Recognition of frequency-modulated signals using the Wigner distribution

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Precision and resolution ability of the frequency-modulated signals time-frequency distributions at the formation of these distributions with the help of pseudo-Wigner processors is investigated. Linear-frequency modulated signals and signals with frequency dependence on time to the higher than the first power are considered. The results of the numerical simulation are presented.

Keywords: Time-frequency distribution; acousto-optical spectrum analyzer; spectrogram; pseudo-Wigner processor; analysis and recognition of similar signals.

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1. Introduction

Time-frequency distributions (TFD), demonstrating the frequency change on time, are used during the medium remote probing, determination of parameters of signals, received from the cosmic space among them, their demodulation and recognition.¹ The sensitivity of methods, using TFD depends upon the precision of forming these distributions and their resolution at the time-frequency plane.

The application of high-speed acousto-optic (AO) devices, such as spectrum analyzer are promising for the broadband signals TFD formation. On the basis of the spectrum analyzer, forming TFD as spectrograms or short-time Fourier transform several devices were constructed for the demodulation and determination of the radiosignal parameters.²⁻⁴ The spectrum analyzer are characterized by the simplicity, but at the high speed tuning the spectrum expansion along the frequency

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axes take place, that can cause the decrease of the precision and resolution at the frequency-time plane. As another acousto-optical device for the TFD formation a pseudo-Wigner distribution processor was suggested.⁵ Experimental investigations of the system showed the possibility to get the considerably greater resolution of the processing signals with the linear frequency modulation on comparison with the spectrum analyzer. It is connected with the compensation of the square phase factor at the pseudo-Wigner processor aperture, that is not compensated at the spectrum analyzer and leads to the defocusing of the output signal.^{6, 7}

Besides the LFM signals it would be interesting to consider the operation of the pseudo-Wigner processor for the signals with the frequency modulation different from the linear law. At the non-linear frequency modulation law the complete compensation of phase factors at the aperture of the pseudo-Wigner processor doesn't take place and it is necessary to estimate the distortions of TFD in that case. The results of the comparative analyzes of precision and resolution ability of the AO pseudo-Wigner processor and that of the spectrum analyzer at the TFD formation, in particular for frequency modulated signal with frequency change proportional to time the second and higher powers are given below.

2. Theoretical Analysis

While comparing two systems we will proceed from the traditional spectrum analyzer including the acoustooptical modulator (AOM), illuminated by the plane light wave and the integrating lens. The pseudo-Wigner processor arrangement for the analysis to be carried out will be represented in the simplified variant consisting of the integrating lens and two AOM's located in front of it with the opposite directions of the ultrasound propagation, with the analyzed signal introduced to their inputs. It can be shown in Ref. 7, that such a scheme arrangement, that is more compact in comparison with the suggested in Ref. 5, can be used for many types of signals.

Momentary lights intensity at the outputs of considered optical processors as the function of time and spatial frequency is proportional to the following quantities:

$$|S_{sp}(t, \omega_x)|^2 = \left| \int_{-\frac{x}{2}}^{\frac{x}{2}} f\left(t + \frac{x}{v}\right) \exp(-j\omega_x x) dx \right|^2. \quad (1)$$

For the processor, forming the pseudo-Wigner distribution:

$$|\tilde{W}_{sp}(t, \omega_x)|^2 = \left| \int_{-\frac{x}{2}}^{\frac{x}{2}} f\left(t + \frac{x}{v}\right) f^*\left(t - \frac{x}{v}\right) \exp(-j\omega_x x) dx \right|^2, \quad (2)$$

where t is time; ω_x is the spatial frequency, proportional to the coordinate x_1 at the output plane of the optical processor ($\omega_x = 2\pi x_1/\lambda F$); λ is the light wave length; F — the focal length of the Fourier transforming lens; X is the width of the input aperture processor; $f(t)$ is the analyzed signal, that is applied to the inputs of the

AOM; x is the spatial coordinate at the input plate; v — the propagation velocity of ultrasound in AOM.

The output signals of the processors (1) and (2) can be represented as the function of time and frequency, i.e. as the time-frequency distribution (TFD):

$$|S(t, \omega)|^2 = \left| \int_{-\frac{x}{2v}}^{\frac{x}{2v}} f(t + \tau) \exp(-j\omega\tau) d\tau \right|^2, \quad (3)$$

$$|\tilde{W}(t, \omega)|^2 = \left| \int_{-\frac{x}{2v}}^{\frac{x}{2v}} f(t + \tau) f^*(t - \tau) \exp(-j\omega\tau) d\tau \right|^2. \quad (4)$$

As follows from the expressions (3) and (4) TFD function $f(t)$ in both arrangements are equal to the square modulus of the short-time Fourier transform along the τ coordinate: for the spectrum analyzer of the function $f(t + \tau)$ and for the pseudo-Wigner processor of the function $f(t + \tau)f^*(t - \tau)$.

Let us consider the cases of TFD derivation for the analytical signals with the quadratic phase dependence (LFM signal) and other signals, whose frequency changes are proportional to time to the higher power, than that for the LFM signals.

For the LFM signal we have:

$$f(t) = \exp \left(j \left(\omega_0 t + \frac{at^2}{2} \right) \right), \quad (5)$$

$$f(t + \tau) = \exp \left(j \left(\omega_0 t + \omega_0 \tau + \frac{at^2}{2} + at\tau + \frac{a\tau^2}{2} \right) \right), \quad (6)$$

$$f(t + \tau)f^*(t - \tau) = \exp(j2(\omega_0 + at)\tau). \quad (7)$$

The signal at the output of the spectrum analyzer can be get from the expression (3):

$$|S(t, \omega)|^2 = \left| \int_{-\frac{x}{2v}}^{\frac{x}{2v}} \exp(j(\omega_0 + at - \omega)\tau) \exp\left(j\frac{a\tau^2}{2}\right) d\tau \right|^2. \quad (8)$$

For the pseudo-Wigner processor distribution we have:

$$|\tilde{W}(t, \omega)|^2 = \left| \int_{-\frac{x}{2v}}^{\frac{x}{2v}} \exp(j[2(\omega_0 + at) - \omega]\tau) d\tau \right|^2. \quad (9)$$

It follows from (9), that at the output of the pseudo-Wigner processor the TFD is formed, the cross section of which along the frequency coordinate is described by the apparatus function $(\sin(X\omega/2\nu)/(X\omega/2\nu))^2$ with the width that is determined by the memory time of the AOM — X/ν . The position of the apparatus function center along the frequency axis, at any moment of time coincides with the double expected value of the current signal frequency, that can be determined for the analytical signal

$$f(t) = \exp(j\varphi(t)), \quad (10)$$

as the derivative of phase

$$\omega(t) = \frac{d\varphi(t)}{dt}. \quad (11)$$

For the LFM signal (5) one can get from (11) the value of the current signal frequency at the moment t : $\omega(t) = \omega_0 + at$. The current frequency doubling appearing due to the double light diffraction in the pseudo-Wigner processor is a positive phenomenon, showing the greater processor sensitivity to frequency changes.

The spectrum analyzer, in accordance with the expression (8) gives the expanded output frequency distribution, equal to the convolution along the frequency coordinate of the required TFD with the apparatus function, defocused due to the quadratic phase factor at the integral symbol. This frequency expansion enlargement of the output distribution is symmetrical with respect to the frequency, so it can be taken into account during the secondary signal processing after the optical processor. But additional time is necessary for this processing, and besides this, the output distribution defocusing leads to the decrease of the signal intensity, and correspondingly to the signal/noise ration decrease, and as a result to the decrease of sensitivity and exactness of the current frequency determination. The resolution ability for the LFM signal, connected with the width of frequency TFD cross-section, can be essentially smaller for the spectrum analyzer because of the defocusing than for the pseudo-Wigner processor. The TFD cross-section width in case of the pseudo-Wigner processor will be smaller than that of the spectrum analyzer not only along the frequency axis, but also along the time axis because they are connected by the direct proportionality through the rate of the frequency rise.⁷ It allows to get the increased resolution along both coordinate using the pseudo-Wigner at the LFM signals processing.

It is necessary to mention that at the resolution of two signals with the help of pseudo-Wigner processor the TFD of the cross modulation signal appears at the processor output with the current frequency equal to half of the sum of current frequencies of resolving signals. Due to this fact the pseudo-Wigner processor resolution ability appears to be two times smaller, but thanks to the doubled sensitivity to the frequency change, its resolution ability still remains not worse than that of the spectrum analyzer, even of not to take into account the spectrogram enlarging because of defocusing. And if we take into consideration the signal enlarging in the spectrum analyzer the resolution ability of the pseudo-Wigner processor appears to be greater.

Now let us consider the operation of two systems for the case when the phase of the analytic signal is proportional to the time to the power higher than second.

For the cubic time-dependence phase law the phase dependence of the analytic signal $f(t) = \exp(j\varphi(t))$ is expressed in the following way:

$$\varphi(t) = \omega_0 t + \frac{a_2 t^2}{2} + \frac{a_3 t^3}{3}. \quad (12)$$

With the help of (11) we get the formula for the signal current circular frequency

$$\omega(t) = \omega_0 + a_2 t + a_3 t^2. \quad (13)$$

Then for the given signal we obtain

$$f(t+\tau) = \exp \left(j \left[\omega_0 t + \omega_0 \tau + \frac{a_2 t^2}{2} + a_2 t \tau + \frac{a_2 \tau^2}{2} + \frac{a_3}{3} (t + 3t^2 \tau + 3t \tau^2 + \tau^3) \right] \right),$$

$$f(t+\tau)f^*(t-\tau) = \exp \left(j 2 \left[(\omega_0 + a_2 t) \tau + \frac{a_3}{3} (3t^2 \tau + \tau^3) \right] \right). \quad (14)$$

Using these expressions from (3), (4) we can find the output signals for the spectrum analyzer and the pseudo-Wigner analyzer:

$$|S(t, \omega)|^2 = \left| \int_{-\frac{\chi}{2v}}^{\frac{\chi}{2v}} \exp(j(\omega_0 + a_2 t + a_3 t^2 - \omega)\tau) \times \exp \left(j \left[\left(\frac{a_2}{2} + a_3 t \right) \tau^2 + \frac{a_3}{3} \tau^3 \right] \right) d\tau \right|^2, \quad (15)$$

$$|\tilde{W}(t, \omega)|^2 = \left| \int_{-\frac{\chi}{2v}}^{\frac{\chi}{2v}} \exp(j[2(\omega_0 + a_2 t + a_3 t^2) - \omega]\tau) \exp \left(j \frac{2a_3}{3} \tau^3 \right) d\tau \right|^2. \quad (16)$$

It can be seen from expressions (15) and (16) that the outputs of both processors TFD are formed, whose cross-sections along the frequency coordinates are displaced with respect to the current frequency by the quantity, that is determined by the cubic phase error at the processor aperture (that is the factor $\exp(j\frac{a_3}{3}\tau^3)$ for the spectrum analyzer and the factor $\exp(j\frac{2a_3}{3}\tau^3)$ for the pseudo-Wigner processor). In contradistinction to pseudo-Wigner processor the spectrogram cross-section is enlarged again because of defocusing, and besides the quadratic phase factor leading to enlarging, one more term with the quadratic phase is added, whose magnitude increases proportional to time to the first power.

If we examine the signals with the non-linear phase proportional to the polynomial with the dependence on time to the higher powers, then additional phase factors, distorting the processor apparatus function will appear in the integrand expressions for the output signals of the spectrum analyzer and pseudo-Wigner processors, and the phase factors, corresponding to the even powers of integrating variable τ in the expression for the pseudo-Wigner processor will be compensated whereas they will cause the additional output signal enlargement along the frequency axis for the spectrum analyzer. This enlargement will increase in the course of time, the more power of the signal phase non-linearity the more will be this increase, because time t will be present in the distortion phase factors to the power increasing with the increase of the non-linearity of the signal phase (for the LFM signal this factor is absent).

Phase factors with odd power of the integrand variable will cause the displacement of the apparatus function maximum along the frequency axis for both

processors. The relative magnitude of this displacement will be approximately of the same order, taking into account the double maximum phase error in the aperture of the pseudo-Wigner processor with its double sensitivity to the change of the signal current frequency. Factors with the higher indices of the integrand variable, that appear in case of signals, containing non-linearity of higher orders, will contain besides that time to the more power the more the non-linearity power of the signal will be. That is why the distortion for the signals with the higher power of the non-linearity will increase with the greater rate for the greater length of these signals.

3. Results of Numerical Simulation

In order to compare the influence of the frequency modulation laws upon the TFD, formed by the spectrum analyzer and pseudo-Wigner processor we performed the numerical calculations of the TFD cross-section along the frequency axis. The problem of the determination of the LFM signal current frequency with the $50\text{MHz}/\mu\text{s}$ frequency rate change, was considered, that is typical for the wide-band systems. In addition to the basic LFM signals have the short time frequency running out according to one of three laws of the non-linear frequency modulation. The maximum frequency running out during the AOM memory time equal to $2\tau_{\text{max}} = 1\mu\text{s}$, was equal to 10% of the LFM signal frequency change, i.e. to 5 MHz.

In our calculations signals with signals with the frequency modulation contained only the higher term of the non-linear phase i.e. they have the following form:

$$\exp\left(ja_{n+1}\frac{t^{n+1}}{n+1}\right), \quad n = 1, 2, 3, 4. \quad (17)$$

Such a simplified law of the frequency running out permitted to determine the coefficient a_{n+1} , necessary for the distortion calculation, for the given maximum value of the frequency running out, and estimate independently the influence of different powers of non-linearity upon the processor output signals. The total influence of different phase factors on the processor apparatus function at the more complicated law of the frequency running out can be estimated easily, taking into account that this function is determined as the convolution of the functions, formed at the independent influence of separate phase factors.

In the Table 1 the expressions are given for the phase components $\Delta\varphi(\tau)$ in the integrand phase factors for the spectrum analyzer that lead to the TFD displacement or enlargement along the frequency coordinate that were received at the theoretical analysis, and maximum values of these components at the aperture edge, received at the above presented parameters of AOM and signal. The maximum steepness values for the frequency modulated signals in the AOM aperture are also given in the Table 1.

As it was already mentioned, the TFD enlargement, connected with even power phase factors of integrated variable, is observed in the output signal of the AO

Fig. 1. TFD sections along the frequency axis (in MHz), formed by the spectrum analyzer (left) and by pseudo-Wigner processor (right) for the LFM signal from Table 1 row 1 (top) and additional non-linear frequency changes 2, 3, 4 (lower). The normalized power is plotted along the vertical axes.

spectrum analyzer only, that is why these factors were omitted during the calculation of the pseudo-Wigner distribution cross-section form, but the phase value proportional to the odd powers of the integrated variable was doubled.

In the Fig. 1 the cross-section of the TFD along the frequency axis that is formed by the spectrum analyzer (the left side of the figure) and by pseudo-Wigner processor (the right side of the figure) for the above mentioned in the Table 1 signals is represented.

It can be seen that distributions at the spectrum analyzer are greatly expanded along the frequency axis and have an essentially smaller magnitude and, consequently, the smaller signal/noise ratio, in comparison with the pseudo-Wigner processor signals. The asymmetry, connected with the odd power phase factors in the

Table 1. Phase components in the integrand phase factors

n	$e^{j\frac{a_{n+1}t^{n+1}}{n+1}}$	a_{n+1}	$(\frac{df}{dt})$	$\Delta\varphi \sim \tau^2$	$\Delta\varphi \sim \tau^3$	$\Delta\varphi \sim \tau^4$	$\Delta\varphi \sim \tau^5$
			$\frac{MHz}{\mu sec}$	$\Delta\varphi_{max}$	$\Delta\varphi_{max}$	$\Delta\varphi_{max}$	$\Delta\varphi_{max}$
1	$e^{j\frac{a_2t^2}{2}}$	100π $\frac{MHz}{\mu sec}$	50	$1/2a_2\tau^2$ $12, 5\pi$	—	—	-
2	$e^{j\frac{a_3t^3}{3}}$	10π $\frac{MHz}{(\mu sec)^2}$	10	$a_3t\tau^2$ $5/4\pi$	$1/3a_3\tau^3$ $5/12\pi$	—	—
3	$e^{j\frac{a_4t^4}{4}}$	10π $\frac{MHz}{(\mu sec)^3}$	15	$3/2a_4t^2\tau^2$ $15/16\pi$	$a_4t\tau^3$ $5/8\pi$	$1/4a_4\tau^4$ $5/32\pi$	-
4	$e^{j\frac{a_5t^5}{5}}$	10π $\frac{MHz}{(\mu sec)^4}$	20	$5a_5t^3\tau^2$ $25/16\pi$	$2a_5t^2\tau^3$ $5/8\pi$	$a_5t\tau^4$ $5/16\pi$	$1/5a_5\tau^5$ $1/16\pi$

expressions of thr analyzer spectrum output signals has a slightly expressed behavior, and the frequency running out value reconstruction by the secondary processing of the these signals seems to be a complicated problem. The difference in the output signals value decrease of the spectrum analyzer while signals processing with the different steepness of the frequency tuning will decrease the processor dynamic range to a considerable extent.

The output signals of the pseudo-Wigner processor are not expanded and have a value, that doesn't change at the change of the frequency running rate of the input LFM signal. Comparing this processor output signals for the different nonlinearities of the frequency running out with the output signal for the net LFM signal, one can see, that the TFD distortions, connected with the frequency running out, appear to be acceptable for the given signal and processor parameters both from the point of view of the maximum displacement and the secondary lobes level. At the smaller rate of the frequency tuning and smaller AOM size the distortions will be still smaller. And the secondary signal processing of the pseudo-Wigner processor for the signal current frequency determination is not necessary in contradistinction to the spectrum analyzer.

In order to compare the influence of different power phase factors upon the processors output distributions the TFD frequency cross-sections are represented in the Fig. 2, for the independent influence of the distorting phase factors and different maximum phase values at the aperture edge. As it is seen from the Fig. 2, at maximum value of non-linear phase of the third and higher powers that doesn't exceed 180 at the processor aperture edge, the displacement and secondary lobes level of the output signal can be considered to be acceptable. This fact can be used for the estimation of the processor output distortion according to this procedure, when the signals parameters will differ from used in the considered case.

Fig. 2. TFD sections along the frequency axis at the optical processor output for different distortion phase factors at the aperture (the integrated variable power of the phase factor is 2 (top), 3, 4, 5 (lower) and different maximum phase magnitudes at the aperture edge equal to 0, $\pi/2$, π , $3\pi/2$, 2π . The normalized power is plotted along the vertical axes.

4. Conclusion

In contradistinction from the pseudo-Wigner formation processor in the spectrum analyzer the TFD frequency cross-section is enlarged due to the presence of the even power phase factors in the processor aperture. The TFD enlargement along frequency, formed by the spectrum analyzer is symmetrical with respect to the current signal frequency, so it is possible to find the current frequency by the TFD averaging for every time moment in case of the unifrequency signal. But this operation requires additional time for the secondary signal processing. Besides that, the apparatus function enlargement in the spectrum analyzer causes the decrease of the

output signal, the signal/noise ratio and the dynamic range, as well as the degradation of the processor resolution ability on the frequency-time plane, that reduces the ability to recognize signals upon their fine structure.

It is necessary to mention, that the errors in the frequency change law determination of the FM signals is a shortcoming not for all problems. The TFD fine structure, even with the distortion in FM law, can be used for the formation of the data bank at the recognition of the proximity radiosignals, as well as in other problems, where it is necessary to know not the exact FM law, but only the high sensitive determination of its variations.

The above represented comparison of TFD formation arrangements was carried out based on the output optical signals of the considered processors. In practical arrangements it is necessary also to take into consideration the distortions of output signals introduced by the used photoreceiver. Its spectrum speed and number of elements determine the realization of the resolution ability advantages of the pseudo-Wigner processor at the frequency-time plane.

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