A Model for the Braking Indices of Pulsars

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Stars known as pulsars are generally modeled as magnetized spheres made of neutrons with high rotation frequency. It is known that such stars are spinning down and this braking is measured by a parameter, n , known as braking index. For the canonical model such parameter should have a single value for all pulsars: $n = 3$. However, from observations it is known that n diverges from 3. In this work, differently from the canonical model, we have hypothesized the existence of a variation of the moment of inertia of the star through a time-varying radius. Using energy conservation we find the values for the variation of the radius of our pulsar sample. Our results indicate that it may be reasonable to consider that the radius of pulsars can be changing with time.

Keywords: Pulsars, Star Rotation, Magnetic Fields.

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1. Introduction

Pulsars are generally modeled as neutron stars, and they are referred to as pulsating because signals received from them arrive in the form of well-defined radio pulses¹. This effect is basically explained assuming a strong magnetic field and high speed rotation of a neutron star, leading to radiation in the form of a rotating beacon² whose emission is given by a rotating dipole with magnetic axis inclined relative to the axis of rotation³.

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Traditionally the canonical model for pulsars is accepted to explain the pulses and it predicts a slow-down quantified by a parameter n (known as braking index) equal to 3. However, for pulsars with braking index calculated from observations (B0531+21, B0833-45, B0540-69, B1509-58, J1846-0258, J1119-6127 and J1734-3333) the values of n are different and less than 3, as shown in Table 1. The complexity in understanding the phenomenon that leads to $n \neq 3$ has been investigated by many authors^{4–11} and is still an open problem. In this work we summarize our proposed modification in the canonical model, where we consider a uniform variation in a pulsar's radius to explain the different values for n .

2. The Canonical Model for Pulsars' Spin-Down

The model called canonical assumes that a pulsar is radiating its rotational energy (E_{rot}) solely through its magnetic dipole (m), i.e., in the form of electromagnetic energy (E_{mr}) and therefore the variation of its energy is given as follows:

$$\dot{E}_{rot} + \dot{E}_{mr} = 0. \quad (1)$$

A magnetic dipole misaligned relative to its axis of rotation radiates energy as¹²:

$$\dot{E}_{mr} = \frac{2}{3c^3} |\ddot{m}|^2, \quad (2)$$

where c is the speed of light in vacuum and m is the magnetic dipole moment that in vector form is:

$$\vec{m} = \frac{B_P R^3}{2} (\cos \alpha \hat{k} + \sin \alpha \cos(\Omega t) \hat{i} + \sin \alpha \sin(\Omega t) \hat{j}). \quad (3)$$

In this canonical model, this equation has the following constants: B_P is the magnetic field at the pole, R is the radius of the pulsar and α is the angle between the axis of the magnetic dipole and the pulsar rotation axis, while Ω is the pulsar's rotation speed, which varies with time. Then the magnetic dipole power becomes:

$$\dot{E}_{mr} = \frac{1}{6c^3} B_P^2 R^6 \Omega^4 \sin^2 \alpha = k I \Omega^4. \quad (4)$$

For the rotational energy of a pulsed star we have

$$E_{rot} = \frac{I \Omega^2}{2}, \quad (5)$$

where I is the moment of inertia that in the canonical model is assumed to be constant ($\dot{I} = 0$). Consequently, the variation of this rotational energy (or rotation power) is given by

$$\dot{E}_{rot} = I \cdot \Omega \cdot \dot{\Omega}, \quad (6)$$

where the dot denotes a time derivative.

From the energy variations in Eq. (4) and Eq. (6) the Eq. (1) results in:

$$\dot{\Omega} = -k\Omega^3, \quad (7)$$

where k is a positive constant. Therefore, the canonical model predicts a gradual decay of the rotation frequency.

A pulsar's spin-down can be quantified by a parameter known as braking index (n), given by the observables Ω , $\dot{\Omega}$ and $\ddot{\Omega}$ according to the definition

$$n \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}. \quad (8)$$

For the canonical model Eq. (7) yields $n = 3$.

3. Modifying the Canonical Model

We propose a uniform time variation in the radius of pulsars. As a consequence, the mathematical expressions of the canonical model change yielding

$$n = \frac{3R\dot{\Omega} + 3\dot{R}\Omega}{R\ddot{\Omega} - \dot{R}\dot{\Omega}}. \quad (9)$$

In Table 2 we present the results from Eq. (9) assuming the existence of a uniform variation of the radius for our sample of pulsars, whose n values are reported in Table 1.

Table 1. Angular velocity ($\dot{\Omega}$) and its first time derivatives for the sample of pulsars.

Pulsar	Ω (rad s ⁻¹)	$\dot{\Omega}$ ($\times 10^{-10}$ rad s ⁻²)	$\ddot{\Omega}$ ($\times 10^{-21}$ rad s ⁻³)	n
B 0531+21	189.912022 ¹³	-24.2674 ¹³	78.075 ¹³	2.51 ¹⁸
B 0833-45	70.4 ¹⁴	-0.986 ¹⁴	0.19 ¹⁴	1.4 ¹⁸
B 0540-69	124.623817 ¹⁵	-11.8365 ¹⁵	25.6 ¹⁵	2.28 ¹⁸
B 1509-58	41.68013054 ¹⁶	-4.24618765 ¹⁶	12.2944 ¹⁶	2.839 ¹⁸
J 1846-0258	19.340994108 ¹⁶	-4.21955 ¹⁶	24.3 ¹⁶	2.64 ¹⁸
J 1119-6127	15.401361301 ¹⁷	-1.517708 ¹⁷	4.014 ¹⁷	2.684 ¹⁸
J 1734-3333	5.37327178 ¹⁹	-0.104742 ¹⁹	0.018 ¹⁹	0.9 ¹⁸

Table 2. Time variation of the pulsar's radius, $\dot{R}$.

Pulsar	$\dot{R}$ (cm s ⁻¹)
B 0531+21	1.14×10^{-6}
B 0833-45	5.1×10^{-7}
B 0540-69	1.30×10^{-6}
B 1509-58	2.809×10^{-7}
J 1846-0258	1.39×10^{-6}
J 1119-6127	5.479×10^{-7}
J 1734-3333	1×10^{-6}

4. Conclusions

By assuming that the radii of pulsars steadily change in time we calculated this rate of change ($\dot{R}$) for pulsars with known braking indices. These values are small and at the present they are not possible to be verified observationally. We speculate that physical mechanisms occurring in the star interior could be responsible for such increase in radius, perhaps similar to a mechanism reported in the literature that deals with the possibility of increasing the moment of inertia from of phase transitions in hybrid stars²⁰.

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