AN IMPROVED MOMENTUM-EXCHANGED IMMERSED BOUNDARY-BASED LATTICE BOLTZMANN METHOD FOR INCOMPRESSIBLE VISCOUS THERMAL FLOWS

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An improved momentum-exchanged immersed boundary-based lattice Boltzmann method (MEIB-LBM) for incompressible viscous thermal flows is presented here. MEIB-LBM was first proposed by Niu et al.², which has been shown later^{3,4} that the non-slip boundary condition is not satisfied. Wang. et al.² and Hu. et al.³ overcome this drawback by iterative method. But it needs to give an appropriate relaxation parameter. In this work, we come back to the intrinsic feature of LBM, which uses the density distribution function as a dependent variable to evolve the flow field, and uses the density distribution function correction at the neighboring Euler mesh points to satisfy the non-slip boundary condition on the immersed boundary. The same idea can also be applied to the thermal flows with fluid-structure interference. The merits of present improvements for the original MEIB-LBM are that the intrinsic feature of LBM is kept and the flow penetration across the immersed boundaries is avoided. To validate the present method, examples, including forced convection over a stationary heated circular cylinder for heat flux condition, and natural convection with a suspended circle particle in viscous fluid, are simulated. The streamlines, isothermal contours, the drag coefficients and Nusselt numbers are calculated and compared to the benchmark results to demonstrate the effective of the present method.

Keywords: Fluid-structure interaction, heat transfer characteristics, moving boundary.

1. Introduction

In the past years, lattice Boltzmann method (LBM) has received growing attentions for its simple concept and easy implementation. For solving boundary condition, the immersed boundary method (IBM) is one of the most promising methods currently. IBM is first introduced by Peskin¹ in 1970s to investigate the blood flows problem in the heart.

Niu et al.² proposed the idea of momentum-exchange LBM for moving boundary problems. The idea indicates that the fluid-structure interaction is simulated by calculating the difference of the momentum, which is the summation of the density

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distribution timing its discrete velocity. However, it has been shown by many later researchers^{3,4} that there is a flow penetration on the immersed boundary.

In this work, we present an improved momentum-exchanged immersed boundary-based lattice Boltzmann method, using the density distribution function as a dependent variable to evolve the flow field, and the density distribution function correction at the neighboring Euler mesh points to satisfy the non-slip boundary condition on the immersed boundary. The same idea of the present work has been applied to the thermal flows. Force convection over a stationary isoflux circular cylinder, and natural convection with a buoyant circle particle in viscous fluid are tested. The present results show a good agreement with those in the literature.

2. Method

2.1. Lattice Boltzmann method

Lattice Boltzmann equation coupled double distribution functions model based on Boussinesq approximation in original MEIB-LBM can be expressed as:

$$f_\alpha(\mathbf{r} + \mathbf{e}_\alpha \delta_t, t + \delta_t) = f_\alpha(\mathbf{r}, t) - \frac{f_\alpha(\mathbf{r}, t) - f_\alpha^{eq}(\mathbf{r}, t)}{\tau_f} + F_\alpha \delta t. \quad (1)$$

$$g_\alpha(\mathbf{r} + \mathbf{e}_\alpha \delta_t, t + \delta_t) = g_\alpha(\mathbf{r}, t) - \frac{g_\alpha(\mathbf{r}, t) - g_\alpha^{eq}(\mathbf{r}, t)}{\tau_g} + Q_\alpha \delta t. \quad (2)$$

where f_α and g_α are the density and temperature distribution function, respectively, f_α^{eq} and g_α^{eq} are the density and temperature equilibrium distribution function, respectively. $\mathbf{r}$ is the spatial vector, δ_t is the time step, τ_f and τ_g are the dimensionless relaxation times for velocity and temperature, respectively. The equilibrium distribution function is:

$$f_\alpha^{eq}(\mathbf{r}, t) = \omega_\alpha (\rho + \rho_0 (\frac{3}{c^2} (\mathbf{e}_\alpha \cdot \mathbf{u}) + \frac{9}{2c^4} (\mathbf{e}_\alpha \cdot \mathbf{u})^2 - \frac{3}{c^2} \mathbf{u}^2)). \quad (3)$$

$$g_\alpha^{eq}(\mathbf{r}, t) = \omega_\alpha T [1 + \frac{3}{c^2} (\mathbf{e}_\alpha \cdot \mathbf{u}) + \frac{9}{2c^4} (\mathbf{e}_\alpha \cdot \mathbf{u})^2 - \frac{3}{c^2} \mathbf{u}^2]. \quad (4)$$

2.2. Velocity and temperature correction by an improved MEIB-LBM

The profile of boundary is denoted by a series of Lagrangian points, and the flow field includes both internal and external of the solid object which is represented by a set of Euler mesh points. The interaction between the immersed boundary and the flow can be simulated by calculating the distribution function change generated by boundary onto the flow. The distribution function on boundary points can be approximated by following

$$f_\alpha(X_l, t) = \sum_{ij} f_\alpha(\mathbf{r}_{ij}, t) D_{ij}(\mathbf{r}_{ij} - \mathbf{X}_l) h^2 \quad (5)$$

$$g_\alpha(X_l, t) = \sum_{ij} g_\alpha(\mathbf{r}_{ij}, t) D_{ij}(\mathbf{r}_{ij} - \mathbf{X}_l) h^2 \quad (6)$$

where i, j are the Euler mesh points in x and y -directions, respectively; $f_{\alpha}(X_l, t)$ and $g_{\alpha}(X_l, t)$ are the distribution functions of Lagrangian points on the immersed boundary, respectively; $f_{\alpha}(r_{ij}, t)$ and $g_{\alpha}(r_{ij}, t)$ are distribution functions in Euler mesh points, respectively, and h is equal to the space of the mesh. $D_{ij}(r_{ij}, X_l)$ is delta function. The density $\rho(X_l, t)$ and temperature $T(X_l, t)$ of solid boundary can be computed as

$$\rho(X_l, t) = \sum f_{\alpha}(X_l, t) \quad (7)$$

$$T(X_l, t) = \sum g_{\alpha}(X_l, t) + \left(-\frac{Q(X(S), t)\delta_l}{\rho c_p}\right) \quad (8)$$

To satisfied the non-slip boundary condition, we must substituting boundary condition of velocity $\mathbf{u}_b$, $T(X_l, t)$ temperature in the boundary into Eqs. (3) and (4) to get the equilibrium distribution function in Lagrangian points. To obtain the non-equilibrium distribution function in Lagrangian points, difference of the distribution function and equilibrium distribution function in Lagrangian points is computed, using the bounce back rules, which is

$$f_{\alpha}(X_l, t + \delta_l) = f_{\bar{\alpha}}(X_l, t) - f_{\bar{\alpha}}^{eq}(X_l, t). \quad (9)$$

$$g_{\alpha}(X_l, t + \delta_l) = -(g_{\bar{\alpha}}(X_l, t) - g_{\bar{\alpha}}^{eq}(X_l, t)). \quad (10)$$

Obtaining a new set of non-equilibrium part of distribution functions on the boundary points and further projected on the neighbouring Euler points for reflecting the boundary effects as following

$$f_{\alpha, b}^{non-eq}(r_{ij}, t) = \sum_l f_{\alpha}^{non-eq}(X_l, t) D_{ij}(\mathbf{r}_{ij} - \mathbf{X}_l) \Delta s_l. \quad (11)$$

$$g_{\alpha, b}^{non-eq}(r_{ij}, t) = \sum_l g_{\alpha}^{non-eq}(X_l, t) D_{ij}(\mathbf{r}_{ij} - \mathbf{X}_l) \Delta s_l. \quad (12)$$

where Δs_l is the arc length of the boundary element. The macroscopic density ρ , velocity $\mathbf{u}$ and temperature T can be corrected as

$$\rho(r_{ij}, t) = \sum_{\alpha} f_{\alpha, ij} + 0.5 \delta_l \sum_{\alpha} f_{\alpha, b}^{non-eq}(r_{ij}, t). \quad (13)$$

$$\rho \mathbf{u} = \sum_{\alpha} f_{\alpha, ij} \mathbf{e}_{\alpha} + 0.5 \delta_l \sum_{\alpha} f_{\alpha, b}^{non-eq}(r_{ij}, t) \mathbf{e}_{\alpha}. \quad (14)$$

$$T(r_{ij}, t) = \sum_{\alpha} g_{\alpha, ij} + 0.5 \delta_l \sum_{\alpha} g_{\alpha, b}^{non-eq}(r_{ij}, t). \quad (15)$$

The LBM equation of the improved momentum-exchanged immersed boundary-based lattice Boltzmann method can be expressed as:

$$f_{\alpha}(\mathbf{r} + \mathbf{e}_{\alpha} \delta_l, t + \delta_l) = f_{\alpha}(\mathbf{r}, t) - \frac{f_{\alpha}(\mathbf{r}, t) - f_{\alpha}^{eq}(\mathbf{r}, t)}{\tau_f} + \omega_{\alpha} \delta_l \left(1 - \frac{1}{2\tau_f}\right) (f_{\alpha, b}^{non-eq}(\mathbf{r}, t) + \left(\frac{3(\mathbf{e}_{\alpha} - \mathbf{u})}{c^2} + \frac{9\mathbf{u}\mathbf{e}_{\alpha}}{c^4} \mathbf{e}_{\alpha}\right) G). \quad (16)$$

$$g_a(\mathbf{r} + \mathbf{e}_a \delta_t, t + \delta_t) = g_a(\mathbf{r}, t) - \frac{g_a(\mathbf{r}, t) - g_a^{eq}(\mathbf{r}, t)}{\tau_g} + \omega_a \delta_t \left(1 - \frac{1}{2\tau_g}\right) g_{a,b}^{non-eq}(\mathbf{r}, t). \quad (17)$$

where f^{non-eq} and g^{non-eq} is correction of the non-equilibrium part of the distribution function nearly the Lagrangian points. G is buoyancy of Boussinesq approximation. The important advantage of this method is that it avoids introducing the forcing term and heat source into the momentum equations, which is generated by the boundary onto the fluid. Instead, the non-equilibrium part of the distribution function in Lagrangian points is calculated to obtain the macro quantity near the boundary.

3. Numerical results and discussions

3.1. Heat flux condition for forced convection over a stationary circular cylinder

In this section, forced convection over a stationary circular cylinder with a constant heat flux boundary condition is considered. The flow behavior and isotherm patterns is related to the Reynolds number $Re = \rho u_\infty D / \mu$ and the Prandtl number $Pr = \mu c_p / k$, where ρ , u_∞ and D means the fluid density, free stream velocity, and cylinder diameter respectively, μ , c_p and k is the dynamic viscosity, specified heat and the thermal diffusivity, respectively. In present study, numerical investigations are carried out for $Re=20$, 40 and $Pr=0.7$. Drag coefficient Cd , streamlines and isotherms, average Nusselt number Nu on the boundary are listed.

The domain size is $40D \times 30D$ channel, in middle of which stands stationary a circular cylinder with a constant heat flux condition $-\partial T / \partial y = 1.0$, and $\partial u / \partial y = \partial T / \partial y = 0$ is defined for the top and bottom wall. The cylinder is $15D$ far away from the inlet, which has $\mathbf{u} = (u_\infty, 0)$ and $T=0$. At the outlet, the homogeneous Neumann boundary condition is applied, which is $\partial u / \partial x = \partial T / \partial x = 0$.

Table 1 Comparison of the Cd at $Re=20$ and 40

References	Re=20	Re=40
Ren et al. [5]	2.126	1.568
Niu et al. [2]	2.144	1.589
Hu et al. [3]	2.213	1.660
Present	2.261	1.647

Table 2 Comparison of Nu at $Re=20$ and 40

References	Re=20	Re=40
Ren et al [5]	2.7413	3.7407
Ahmad et al [4]	2.6620	3.4720
Dhiman et al [6]	2.8630	3.7930
Present	2.8414	3.6872

Fig. 1. Streamlines at $Re=20$ and 40

Fig. 2. Isotherm at $Re=20$ and 40 .

Figure 1 shows that penetration of streamlines does not happen near the boundary, meaning that the non-slip boundary condition is well satisfied. The outcome indicated that present method improves the accuracy of the boundary. The drag coefficient C_d is listed in Table 1, agreeing well with the literature. Figure 2 shows the isotherms nearby the cylinder boundary for $Re=20$ and 40 . It is obviously to found that the isotherms in the front surface of the cylinder are dense, while the rear surface is relatively sparse. Table 2 shows the average Nu on the cylinder surface, according well with reference data.

3.2. Natural convection with a neutrally buoyant particle in viscous fluid for moving boundary

Natural convection has been investigated by many researchers⁷⁻⁹. We extended it to investigate the case in a square cavity of natural convection with a suspended circle particle in viscous fluid. The grid is taken as 200×200 . In the position of $(0.5, 0.9)$ stands a neutrally particle, with the isoflux of $-\partial T / \partial n = 5.0$, density of $\rho = 1.005$, and diameter of $d = 0.05$. For natural convection, the flow behavior is related to the Prandtl number $Pr = \nu / \chi$ and Rayleigh number $Ra = g\beta\Delta TPrh^3/\nu^2$. Where ρ , ν , χ , g , β , and h means the fluid density, kinematic viscosity, thermal diffusivity, acceleration of gravity, thermal expansion coefficient, reference length respectively, ΔT is the difference of high and low temperature wall. In present case, $Pr = 0.71$. The Boussinesq approximation is applied.

Fig. 3. Isothermal at $Ra=10^3$ with a moving particle.

Fig. 4. Isotherm at $Ra=10^4$ with a moving particle.

Fig. 5. Isotherm at $Ra=10^5$ with a moving particle.

Figures 3 to 5 show the temperature with a moving particle when the flow evolve at the time from 0 second to 45 seconds with the $Ra=10^3, 10^4, 10^5$ separately. As we can see, the particle fall in the lower right, move to the left side in the corner, then rise to the upper left, and finally travel to the top right. And move again and again with the similar trajectory. Fig. 6 shows the routes in $Ra=10^3, 10^4, 10^5$. The particle has a different track as the change of Rayleigh number, which can be observed in Fig. 6. It can be seen that when the $Ra=10^3$, the trajectory of the particle is nearly a circular. However, when at $Ra=10^4, 10^5$, the particles are gradually shrinking their routes and become stable after a time.

Fig. 6. Position trajectory at $Ra=10^3, 10^4$ and 10^5 .

Fig. 7. Force in x and y direction of the particle at $Ra=10^3, 10^4$ and 10^5 .

Fig. 8. Kinetic energy at $Ra=10^3, 10^4$ and 10^5 .

Table 3 Comparison of Nu on cold wall with a particle

References	10^3	10^4	10^5
Peng et al. [7]	1.117	2.241	4.511
Wang et al. [8]	1.115	2.232	4.491
Shu et al.[9]	1.118	2.245	4.523
Present	1.268	2.416	4.707

Figure 7 shows the force in x and y directions acting on the particles varied with time. The force changes at first, and finally becomes periodic change when the particle reaches a steady state at every case. At $Ra=10^4$, the amplitude of oscillation is larger and the circling period is smaller than other two cases. The possible reason is due to the gradient of temperature greater.

Figure 8 shows the kinetic energy in different Rayleigh numbers, which shows that when $Ra=10^4$, the oscillation amplitude is greatest, which agrees well with the force and motion behavior.

Table 3 lists the Nusselt number at $Ra=10^3, 10^4, 10^5$, showing good agree with those in referent literature⁷⁻⁹. It is easy to found that Nu becomes greater as the increase of Ra . Compare the Nu in the same Ra , the Nu improve in present method, indicating that the heated particle can improve heat transfer characteristic of the natural convection.

4. Conclusions

An improved momentum-exchanged immersed boundary-based lattice Boltzmann method for incompressible viscous thermal flows is presented. We come back to the intrinsic feature of LBM, using the density distribution function as a dependent variable to evolve the flow field. This method has been applied to simulated static boundary of forced convection over a stationary isoflux circular cylinder, natural convection with a buoyant circle particle moving in viscous fluid. Note that the results validate its capability and efficiency to simulation of incompressible viscous thermal flows.

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