Excitation Magnetohydrodynamic Wave by Gravitational Wave Produced by Binary of Neutron Stars

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The gravitational wave, through the strongly magnetized plasma surrounding the neutron stars, in the z -direction, deforms plasma particle rings in ellipses, alternating axes periodically along the direction of the magnetic field (x -axis) and of the y -axis. The uniform field leads to a modulation of the magnetic field, which results in magnetic pressure gradients (magneto-acoustic mode) or in the shear of the magnetic field lines (Alfvén mode). The gravitational wave drives MHD modes and transfers energy to the plasma, can become an important alternative process for the acceleration of baryons to high Lorentz factors observed in short GRBs. The total amount of energy that is transferred from the gravitational wave to the plasma is estimated ($\simeq 10^{40} \text{ J} - 10^{41} \text{ J}$), with $\gamma \simeq 10^2$. We compare our results with previously obtained results by other works.

Keywords: Magnetohydrodynamics Wave, Gravitational Wave, Gamma-Ray Bursts.

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1. Introduction

Binary systems that include Neutron Stars (NSs) are considered the most likely progenitors for short Gamma-Ray Bursts (sGRBs).¹ These systems must be able to generate an outflow, consisting of photons, matter and magnetic fields, with high luminosity ($\sim 10^{54} \text{ erg s}^{-1}$), total energy ($\sim 10^{55} \text{ erg}$) and high Lorentz factors² ($\sim 10^2$). The GRBs' irradiation processes are described by the fireball model. This model considers the object to have an intense magnetic field³ and suggests that its baryons contamination must be small enough² ($M \ll E/c^2$) to reach relativistic speed. This is related to the compactness problem. That is, if the baryon mass is large enough, nearly all the radiated energy will be converted to the baryons, in the form of kinetic energy, and there will be no observed γ -ray signal. The presence

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of a mass of $10^{-9}M_{\odot}$ of baryons is sufficient to significantly reduce the radiation; moreover, with a mass of $10^{-5}M_{\odot}$ of baryons, the expansion will not reach relativistic speeds.⁴ Therefore, the Lorentz factor of matter during GRB prompt emission is a compactness parameter, which limits the amount of baryonic pollution in the outflow and can be determined, for example, by the maximum of the light curve of the observed initial afterglow.²

There are several types of astronomical objects that can produce significant amounts of gravitational waves (GWs) together with ultra relativistic free-force winds.⁵ Coalescence of NSs (see Ref.⁵), for example, emits high frequency GW through the magnetosphere – composed by highly magnetized electron-positron plasma – of these objects. The gravitational radiation produced by these compact stars travels through their heavily magnetized ($\sim 10^{12}$ G) magnetospheres with density up to $n \sim 10^{12} \text{ cm}^{-3}$, and indirectly interacts with the electromagnetic fields coupled to the matter. The coupling excites the magnetohydrodynamic (MHD) modes, transferring energy to MHD waves, and may generate even higher harmonics, producing observable electromagnetic radiation in the radio frequency band.⁶ The mechanism of GW energy deposition into the plasma, exciting MHD modes, may become an alternative process for GRB's fireball ignition, such as it accelerates the baryonic matter to high Lorentz factors.⁷

The interaction of GW with electromagnetic fields was investigated by several works (e.g., see Refs.^{6,8}), mainly the Ref.,⁹ which estimated the amount of energy transferred to MHD wave, by propagation of the monochromatic GW produced by the merger of NSs. The Alfvén wave (AW) is excited by $h_{\times}$, whereas the slow and fast magnetosonic wave (MSW) are coupled to h_{+} of the GW.

The present work wants to analyze the excitation of the MHD waves through the GWs irradiated to the surrounding plasma to the binary NSs during the coalescence of the system.

2. General Relativistic Magnetohydrodynamics (GRMHD) Waves

Assume, without loss of generality, that a binary system, whose chirp mass is

$$M_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5},$$

coalesce in the plane (x, y) of a reference (x, y, z). Initially, any loss of energy due to the emission of GWs is negligible. As the system evolves, the spiral phase evolves through a succession of quasi-circular orbits controlled by the emission of gravitational radiation up to distances $r \gtrsim r_{(ISCO)}$ (Inner-Most Stable Circular Orbit).¹⁰ Close to $r_{(ISCO)}$, the system dynamics is dominated by strong field effects and the two stars fall towards each other. The spiral phase ends with the beginning of the merger. Up to the merger, the bonding energy decreases, releasing high-frequency GWs (f_g) in the surroundings.

In fact, GW induces small perturbations in space-time, assuming the linearized theory of gravity. It modifies the quantities that describe the plasma supposedly

in equilibrium ^a. If, based on the 3+1 formalism, ideal MHD theory approach ^b is considered, no consideration *priori* is made on temperature and $p^{(0)}$ is not neglected, Maxwell's equations can be described in a co-moving frame to the plasma. With generalized Ohm law and fluid equations, one can obtain a closed set of partial differential equations as a function of (z, t) for the 16 variables: $\vec{B}$, $\vec{E}$, $\vec{j}$, ρ_m , $\vec{v}$, μ , ρ , p ^c. They constitute the MHD description of the general relativity of a wave that propagates through a magnetized relativistic plasma. The general dispersion equation that describes the coupled system of GWs in a magnetized plasma is given by Eq. 1. Where ω and k are respectively the frequency and the wave vector of the MHD wave ^d

$$\begin{pmatrix} \omega^2(1 - u_{A\perp}^2) - k^2 u_{A\parallel}^2 & 0 & -(\omega^2 - k^2)u_{A\parallel}u_{A\perp} \\ 0 & \omega^2 - k^2 u_{A\parallel} & 0 \\ -(\omega^2 - k^2)u_{A\parallel}u_{A\perp} & 0 & \omega^2(1 - u_{A\parallel}^2) - k^2(u_m^2 - u_{A\parallel}^2) \end{pmatrix} = -u_{A\perp}\omega k \begin{pmatrix} u_{A\parallel}h_+ \\ u_{A\parallel}h_\times \\ -u_{A\perp}h_+ \end{pmatrix}. \quad (1)$$

The generalized AW phase speed ($\vec{u}_A \parallel \vec{B}^{(0)}$) and the MSW speed ($u_{f,s}$) are defined by Eq. (2):

$$u_A^2 = \frac{|\vec{B}^{(0)}|^2}{4\pi\eta} \quad \text{and} \quad u_{f,s}^2 = c_s^2 \frac{u_{A\parallel}}{\sqrt{1 - u_A^2}} \left(\frac{1 \pm \sqrt{1 - \sigma}}{\sqrt{\sigma}} \right), \quad (2)$$

where $u_m^2 = u_A^2 + c_s^2$, $c_s = \Gamma p^{(0)}/\eta$ is sound speed with the polytropic index of the state equation Γ and

$$\eta = \varepsilon^{(0)} + \frac{|\vec{B}^{(0)}|^2}{4\pi}$$

with the enthalpy

$$\varepsilon^{(0)} = \mu^{(0)} + p^{(0)},$$

and

$$\sigma(\theta) \equiv 4c_s^2 \frac{u_{A\parallel}^2}{1 - u_A^2} / \left(u_m^2 + c_s^2 \frac{u_{A\parallel}^2}{1 - u_A^2} \right)^2.$$

The left side of Eq. (1) is purely magnetohydrodynamic. The right side, that is purely gravitational and non-zero, modifies the behavior of MHD modes as a source

^aIndices (0) refer to the background quantities and (1) are the perturbed quantities.

^bDoes not take into account any dissipative effect - viscosity, Joule effect, heat flux and resistivity - and the quasi neutrality condition ($\vec{E}^{(0)} = \vec{j}^{(0)} = \rho^{(0)} = 0$) must be satisfied.

^cMagnetic field, electric field, current density, density of charge, velocity of the plasma, density of energy-matter relativistic, density of matter, pressure. Where $\mu = \rho c^2 - p/(\Gamma - 1)$.

^dThe derivation of the expressions of the closed set, in Gaussian units, in this section, it compares the method used by Ref.⁹

of perturbation of plasma quantities. The function $h_{+, \times}$ depends on the behavior of gravitational emission. In Refs. 7 and 9, the effects of a pulse function ^e

$$h_{+, \times}(z, t) = h_{+, \times} e^{if_g(z-t)}$$

was studied.

Following, the solutions for the AW and MSW in the laboratory frame are derived, since the Lorentz factor is $\gamma = 1/(1 - \beta^2)^{1/2}$, with speed of the plasma β .

By developing the Eq. (1), AW is derived by the interaction with the $\times$ polarization of GW. The perturbation of the magnetic field caused by the coupling of the GW during the coalescence of the binary system is given by Eq. (3):

$$B_y^{(1)}(k, \omega) = \frac{h_{\times}(f_g)B_x^{(0)}}{2} \frac{u_{A\parallel}}{u_{A\parallel} - \beta} \frac{[1 - \beta u_{A\parallel}(1 + \beta)]}{1 - \beta u_{A\parallel}} \frac{[\omega(\omega - ku_{A\parallel}^2)]}{\omega^2 - k^2 u_{A\parallel}^2}. \quad (3)$$

Note that the perturbed magnetic field $B_y^{(1)} \propto h_{\times}$ and $B_y^{(1)} \propto B^{(0)}$. In this situation, the coupling does not cause perturbation at the pressure $p^{(1)}$ and the density of energy-matter relativistic $\mu^{(1)}$, since the AW are non-compressional. However, the electric field divergence is not zero, hence the AW are accompanied by perturbations in the charge density $\rho^{(1)}$.

On the other hand, MSW is excited by $+$ polarization of GW. The perturbed magnetic field $B_x^{(1)}$ in this mode is given by Eq. (4), defining Ξ and Π given in Eqs. (A.1) and (A.2), respectively:

$$B_x^{(1)}(k, \omega) = -\frac{h_{+}(f_g)}{2\gamma^2} \omega \frac{(1 - \beta u_f)^2 (1 - \beta u_s)^2}{(u_f - u_s)[(u_f + u_s)(1 + \beta^2) - 2\beta(1 + u_f u_s)]} [\Xi(k, \omega) + \Pi(k, \omega)]. \quad (4)$$

The opposite of AW, MSW cause perturbations in the plasma pressure $p^{(1)}$ and, consequently, it causes perturbations in $\mu^{(1)}$. The divergence of the electric field is zero, so there is no disturbance in charge density $\rho^{(1)}$. Note that the term $\Xi(k, \omega)$ becomes significant when the GW propagates perpendicular to the surrounding magnetic field and the term $\Pi(k, \omega)$, becomes neglected. If the angle is $\theta = 0^\circ$, exactly the opposite happens. This means that the fast mode, with electromagnetic and compressional characteristic, predominates, when $\theta = \pi/2$. Otherwise, the slow mode, purely compressional, predominates. Commonly $u_f > u_s$, therefore the fast mode interacts coherently with the GW more easily than the slow mode.

The conditions for the evolution and energy transfer by magnetic field disturbance of AW solutions are less favorable, compared to MSW, because of the angular dependence of the resonance condition. In order to interact coherently with GW ($\Delta k = k_A - \omega_g \approx 0$), AW phase speed, $u_{A\parallel}$, has to approximate the speed of light ($u_{A\parallel} = u_A \cos \theta \rightarrow 1$). This means that AW vector, k_A , must be nearly parallel to the

^eAn approximation of the gravitational emission in instant before the merger of compact binary systems.

background magnetic field (since $B_z^{(0)} = B^{(0)} \cos \theta$). On the other hand, the amplitude of B_y is proportional to the transverse component $B_x^{(0)} = B^{(0)} \sin \theta \simeq B^{(0)} \theta$, at low angles, by Eq. (3). Therefore, at the ultra-relativistic limit, where the phase speed $u_{A\parallel} \rightarrow 1$, the coherent interaction AW with the GW is possible, but less efficient than in the magnetosonic mode. Because magnetic field disturbance of MSW $B_x^{(1)} \propto B_x^{(0)} = B^{(0)}$ is possible for ultra-relativistic limit $u_{A\parallel} \rightarrow 1$.

3. Energy Transfer

In single NS, the magnetic field anchored in the polar caps of the star evolves as a dipole, $B(r) = B_*(R_*/r)^3$, where B_* and R_* are parameters of the star on surface. However, in a binary system, the magnetic field can be considered constant within the radius of the light cylinder. This behavior is observed in numerical simulations (e.g., see Ref. 11). The magnetosphere of a NS with rotation Ω , possibly, extends up to $R_L = c/\Omega \approx 10^6$ m, which corresponds GW frequency of 300 Hz, away from the merger. Close to the merger, GW frequency is 1.5 kHz, whose distance from the light cylinder reaches 10^5 m. There is no imposition for the β parameter, just be sure to consider that it depends the evolution of stars, because it describes the velocity of the plasma that is influenced by the rotation of stars. The magnetic energy associated with magnetic fields disturbed can be being calculated by Eq. (5), where μ_0 is vacuum permeability:

$$W^{(B)} = \int_{\mathcal{R}_{\vec{k}}} \frac{|B(\vec{k})|^2}{2\mu_0} d^3k. \quad (5)$$

By replacing in the integral of Eq. (5), the disturbed magnetic fields of Eqs. (3),(4) and set the values – $B^{(0)} = 10^{12}$ T, $\beta = 0.9999$ ou $\gamma = 10^2$, $\omega \simeq 1.5 \times 10^3$ Hz, $u_A \approx 1$, $\theta = 0,01^\circ$ – it is possible to solve that the energy deposition during the spiral phase and to verify that the coupling is more efficient in the final stages of the spiral phase. (See Fig. 1). In the final stages of this phase, the energy transferred

Fig. 1. (a) Left: the magnetic energy as a function of the frequency of the Alfven wave of an ultra-relativistic plasma ($\beta = 0.9999$), during a coalescence. (b) Right: now, as a function of the frequency of the magnetosonic wave.

by the GW to the plasma that excite AW is $W_{AW}^{(B)} \simeq 10^{40} \text{ J}$, while for MSW, with $\theta \approx \pi/2$ and $u_A \approx u_f \approx 1$, the energy approximates to $W_{MSW}^{(B)} \sim 10^{41} \text{ J}$.

These values are seven orders of magnitude greater than the values obtained for the Alfvén and magnetosonic modes by the Ref. 9, $W_{AW}^{(B)} \sim 10^{33} \text{ J}$ and $W_{MSW}^{(B)} \sim 10^{34} \text{ J}$, respectively. A first interpretation for the energy difference, given Fig. 1, is that the energy dissipated in the plasma has important values for lower frequencies, since almost nothing differs from the order of magnitude of the energy associated with the larger frequencies, if β remains constant.

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Appendix A. Definitions

$$\begin{aligned} \Xi(k, \omega) \equiv & \frac{B_z^{(0)}}{1-\beta} \left\{ \frac{u_s}{\omega^2 - k^2 u_s^2} \left[\omega \frac{u_s - \beta}{1-\beta u_s} + \frac{k u_s}{(1-\beta u_{A\parallel})^2} \left[(u_{A\parallel} - \beta)^2 + \frac{u_{A\perp}^2}{\gamma^2} \right] \right] \right. \\ & - \frac{u_f}{\omega^2 - k^2 u_f^2} \left[\omega \frac{u_f - \beta}{1-\beta u_f} + \frac{k u_f}{(1-\beta u_{A\parallel})^2} \left[(u_{A\parallel} - \beta)^2 + \frac{u_{A\perp}^2}{\gamma^2} \right] \right] + 2 \frac{(u_s - \beta)^2 (u_f - \beta)^2}{(1-\beta u_f)^2 (1-\beta u_s)^2} \\ & \left[\frac{u_s}{\omega^2 - k^2 u_s^2} \frac{1-\beta u_s}{(u_s - \beta)^2} \left[\omega \frac{u_s - \beta}{(u_{A\parallel} - \beta)^2} \left[1 - \frac{u_{A\perp}^2}{\gamma^2 (1-\beta u_{A\parallel})^2} \right] + k u_s \frac{1-\beta u_s}{(1-\beta u_{A\parallel})^2} \right] \right. \\ & \left. \left. - \frac{u_f}{\omega^2 - k^2 u_f^2} \frac{1-\beta u_f}{(u_f - \beta)^2} \left[\omega \frac{u_f - \beta}{(u_{A\parallel} - \beta)^2} \left[1 - \frac{u_{A\perp}^2}{\gamma^2 (1-\beta u_{A\parallel})^2} \right] + k u_f \frac{1-\beta u_f}{(1-\beta u_{A\parallel})^2} \right] \right] \right\}. \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \Pi(k, \omega) \equiv & -\frac{\beta}{\gamma} B_z^{(0)} \frac{\omega}{(1-\beta u_{A\parallel})^2} \frac{1-u_A^2}{(u_{A\parallel} - \beta)^2} \frac{(u_f - u_{A\parallel}) [(u_f + u_{A\parallel})(1+\beta^2) - 2\beta(1+u_f u_{A\parallel})]}{(1-\beta u_f)^2 (1-\beta u_s)^2} \\ & \left[\frac{u_s (u_s - \beta) (1-\beta u_s)}{\omega^2 - k^2 u_s^2} - \frac{u_f (u_f - \beta) (1-\beta u_f)}{\omega^2 - k^2 u_f^2} \right]. \end{aligned} \quad (\text{A.2})$$

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