

$$P(D|H_k, \theta) = \frac{1}{(\sqrt{2\pi})^N \prod_i \sigma_i} \exp \left[-\frac{1}{2} \sum_{i=1}^N \frac{(d_i - \dot{\Omega}_i)^2}{\sigma_i^2} \right], \quad (4)$$

where σ_i is the standard deviation for each measurement.

4. Bayesian Evidence

Bayesian Evidence (BE) derives from Bayes' theorem and allows the comparison of different models by marginalizing the likelihood function 4. Integrating over all the parameters space

$$P(D|H_i) = \int P(D|H_i, \theta)P(\theta)d\theta. \quad (5)$$

BE is based on Occham’s razor principle and its test of different models is performed through the Bayes factor

$$B_{ij} = \frac{P(D|H_i)}{P(D|H_j)}.$$
(6)

In this case, $E = B_{ij}$ and the interpretation of BE is shown in Table 1 (see Ref. ⁹).

Table 1. Bayesian Evidence.

Support	
$0 < 2 \ln E \leq 2$	Not worth more than a bare mention
$2 \leq 2 \ln E \leq 6$	Significant/Weak
$6 \leq 2 \ln E \leq 10$	Strong to very strong/Significant
$10 < 2 \ln E$	Decisive/Strong

5. Results

By considering the data of the 9 pulsars with known braking index: B0531+21 (Crab), B0540-69, B0833-45 (Vela), B1509-58, J1846-0258, J1833-1034, J1119-6127 and J1734-3333, we have evaluated the posterior PDF for the magnetic field B of the pulsar. Examples of results with two EoS are shown in Figures 3 and 4. Note that the magnetic field distribution and its mean value vary with the chosen EoS.

Fig. 2. Posterior PDF for the mass of the pulsars (in units of solar masses). The posterior is very similar to the bimodal prior distribution that was used.

Fig. 3. The posterior PDF for the magnetic field of the neutron star obtained by considering the parametrization TM1HT.

Fig. 4. The posterior PDF for the magnetic field of the neutron star obtained by considering the parametrization TM1L. Notice that different EoS lead to different estimates for the magnetic field.

On the other hand, the posterior PDF for the mass distribution of pulsars^{10,11} has a weak dependence on the EoS, as Figure 2 illustrates. Furthermore, this posterior is very similar to the prior mass distribution, indicating that the former is not significantly affected by the choice of the EoS.

Table 2. The BE for the EoS we are considering. The ratios are calculated by using the models in the first column in the numerator and the models in the first line in the denominator.

	GM1L	GM1HT	NL3L	NL3HT	TM1L	TM1HT
GM1L	0	-0.16	0.93	0.78	-0.18	0.65
GM1HT	0.16	0	1.09	0.94	-0.02	0.81
NL3L	-0.93	-1.09	0	-0.15	-1.11	-0.28
NL3HT	-0.78	-0.94	0.15	0	-0.96	-0.13
TM1L	0.18	0.02	1.11	0.96	0	0.83
TM1HT	-0.65	-0.81	0.28	0.13	-0.83	0

Finally, by marginalizing over M and B we find the BE for all the pairs of EoS considered in this work. As can be seen in the Table 2, none of the models are strongly favored when compared with the others. This can be explained in one hand by the few data considered and, on the other hand, by the similar predictions of the different models regarding the mass-radius relation.

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