WAKE NUMERICAL SIMULATION BASED ON THE PARK-GAUSS MODEL AND CONSIDERING ATMOSPHERIC STABILITY

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Published 22 June 2016

In this paper, a new Park-Gauss model based on the assumption of the Park model and the Eddy-viscosity model is investigated to conduct the wake numerical simulation for solving a single wind turbine problem. The initial wake radius has been modified to improve the model's numerical accuracy. Then the impact of the atmospheric stability based on the Park-Gauss model has been studied in the wake region. By the comparisons and the analyses of the test results, it turns out that the new Park-Gauss model could achieve better effects of the wind velocity simulation in the wake region. The wind velocity in the wake region recovers quickly under the unstable atmospheric condition provided the wind velocity is closest to the test result, and recovers slowly under stable atmospheric condition in case of the wind velocity is lower than the test result. Meanwhile, the wind velocity recovery falls in between the unstable and stable neutral atmospheric conditions.

Keywords: Park model; Park-Gauss model; wake region; atmospheric stability; wind turbine.

1. Introduction

The green pollution-free wind energy becomes one of the hot spots in the field of new energy for the sustainable development in the future. The site selection is an important work which is prior to the construction of the wind farm, and also plays an important role in the wind farm's construction and the beneficial prediction in the future. In the micro-sitting selection of the large wind farm, the costs would be very high for the full usage of the CFD method to the simulations due to the lack of the computing hardware. In the field of wind powering projects, some wake models for the projects with acceptable numerical accuracy and the simple forms have very strong practicability¹. On account of the wake effect will affect the power outputs and the mechanical loads of the wind farm, a fine wake model can predict the fatigue loads and the performance of the wind farm layout² accurately and precisely. Jensen³ and Katic et al.⁴ proposed the Park model which is an empirical model with a simple form and the wake attenuation constant k is equal to the linear expansion rate of the wake. For the onshore wind turbine occasion, the wake attenuation constant k is taken as 0.075 or 0.05 for the offshore wind turbine, and take a half empirical value of $k = 0.5 / \ln(z / z_0)$ in case of the surface roughness of the wind turbine is considered in Ref. [5]. And Peña et al.⁶ put forward a new wake expansion coefficient as $k = \kappa / (\ln(h / z_0) - \psi_m(h / L))$ in the study of the atmospheric stability effects in the wake region. The Park model assumed that the wind velocity which was inconsistent with the actual situation had a constant distribution in the cross-wind direction in the wake region. By the test measurement⁷, the wake wind velocity was in multinomial distribution or a Gauss distribution in the cross-wind direction in the real flow field. In 1988, Ainslie⁸ put forward the Eddy-viscosity model for solving the Navier-Stokes equations based on the axis-symmetric coordinate system. Therefore, it is also considered to be a simple RANS model and the wake region is assumed to be the Gauss distribution. Yet the wind velocity loss was underestimated by the Park model and the Eddy-viscosity model in the wake region. For the remedy the drawback, recently, Brower et al.⁹ proposed the Deep-Array wake model based on the method considering internal boundary layers by the surface resistance. In 2011, Ott et al.¹⁰ reconstructed the Fuga model, which is a linear RANS model and uses the wind turbine of the actuator disk to simulate the wake region. Then Montavon et al.¹¹ gave the Wind Modeller model which adopted the $k - \varepsilon$ turbulence model and the actuator disk to simulate the wind turbine in 2011. Thereafter, Xue et al.¹² proposed the LES wake model with a very high demanding for the near wall grid of the real objects.

For the purpose of further improving the simulations' numerical accuracy, we would like to modify the initial wake radius on some occasions. In this paper, a new Park-Gauss model in conjunction with the assumption of the Park model whose wake region is in linear expansion and the assumption of the Eddy-viscosity model whose cross-wind is in Gauss distribution in the wake region is proposed in order to possess the characteristics of the high resolution, the simple form and easy-to-code fashion simultaneously maintaining the wind velocity conforming to the real flow field distribution in the wake region. This paper is organized as follows: we describe the details of the mathematical models

including the new Park-Gauss model in Section 2 and present some numerical tests and discussions in Section 3 to verify the advantages of this new approach. Concluding remarks are then given in Section 4.

2. Mathematical model

2.1. The Park model

The Park wake model is applied to the wind resource analysis software WAsP, which is proposed by Jensen^[3] based on the principle of the momentum conservation:

$$\pi r_0^2 v_0 + \pi (r_x^2 - r_0^2) u_0 = \pi r_x^2 u_*. \quad (0.1)$$

In the above formula, r_0 is the initial wake radius in the rotor; v_0 is the wind velocity at the initial wake radius; r_x is the wake radius from the disk x ; u_* is the wind velocity at the wake radius from the disk x ; and u_0 is the incoming wind velocity. Wherein, $r_x = kx + r_0$ and k is the rate of the expansion in the wake region. The Park model assumes the wake region is in a fully turbulent flow state while the tip vortex and the other phenomena are ignored. Therefore, the Park wake model is unable to accurately simulate the characteristics nearby the vicinities of the wake region, which applies to the transition region and the far wake region (the region behind $3D$ in the downstream of the wind turbine). In the literature³, it was believed that the initial wake radius r_0 equals to the rotor radius r_d . After taking into account the existence of the phenomenon such as the tip vortex, the initial radius is certainly greater than the rotor radius r_d . This paper modifies this condition and takes the wake radius close to the downstream of the rotor as the initial wake radius. According to the actuator disk theory, it can be deduced that:

$$u_* = (1 - 2a)u_0, \quad (0.2)$$

$$r_0 = r_d \sqrt{(1 - a) / (1 - 2a)}. \quad (0.3)$$

where a is the axial induced factor and r_d is the rotor radius. Substitute (2.2) and (2.3) into (2.1):

$$u_* = u_0 (1 - 2a / (1 + kx / r_0)^2). \quad (0.4)$$

2.2. The k model (rate of expansion)

When the surface roughness in which the wind turbine is previously installed is known, we can take the k value of 0.075 for the onshore wind turbine and 0.05 for the offshore wind turbine according to suggestions of Barthlmie et al⁷. If the surface roughness is able to be measured, the empirical formula proposed by Frandsen et al.⁵ could be adopted:

$$k = 0.5 / \ln(z / z_0). \quad (0.5)$$

In the formula, z is the hub height, and z_0 is the surface roughness.

How to determine the wake expansion coefficient k is an important problem for the Park model. Some scholars think that the assumption that k is directly set to be equal to a constant or just a constant related to the roughness is too idealistic¹⁴. Attenuation and

expansion characteristics of the wake in real wind farm relate to many factors, such as incoming turbulence intensity, additional wake turbulence intensity and the atmospheric stability, etc. There are three conditions for the atmospheric stability: unstable, neutral and stable. Unstable layer: the unstable boundary layer is also called the convective boundary layer, which is directly affected by surface forces, such as the frictional resistance, thermal transmission and evaporation. After sunrise, the earth's surface heats, the hot air rises and the cold air descends, the convection enhances gradually and various characteristics are mixed in a nearly homogeneous manner. The unstable atmosphere is in the mixing layer in which the turbulent convection mainly plays a leading role. Neutral layer: the neutral atmospheric boundary layer will be with a highest mixture around one or two o'clock in the afternoon since the troposphere develops very fast after sunrise. After sunset, the earth's surface stops heating which weakens the turbulent intensity within the troposphere. The unstable atmosphere originally will convert into the neutral atmosphere gradually. This is the residual for troposphere during the day, which is also known as the residual layer. Stable layer: as for the stable boundary layer, the earth's surface cools down in a long-wave radiation and becomes cold gradually after sunset. Then a temperature inversion forms at the surface and develops into ground inversion layer at night within which the atmosphere is very stable, so it is known as the stable boundary layer. The turbulence intensity is very wake within the stable layer.

A new wake expansion coefficient has been proposed after considering the effects of atmospheric stability on wake region⁶:

$$k = \kappa / (\ln(z / z_0) - \psi_m(z / L)). \quad (0.6)$$

In above formula, z is the hub height of the wind turbine, $\kappa=0.4$ is Von Kármán constant, L is the length of Monin-Obukhov and $\psi_m(z / L)$ is the diabatic correction function¹⁵.

$$\psi_m = \int_0^{z/L} (1 - \phi_m(\xi)) / \xi d\xi. \quad (0.7)$$

where $\phi_m(\xi)$ is the dimensionless wind shear function:

$$\phi_m = \begin{cases} (1 - a_1 z / L)^{p_1}, & -2 < z / L < 0, \\ 1, & \frac{z}{L} = 0, \\ 1 + b_1 z / L, & 0 < z / L < 1. \end{cases} \quad (0.8)$$

Emeis¹⁸ notes that it is unstable under $z / L < 0$, neutral under $z / L = 0$ and stable under $z / L > 0$. In order to facilitate the intuitive study of the influences of the atmospheric stability on the wake region, special nodal values of three states have been taken respectively: $z / L = -1$ represents the unstable state, $z / L = 0$ represents the neutral state and $z / L = 1$ represents the stable state. The expression of diabatic correction function $\psi_m(z / L)$ under unstable condition depends on the value of p_1 ^{16,17}:

$$\psi_m = 2 \ln((1 + x) / 2) + \ln((1 + x^2) / 2) - 2 \arctan(x) + \pi / 2, p_1 = 1 / 4. \quad (0.9)$$

$$\psi_m = 3/2 \ln((1+x+x^2)/3) - \sqrt{3} \arctan((2x+1)/\sqrt{3}) + \pi/\sqrt{3}, p_1 = 1/3. \quad (0.10)$$

In the formula above, $x = (1 - a_1 z / L)^{-p_1}$. Under the neutral condition, $\psi_m = 0$; Under the stable condition $\psi_m = -b_1 z / L$. Wherein, $a_1 = 19.3$, $b_1 = 6$, $c = 0.95$, and $p_1 = -1/4$, which are obtained by Höglström¹⁹ through long-term observation of the flat topography in Sweden.

2.3. The Park-Gauss model

The Park wake model assumes the wake velocity only varies from the distance of the wind direction x , and the wind velocity is a constant in the cross-wind direction. Since the cross-wind velocity is in hat shape distribution, the Park model is also known as the top-hat model. The Eddy-viscosity model assumes the wake is in symmetrical and similar Gauss distribution in the cross-wind direction. By the test measurement⁷, the wake wind velocity in real flow field is in multinomial distribution or Gauss distribution in the cross-wind direction. As a result, in combination with the Park model and the Eddy-viscosity model, we propose a new wake model that the wake region is in linear expansion with an expansion rate of k , and the wind velocity in the cross-wind direction is in symmetrical and Gauss distribution, known as the Park-Gauss model. Fig. 1 is the sketch of the Park model and the Park-Gauss model.

Fig. 1. Sketch of the Park model and Park-Gauss model.

The Park-Gauss model is assumed as:

$$u = A \exp(-((r - B) / r_x)^2) + C. \quad (0.11)$$

In the formula, A , B and C are undetermined coefficients, r_x is the wake radius at x in the downstream of the wind turbine, and r is the distance to rotor center in the cross-wind direction. Since the Park-Gauss model is proposed based on the assumption of the Park model and the assumption of Eddy-viscosity model, following assumptions are met:

- (1) The assumption that when the cross-wind distance r expands to the wake radius r_x , the wind velocity at D and E recovers to the incoming wind velocity u_0 .

$$A \exp(-((r_x - B) / r_x)^2) + C = u_0. \quad (0.12)$$

$$A \exp(-((-r_x - B) / r_x)^2) + C = u_0. \quad (0.13)$$

- (2) The assumption that the wake radius as well as mass flux (i.e. area covered by the velocity is identical) of both models are the same.

$$\int_{-r_x}^{r_x} A \exp(-((r - B) / r_x)^2) + C dr = 2r_x u_*. \quad (0.14)$$

By (2.13) and (2.14):

$$B = 0, A / e + C = u_0. \quad (0.15)$$

Substitute $B = 0$ into (2.16) and then lead to:

$$\int_{-r_x}^{r_x} A \exp(-(r / r_x)^2) dr + 2r_x C = 2r_x u_*. \quad (0.16)$$

Since that $\exp(-(r / r_x)^2)$ does not have a primitive function, here the approximate value of power series expansion is used to calculate.

$$\exp(-(r / r_x)^2) = 1 - r^2 / r_x^2 + r^4 / (2!r_x^4) - r^6 / (3!r_x^6) + O(r^8). \quad (0.17)$$

$$\int_{-r_x}^{r_x} \exp(-(r / r_x)^2) dr = \int_{-r_x}^{r_x} 1 - r^2 / r_x^2 + r^4 / (2!r_x^4) - r^6 / (3!r_x^6) dr. \quad (0.18)$$

$$52 / 35 r_x A + 2r_x C = 2u_* r_x. \quad (0.19)$$

Consider (2.15) and (2.19), A , B and C could be solved:

$$A = (u_0 - u_*) / (1 / e - 26 / 35), B = 0, C = u_0 - (u_0 - u_*) / (1 - 26e / 35). \quad (0.20)$$

Then substitute (2.22) into (2.13) and lead to:

$$u = (u_0 - u_*) / (1 / e - 26 / 35) \exp(-(r / r_x)^2) + u_0 - (u_0 - u_*) / (1 - 26e / 35). \quad (0.21)$$

It can be concluded that the Park-Gauss model mainly has two steps:

Prediction Step:

$$u_* = u_0 (1 - 2a / (1 + kx / r_0)^2),$$

Revision Step:

$$u = (u_0 - u_*) / (1 / e - 26 / 35) \exp(-(r / r_x)^2) + u_0 - (u_0 - u_*) / (1 - 26e / 35).$$

3. Numerical Tests

In this paper, we mainly test and verify the validity of the newly proposed Park-Gauss model by comparing with the Park model. The wind field test results with the initial wake radius $r_0 = r_d$ and the revised $r_0 = r_d \sqrt{(1-a)/(1-2a)}$ in conjunction with the condition of the wake expansion coefficient $k = 0.5 / \ln(z / z_0)$ are proposed in detail respectively.

Example 3.1. The Sexbierum wind farm measured data. The Sexbierum^{1,19} wind field has been arranged with 18 wind turbines with following parameters: the rated power =

310 kW; the rotor diameter $D = 30.1\text{m}$ (round to 30m); the hub height $z = 35\text{ m}$; the switch on speed $u_{in} = 5\text{ m/s}$; the rated speed $u = 14\text{ m/s}$ and the switch off speed $u_{out} = 20\text{ m/s}$. 7 meteorological masts have been installed around the wind field and used to measure the wind velocity, wind direction, air pressure and temperature, etc. In addition, the wake data has been measured particularly at $2.5D$, $5.5D$ and $8.0D$ in the downstream of the one of the wind turbines. The incoming wind velocity $u_0 = 8.5\text{ m/s}$, the wind turbine thrust coefficient $C_T = 0.75$ and the incoming turbulence intensity $I_0 = 10\%$. $F\text{-}M2.5D\text{-}S$ represents the cross-wind wake measured data at $2.5D$ in the downstream of the wind turbine in the Sexbierum wind field; $P\text{-}U2.5D\text{-}rd$ represents the cross-wind wake wind velocity of the Park model at $2.5D$ in the downstream of the wind turbine with $r_0 = r_d$; $P\text{-}G2.5D\text{-}rd$ represents the cross-wind wake wind velocity of the Park-Gauss model at $2.5D$ in the downstream of the wind turbine with $r_0 = r_d$; $P\text{-}U2.5D\text{-}r0$ represents the cross-wind wake wind velocity of the Park model at $2.5D$ in the downstream of the wind turbine with $r_0 = r_d \sqrt{(1-a)/(1-2a)}$; $P\text{-}G2.5D\text{-}U$, $P\text{-}G2.5D\text{-}N$ and $P\text{-}G2.5D\text{-}S$ represent the cross-wind wake wind velocity of the Park-Gauss model at $2.5D$ under the unstable neutral and the stable atmospheric conditions respectively.

Fig. 2. Cross-wind velocity distributions at $2.5D$ in the downstream of the Sexbierum wind turbine.

Fig. 3. Cross-wind velocity distributions at $5.5D$ in the downstream of the Sexbierum wind turbine.

Fig. 4. Cross-wind velocity distributions at 8.0D in the downstream of the Sexbierum wind turbine.

As for the Park model, the wake effect has been underestimated and the wind velocity is higher than the test data under the condition that the initial wake radius $r_0 = r_d$. At 2.5D, 5.5D and 8.0D in the downstream of the wind turbine, the wind velocity is 43.9%, 23.1% and 15.2% higher than the test data respectively. The underestimation of wake effect has been improved significantly under the condition that the modified wake radius $r_0 = r_d \sqrt{(1-a)/(1-2a)}$. At 2.5D, 5.5D and 8.0D in the downstream of the wind turbine, the wind velocity is 38.3%, 17.2% and 12.3% higher than the test data respectively. As for the Park-Gauss model, as well as the Park model, the wake effect has been underestimated under the condition that the initial wake radius $r_0 = r_d$. However, compared to the latter, the Park-Gauss model has made a great improvement in the wake prediction. Not only the trend of the cross-wind velocity is approximately in agreement with the test data, but also the underestimation of the wake effect has been improved significantly. At 2.5D, 5.5D and 8.0D in the downstream of the wind turbine, the cross-wind velocity is 10.5%, 9.5% and 7.0% higher than the test data respectively. Trend of cross-wind velocity of the Park-Gauss model is more close to the test data under the condition that the modified wake radius $r_0 = r_d \sqrt{(1-a)/(1-2a)}$. At 2.5D, 5.5D and 8.0D in the downstream of the wind turbine, the cross-wind velocity is almost consistent with the test data. The analysis shows the simulation precision of wind velocity in the wake region for the Park-Gauss model is very high under the condition of modified initial wake radius. The further discussion of the effects of atmospheric stability on wake is under the condition of the Park-Gauss model and the revised wake radius. The results are shown in Figs. 2-4 in details.

Fig. 5. Cross-wind velocity distributions at 2.5D in the downstream of the Sexbierum wind turbine under different atmospheric stability conditions.

Fig. 6. Cross-wind velocity distributions at 5.5D in the downstream of the Sexbierum wind turbine under different atmospheric stability conditions.

Fig. 7. Cross-wind velocity distributions at 8.0D in the downstream of the Sexbierum wind turbine under different atmospheric stability conditions.

At $2.5D$, $5.5D$ and $8.0D$ in the downstream of the wind turbine, the cross-wind velocity is almost consistent with the test data. Within the unstable atmospheric layer, the turbulence intensity is relatively strong, which facilitates the recovery of the wake velocity. At $2.5D$, $5.5D$ and $8.0D$ in the downstream of the wind turbine, losses of wake wind velocity have been overestimated for all, which is 33.6%, 31.8% and 25.2% lower than the test data, respectively. Within the unstable atmospheric layer, the turbulence intensity is very weak, which does not facilitate the recovery of wake wind velocity. At $2.5D$ in the downstream of the wind turbine, the cross-wind velocity is also very close to the test data. At $5.5D$ and $8.0D$ in the downstream of the wind turbine, losses of wake wind velocity have both been overestimated, which is 8.4% and 4.1% less than the test data respectively. Within the neutral atmospheric layer, the turbulence intensity is between stable and unstable, so the wind velocity recovery in wake region falls in between. The results are shown in Figs. 5-7 in details.

Example 3.2. The Garrad Hassan wind tunnel experimental data. In 1989, the Garrad Hassan Company has conducted a series of wind tunnel experiments for wind turbine wake²⁰. Objects selected for the test are horizontal axis wind turbine models with a rotor diameter of 0.27m. The compression ratio is 1:160, the corresponding simulative wind turbine is with a rotor diameter $D = 43.2$ m and a hub height $z = 50$ m, and the operation environment is a flat terrain whose surface roughness is 0.075 m and area is $2\text{km} \times 2\text{km}$. The incoming wind velocity $u_0=5.3\text{m/s}$, the tip speed ratio $\lambda_1=2.9$ and the thrust coefficient $C_T=0.62$.

Fig. 8. Cross-wind velocity distributions at 5.0D in the downstream of the Garrad Hassan wind turbine.

Fig. 9. Cross-wind velocity distributions at 7.5D in the downstream of the Garrad Hassan wind turbine.

Fig. 10. Cross-wind velocity distributions at 10D in the downstream of the Garrad Hassan wind turbine.

As for the Park model, the wake effect has been underestimated and the wind velocity is higher than the test data under the condition that the initial wake radius $r_0 = r_d$. At 5.0D, 7.5D and 10D in the downstream of the wind turbine, the cross-wind velocity is 19.7%, 10.0% and 2.1% higher than the test data respectively. The underestimation of wake effect has been improved significantly under the condition that the modified wake radius $r_0 = r_d \sqrt{(1-a)/(1-2a)}$. At 5.0D, 7.5D and 10D in the downstream of the wind turbine, the cross-wind velocity is 17.5%, 8.1% and 0.6% higher than the test data respectively. As for the Park-Gauss model, under the condition that the initial wake radius $r_0 = r_d$, the Park-Gauss model has made a great improvement in the wake prediction. Not only trend of the cross-wind velocity is approximately in agreement with the test data, but also the underestimation of wake effect has been improved significantly. At 5.0D and 7.5D in the downstream of the wind turbine, the cross-wind velocity is slightly higher than the test data, and at 10D, it is slightly lower than the test data. Trend of cross-wind velocity of the Park-Gauss model is more close to the test data under the

condition that the modified wake radius $r_0 = r_d \sqrt{(1-a)/(1-2a)}$. At $5.0D$ and $7.5D$ in the downstream of the wind turbine, the cross-wind velocity is almost consistent with the test data, while at $10D$, it is slightly lower than the test data. The results are shown in Figs. 8-10 in details.

Fig. 11. Cross-wind velocity distributions at $5.0 D$ in the downstream of the Garrad Hassan wind turbine under different atmospheric stability conditions.

Fig. 12. Cross-wind velocity distributions at $7.5 D$ in the downstream of the Garrad Hassan wind turbine under different atmospheric stability conditions.

Fig. 13. Cross-wind velocity distributions at 10 D in the downstream of the Garrad Hassan wind turbine under different atmospheric stability conditions.

At $5.0D$ and $7.5D$ in the downstream of the wind turbine, the cross-wind velocity is approximately consistent with the test data, while at $10D$, it is slightly lower than the test data. Within the unusable atmospheric layer, the turbulence plays a leading role which facilitates the recovery of wake wind velocity and can improve the output of wind energy for the wind turbine in the downstream wake region. At $5.0D$, $7.5D$ and $10D$ in the downstream of the wind turbine, losses of wake wind velocity have all been overestimated, which is 21.9%, 23.7% and 25.5% lower than the test data respectively. Within this atmospheric layer, the turbulence intensity is weak and the wind velocity in wake region recovers slowly. Therefore, the output of wind energy for the wind turbine in the downstream wake region could be reduced. At $5.0D$, $7.5D$ and $10D$ in the downstream of the wind turbine, losses of wake wind velocity have all been overestimated, which is 6.0%, 8.4% and 11.6% lower than the test data respectively. The results are shown in Figs. 11-13 in details.

4. Concluding Remarks

In this paper, a new Park-Gauss model based on the assumption of the Park model and the Eddy-viscosity model is investigated to conduct the wake numerical simulation for the single wind turbine problem. The initial wake radius has been modified to improve the new model's numerical accuracy. Then the impact of the atmospheric stability based on the Park-Gauss model has been studied in the wake region. By the comparisons and the analyses of the test examples, we can obtain these conclusions: comparing with the condition of $r_0 = r_d$, the modified initial wake radius $r_0 = r_d \sqrt{(1-a)/(1-2a)}$ could significantly improve the wake prediction accuracy. The proposed new Park-Gauss model could simulate the wind velocity in the wake region precisely, and meanwhile the cross-wind velocity trend and value are nearly consistent with the test results in the wake region. When in the face of the unstable atmosphere condition, it is helpful for the recovery of the wake wind velocity for the turbulence intensity is relatively strong in air.

When the special node $z/L = -1$ is inferior to the unstable atmosphere, the cross-wind velocity is relatively close to the test data. It is not helpful for the recovery of the wind velocity in the wake region on account of the turbulence intensity is weak in air in the stable atmosphere. When the special node $z/L = 1$ is under the stable atmosphere, the cross-wind velocity is lower than the test data since the wind velocity recovers relatively slowly and the loss of velocity is unable to recover in time in the wake region. If it is in the neutral atmosphere, the wind velocity recovery falls in between the stable and unstable statuses in the wake region. The wind velocity is slightly lower than the test data on condition that $z/L = 0$ in the wake region.

Acknowledgement

The research was partially supported by the National Basic Research Program of China ("973" Program) grant 2014CB046200, NSFC grants 91130030, 11372005, BK2014059, and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

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