Suppression of the Primordial Gravitational Waves

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We study the primordial gravitational waves induced by space-space condensate inflation model. For modes that cross the comoving horizon during matter dominated era, we calculate the energy spectrum of gravitational waves. The energy spectrum of gravitational waves for our model has significantly suppressed in the low frequency range. The suppression occurs due to the phase transition during the early evolution of the Universe and depends on model parameter.

Keywords: Inflation; primordial gravitational waves.

1. Introduction

Inflation paradigm, an early period of accelerated expansion, is the leading candidate of explaining the primordial Universe¹⁻³. As a result, inflation models predict that this acceleration expansion would create ripples in the curvature of space-time, generating gravitational waves, that would remain energetic enough to leave an imprint on the Cosmic Microwave Background(CMB) radiation, approximately 380,000 years later⁴⁻⁶. The nearly scale-invariant, adiabatic, and almost Gaussian primordial curvature perturbation predicted by slow-roll inflation is consistent with the recent observations by Planck⁷ while the primordial tensor perturbation, also

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known as the primordial gravitational waves, is still on the way to be observationally detected.⁸ The observational detection of primordial gravitational waves would not only verify a success of inflation, but would also open a new window to understand the physics of the early Universe.

We calculate the energy spectrum of primordial gravitational in inflation with nonlinear sigma fields^{9–11} having $\sigma^a \sim x^a$ waves for the modes that re-entered the comoving horizon during the matter dominated era. It is assumed that the phase transition between two consecutive regimes to be abrupt. In such case, the cosmic expansion is the only damping effect after modes cross the comoving horizon.

The outline of our paper is the following. The basic setup is introduced in Section 2 while main calculation of the energy spectrum of primordial gravitational waves generated by the space-condensate inflation is provided in Section 3. In section 4, we conclude our results.

2. Setup

In this section, we construct the background setup for space-condensate inflation model^{9–11} which is considered as a source to the primordial gravitational waves in this work. We start with the following action,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \delta_{ab} \partial_\mu \sigma^a \partial_\nu \sigma^b - V(\phi) \right], \quad (1)$$

where we have an additional kinetic term for the triad of scalar fields, σ^a , in addition to action for scalar field, ϕ . To construct the spatial-condensate inflation model, we consider the flat FLRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$, along with the spatially linear configuration for σ fields, $\sigma^a = \xi x^a$ where ξ is the arbitrary constant. Thus, the background equations of motion are obtained as

$$H^2 = \frac{1}{3M_p^2} \left(\frac{1}{2} \dot{\phi}^2 + V + \frac{3\xi^2}{2a^2} \right), \quad (2)$$

$$0 = \ddot{\phi} + 3H\dot{\phi} + V_\phi. \quad (3)$$

If ξ -term in Eq. (2) dominates over both potential and kinetic terms, then we obtain constant comoving horizon,¹⁷

$$(aH)^{-1} \simeq \left(\frac{\xi^2}{2M_p^2} \right)^{-\frac{1}{2}}, \quad (4)$$

during the early evolution of the Universe. On the other hand, as the scale factor in Eq. (2) increases exponentially during inflation, the effect of ξ -term decreases rapidly such that at the end inflation our model asymptotically approaches to the standard single field inflation model with the negligible small contribution from ξ -term.

Therefore, there exist the early phase transition during early evolution of the Universe and usual slow-roll inflation takes place after the phase transition.

3. The primordial gravitational waves signature

The gravitational waves are characterized by the tensor perturbation, h_{ij} , of the flat FLRW metric^{18–20}

$$ds^2 = a^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]. \quad (5)$$

where δ_{ij} is the flat space metric and $h_{ij} \ll \delta_{ij}$. The conformal time τ relates to the cosmic time t via scale factor a , and it is $d\tau = dt/a$. With this metric, we expand Eq. (1) in second order in h_{ij} and obtain

$$S^{(2)} = \int d\tau d^3x \frac{a^2}{8\kappa^2} (h_{ij}'^2 + h_{ij} \partial_k \partial_k h_{ij} - 2\kappa^2 \xi^2 h_{ij} h_{ji}) \quad (6)$$

where $h_{ii} = 0 = \partial_i h_{ij}$. The power spectrum of primordial gravitational waves is calculated as follows. The tensor perturbation, h_{ij} , can be transformed in the Fourier space as

$$h_{ij}(\tau, \mathbf{x}) = \frac{\sqrt{2}}{M_p} \sum_{\lambda} \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \epsilon_{ij}^{(\lambda)}(\mathbf{k}) h_{\lambda, \mathbf{k}} e^{i\mathbf{k}\mathbf{x}} \quad (7)$$

where superscript λ denotes each polarization states of the tensor perturbation and $\epsilon_{ij}^{(\lambda)}$ is the polarization tensors which satisfy $k_j \epsilon_{ij}^{(\lambda)}(\mathbf{k}) = 0$, $\epsilon_{ii}^{(\lambda)}(\mathbf{k}) = 0$, and $\epsilon_{ij}^{(\lambda)}(\mathbf{k}) \epsilon^{ij(\lambda')}(\mathbf{k}) = \delta^{\lambda\lambda'}$.

Thus we have the linearized equation of motion for tensor perturbations in Einstein gravity is¹⁷

$$h''_{\lambda, k} + 2\mathcal{H}h'_{\lambda, k} + \left(k^2 + \frac{2\xi^2}{M_p^2}\right) h_{\lambda, k} = 0, \quad (8)$$

where $\mathcal{H} = a'/a$ and $' \equiv d/d\tau$. The mode solutions of Eq. (8) evolve as constant before they cross the horizon ($k \ll aH$); then they damp, as $h_{\lambda, k} \sim 1/a$, well inside the horizon ($k \gg aH$). As we mentioned in previous section that ξ -term would be negligible small at the end of inflation, therefore, the effect of ξ -term in Eq. (8) is also negligible.

The strength of the primordial gravitational waves is characterized by their energy spectrum^{18–20}

$$h_0^2 \Omega_{GW}(k) = \frac{h_0^2 k^2}{12H_0^2} \mathcal{T}^2(k) \mathcal{P}_T(k), \quad (9)$$

where H_0 is the present value of the Hubble parameter, $H_0 = 100h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and h_0 is dimensionless Hubble parameter. Transfer function, $\mathcal{T}$, reflects the damping effect for the modes inside the Hubble horizon. Therefore, it can be calculated from Eq. (8) and good fit to the transfer function for the modes well inside the horizon is^{21–22}

$$\mathcal{T}(k) = \frac{3}{k^2 \tau_0^2} \frac{\Omega_m}{\Omega_\Lambda} \sqrt{1 + \frac{4}{3} \left(\frac{k}{k_{eq}}\right) + \frac{5}{2} \left(\frac{k}{k_{eq}}\right)^2}, \quad (10)$$

where k_{eq} is the wavenumber when matter and radiation have equal energy density, $k_{eq} = 0.073 \Omega_m h_0^2 \text{ Mpc}^{-1}$ where h_0 is the dimensionless Hubble parameter, τ_0 is the

present conformal time, and Ω_m and Ω_Λ are the present energy density fractions of the matter and the vacuum, respectively. For modes re-entered the horizon during the matter dominated era, Eq. (10) evolves as $\mathcal{T}(k) \sim k^{-2}$.

The inflationary power spectrum, $\mathcal{P}_T(k)$, of tensor modes in Eq. (9) is assumed to be power-law form as

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_*) \left(\frac{k}{k_*} \right)^{n_T + \frac{\alpha_T}{2} \ln(k/k_*)}, \quad (11)$$

where $\alpha_T \equiv dn_T/d \ln k$ is the running of tensor spectral index n_T and $\mathcal{P}_T(k_*)$ is the amplitude of tensor power spectrum at the pivot scale k_* . The amplitude of tensor power spectrum, $\mathcal{P}_T(k_*)$, can relate to that of the scalar power spectrum, $\mathcal{P}_S(k_*)$, which is very well constrained by observational data⁷ via tensor-to-scalar ratio, r , which is another observable parameter as $\mathcal{P}_T(k_*) \equiv r\mathcal{P}_S(k_*)$.

The observable quantities in Eq. (11) for the space-condensate inflation model are obtained in Ref. 10 as follows

$$n_T \simeq -2\epsilon_V + \frac{31\xi^2}{6k^2M_p^2}, \quad \alpha_T = -\frac{31\xi^2}{3k^2M_p^2}, \quad r \simeq 16\epsilon_V + \frac{16\xi^2}{k^2M_p^2}. \quad (12)$$

where ϵ_V and η_V are the potential based slow-roll parameters^a.

Therefore, by substituting Eqs. (10)–(12) into Eq. (9), we obtain the energy spectrum of the primordial gravitational waves for modes that cross the horizon during matter dominated era,

$$h_0^2 \Omega_{GW} = \frac{3h_0^2 \mathcal{P}_S(k_*)}{16\pi^2 H_0^2 \tau_0^4} \left(\frac{\Omega_m}{\Omega_\Lambda} \right)^2 \frac{r}{f^2} \left(\frac{f}{f_*} \right)^{-\frac{r}{8} + \frac{43\xi^2}{24\pi^2 M_p^2 f^2} - \frac{31\xi^2}{24\pi^2 M_p^2 f^2} \ln\left(\frac{f}{f_*}\right)}, \quad (13)$$

where we consider $k \ll k_{eq}$ in Eq. (10) and f is the physical frequency observed today and defined as $f \equiv k/2\pi$, while f_* is the reference frequency $f_* \equiv k_*/2\pi$.

In our numerical plots below, we use the following cosmological parameters, provided by Ref. 7, where $\Omega_m = 0.315$, $\Omega_\Lambda = 0.685$, $\mathcal{P}_S(k_*) \simeq 2.19 \times 10^{-9}$, $k_* = 0.002 \text{Mpc}^{-1}$, $h_0 \simeq 0.6731$, and $\tau_0 = 1.41 \times 10^4 \text{Mpc}$.

In Fig. 1, we plot Eq. (13) as function of f for fixed r . We set $r = 10^{-1}$ in Fig. 1 because recent observational data⁷ predicts $r_{0.002} < 0.11$ as a upper bound for r . For nonzero value of ξ one can see that the energy spectrum of the primordial gravitational waves suppresses. As we increase the value of ξ parameter from smaller to larger, $\xi = 4 \times 10^{-20}$ (dashed), $\xi = 10^{-19}$ (dot-dashed), and $\xi = 4 \times 10^{-19}$ (dotted), the we see even more suppression.

In Fig. 2, we plot Eq. (13), but this time we fix ξ parameter and change the value of r to see its effect to the energy spectrum. We set $\xi = 0$ for solid line and $\xi = 10^{-19}$ for dot-dashed line, then change r value $r = 10^{-1}$, $r = 10^{-2}$, and $r = 10^{-3}$, respectively from top to bottom. As seen in Fig. 2 that the change in

^asee Ref. 10 for details.

Fig. 1. The plot of Eq. (13) for fixed $r = 0.1$. Four different values of ξ are considered; $\xi = 0$ (solid), $\xi = 4 \times 10^{-20}$ (dashed), $\xi = 10^{-19}$ (dot-dashed), and $\xi = 4 \times 10^{-19}$ (dotted).

Fig. 2. The plot of Eq. (13) for fixed $\xi = 0$ (solid) and $\xi = 10^{-19}$ (dot-dashed). Three different values of r are plotted from top to bottom; $r = 10^{-1}$, $r = 10^{-2}$, and $r = 10^{-3}$, respectively.

r value does not have any influence on the suppression of the energy spectrum. Instead, it only affects the amplitude of the energy spectrum.

Therefore, one can see that the suppression of the energy spectrum depends only on the model parameter ξ . In order to realize the suppression of the energy spectrum of the primordial gravitational waves induced by the space-condensate inflation model, we need to understand the phase transition during the early evolution of the Universe.¹¹ The phase transition occurs when $V \sim 3\xi/2a^2$. Before the phase transition, the comoving horizon remains constant, $(aH)^{-1} \simeq (\xi/\sqrt{2}M_p)^{-1}$; after the phase transition, the potential energy of the Universe dominates so that canonical slow-roll inflation takes place.

The existence of constant comoving horizon means that there also exists the minimum for comoving wavenumber.¹⁰ Therefore observational lower bound for the comoving wavenumber is $k_{\min} \simeq \xi/\sqrt{2}M_p$. The modes satisfying $k \geq \xi/\sqrt{2}M_p$ can only cross the comoving horizon during the early phase transition, and the modes that satisfy $k < \xi/\sqrt{2}M_p$ never cross the comoving horizon and stay outside the horizon.

4. Conclusion

In this work, we have studied the primordial gravitational waves from the space-condensate inflation model. The energy spectrum of gravitational waves for the modes that re-entered the horizon during matter dominated era is obtained in Eq. (13). Therefore, our result is true only for the modes crosses the horizon during matter dominated era or stay well inside comoving horizon.

In Fig. 1 and Fig. 2, we plot Eq. (13) as a function of frequency. The figures show that the energy spectrum of primordial gravitational waves has the significant suppression that occurs only for the non-zero values of model the parameter. The tensor-to-scalar ratio has nothing to with the suppression effect instead it does affect to the amplitude of the energy spectrum.

To realize the suppression of gravitational-waves energy spectrum, we have discussed the feature of the model that we consider. The feature of the model is that there exists the phase transition during the early evolution of the Universe which followed by the canonical slow-roll inflation. The existence of this phase transition leads to the existence the observational lower bound for the comoving wavenumber, $k_{\min} \simeq \xi/\sqrt{2}M_p$. The modes with $k < k_{\min}$, would never cross the comoving horizon, and stay outside the horizon. Since those modes causally disconnected to our Universe they suppress the energy spectrum of the primordial gravitational waves.

If the energy spectrum of primordial gravitation waves suppresses in the low-frequency range, it is then expected that such suppression should be imprinted in the CBM measurement. Therefore, in our next project, we would like to see effects of model parameter to the CMB angular power spectrum for BB-mode and to generalize of current result to the modes that cross the coming horizon during radiation dominated era and still stay well inside the horizon.

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