Studies of Systematic Limitations in the EDM Searches at Storage Rings

Artem Saleev

*General and Theoretical Physics Department,
Samara State University,
Samara, 443063, Russia
a.saleev@fz-juelich.de*

Nikolai Nikolaev

*L. D. Landau Institute for Theoretical Physics
Moscow region, Chernogolovka, 142432, Russia
nikolaev@itp.ac.ru*

Frank Rathmann

On behalf of JEDI collaboration

*Institut für Kernphysik and Jülich Center for Hadron Physics,
Forschungszentrum Jülich
Jülich, 52425, Germany
f.rathmann@fz-juelich.de*

Published 29 February 2016

Searches of the electric dipole moment (EDM) at a pure magnetic ring, like COSY, encounter strong background coming from magnetic dipole moment (MDM). The most troubling issue is the MDM spin rotation in the so-called imperfection, radial and longitudinal, B-fields. To study the systematic effects of the imperfection fields at COSY we proposed the original method which makes use of the two static solenoids acting as artificial imperfections. Perturbation of the spin tune caused by the spin kicks in the solenoids probes the systematic effect of cumulative spin rotation in the imperfection fields all over the ring. The spin tune is one of the most precise quantities measured presently at COSY at 10^{-10} level. The method has been successfully tested in September 2014 run at COSY, unravelling strength of spin kicks in the ring's imperfection fields at the level of $10^{-3} rad$.

Keywords: Spin tune; imperfections; electric dipole moment.

PACS numbers: 13.40.Em, 11.30.Er, 29.20.Dh, 29.27.Hj

This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 3.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

1. EDM Searches at All-Magnetic Storage Rings

Spin motion in the pure magnetic ring with a vertical guiding field $\vec{B}$ is governed by T-BMT equation^{1,2}

$$\begin{aligned}\frac{d\vec{S}}{dt} &= \vec{\Omega} \times \vec{S} \\ \vec{\Omega} &= -\frac{e}{m} \left\{ G\vec{B} + \eta\vec{\beta} \times \vec{B} \right\}\end{aligned}\quad (1)$$

where G is a magnetic anomaly and $\eta = \frac{dm}{e}$ has a meaning of the ratio of the EDM, d , to the MDM. The generic signal of the EDM is a rotation of the spin in an electric field. The EDM interacts with the motional electric field, which tilts the stable spin axis,

$$\vec{\Omega} = \Omega_R \frac{G\gamma}{\cos \xi} \{ \vec{e}_x \sin \xi + \vec{e}_y \cos \xi \}, \quad (2)$$

and modifies the spin tune, $Q_S = \frac{G\gamma}{\cos \xi}$. Here $\tan \xi = \frac{\eta\beta}{G}$ and $\frac{\Omega_R}{2\pi}$ is cyclotron frequency.

In a realistic ring, the MDM spin rotation in the imperfection fields all over the ring adds to the EDM rotation. In general case, the vector of the stable spin axis at some point of the ring can be defined as

$$\vec{\Omega} = \Omega_R Q_S \{ c_1 \vec{e}_x + c_2 \vec{e}_y + c_3 \vec{e}_z \}, \quad (3)$$

where $c_{1,2,3}$ are directional cosines of the vector of spin closed orbit. If EDM is zero and no are imperfections present in the ring, $c_1 = c_3 = 0$, $c_2 = 1$, and spin tune $Q_S = G\gamma$.

The spinor equation of motion for the ring with imperfections have form³

$$\Psi(\theta) = e^{-\frac{i}{2} \vec{\sigma} \cdot \vec{c} Q_S \theta} \Psi(0) \quad (4)$$

where $\theta = \Omega_R t = 2\pi k$ is an angular coordinate of the particle after k turns and $\vec{\sigma}$ is a vector of Pauli matrices,

$$\vec{\sigma} \cdot \vec{c} = c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3 \quad (5)$$

Here Ψ is a two-component spinor and the i -th component of the spin vector is given by

$$S_i = \langle \Psi | \sigma_i | \Psi \rangle \quad (6)$$

1.1. JEDI September 2014 run

In general case, spin rotation, defined by “Eq. 4”, is a product of consecutive spin rotations all over the ring. The vector of spin rotation $\vec{c}$ depends on the point of origin and is defined relative to the ideal orbit at that point of the ring.

At September 2014 run, conducted by JEDI (Jülich Electric Dipole moment Investigations) collaboration at COSY, two solenoids from the compensation magnets of the electron coolers have been used as static spin rotators which produce

Fig. 1. Scheme of the ring configuration. The solenoids are located at points 1 and 2. Vectors $\vec{c}$ and $\vec{c}^*$ define direction of spin closed orbit when solenoids are turned off.

artificial imperfection spin kicks. The solenoids are located at straight sections of the ring opposite to each other (see Fig. 1). They effectively split the ring in two halves, and each half is described by the corresponding spin rotation matrix, $\hat{R}_1$ and $\hat{R}_2$:

$$\hat{R}_1 = e^{-\frac{i}{2} \vec{\sigma} \cdot \vec{m} \pi Q_1} \quad (7)$$

$$\hat{R}_2 = e^{-\frac{i}{2} \vec{\sigma} \cdot \vec{n} \pi Q_2} \quad (8)$$

The biggest contribution to the spin rotation in each half of the ring comes from the vertical guiding field of the main dipoles in the arcs. Vertical shifts of the quadrupoles, inclinations of the dipole field and other misalignments of magnets produce small horizontal imperfection fields. Spin kicks in the imperfection fields slightly perturb spin rotation in the vertical guiding field, and give rise to the c_1 and c_3 in Eq. 4. But misalignments differ in each half of the ring, so that corresponding spin rotations for these two halves of the ring are slightly different as well:

$$Q_1 \cong Q_2 \approx G\gamma \quad (9)$$

$$m_2 \cong n_2 \approx 1 \quad (10)$$

Let us consider the case when the solenoids are switched off. Then the total spin rotation matrix at the point downstream the first solenoid:

$$\hat{R} = \hat{R}_1 \hat{R}_2 = \exp \left\{ -\frac{i}{2} \vec{\sigma} \cdot \vec{c} 2\pi Q_S \right\}, \quad (11)$$

whereas downstream the second solenoid it is different:

$$\hat{R} = \hat{R}_2 \hat{R}_1 = \exp \left\{ -\frac{i}{2} \vec{\sigma} \cdot \vec{c}^* 2\pi Q_S \right\}. \quad (12)$$

The rotation axis, $\vec{c}^* = c_1^* \vec{e}_x + c_2^* \vec{e}_y + c_3^* \vec{e}_z$, is different from that one in “(11)”, $\vec{c}^* \neq \vec{c}$ due to non-commuting property of the rotations. The spin tune, $\nu_s = Q_S$, does not depend on the order of rotations and it is the same in both cases.

When the e-cooler's solenoids are switched on, they produce spin kicks around the longitudinal axis,

$$\chi_i = (1 + G) \frac{\int B_i dl}{B\rho} \quad (13)$$

where $B\rho$ represents magnetic rigidity of the ring. The magnitude of $\int B dl$ has linear dependence with respect to the applied current J_1 or J_2 in each solenoid. It is calculated by using corresponding calibration factors f_1 or f_2 :

$$\int B_i dl = f_i J_i \quad (14)$$

The spin kicks in the solenoids produce perturbation of the spin tune in the ring, $\nu_s = Q_S + \Delta\nu_s$. The total spin rotation matrix, defined downstream the first solenoid, becomes

$$\hat{R} = \hat{R}_2 \hat{R}_z(\chi_2) \hat{R}_1 \hat{R}_z(\chi_1) \quad (15)$$

Using the relation $\cos(\pi\nu_s) = Tr[\hat{R}]/2$, perturbation of the spin tune is given by

$$\begin{aligned} \cos(\pi Q_S) - \cos(\pi(Q_S + \Delta\nu_s)) = \\ (E + \cos(\pi Q_S)) \sin^2\left(\frac{y_+}{2}\right) + \frac{1}{2}(c_3 + c_3^*) \sin(\pi Q_S) \sin y_+ \\ - (E - \cos(\pi Q_S)) \sin^2\left(\frac{y_-}{2}\right) + \frac{1}{2}(c_3 - c_3^*) \sin(\pi Q_S) \sin y_- \end{aligned} \quad (16)$$

where the solenoid's spin kicks are defined as

$$y_{\pm} = \frac{\chi_1 \pm \chi_2}{2} \quad (17)$$

and parameter E is

$$E \approx \cos \frac{\pi(Q_1 - Q_2)}{2} \approx 1 \quad (18)$$

The experiment consists of multiple spin tune measurements.⁴ At first, the polarized deuteron beam is prepared for the spin tune measurement: after injection and acceleration to $T = 270$ MeV, the beam is cooled, then bunched. Then RF-solenoid flips the initial vertical polarisation to the horizontal plane by partial Froissart-Stora scan and time clock starts. After 20 seconds the static solenoids are switched on at specified currents J_1 and J_2 . The spin tune shifts and becomes $\nu_s = Q_S + \Delta\nu_s(\chi_1, \chi_2)$. To suppress systematic effects related to the instability of spin tune during the cycle, at 45 seconds the solenoids are switched off and spin tune becomes $\nu_s = Q_S$ again (see Fig. 2). Special data analysis software determines the spin tune shift $\Delta\nu_s$ and base spin tune Q_S with the precision of $\sim 10^{-10}$. To maintain a long spin coherence time of horizontal polarization, the sextupole magnets have been set up at specified settings which compensate decoherence effects from emittance and momentum spread of the beam.⁵

Fig. 2. a) Expected spin tune behaviour in each measurement. b) Typical output of the spin tune analysis software, here the spin tune shift $\Delta\nu_s \simeq 10^{-5}$ has been observed.

The applied currents for the solenoids have been chosen such that corresponding spin kicks χ_1^i and χ_2^j form a rectangular mesh of $i \times j$ points. The mesh has equal steps $\Delta\chi$ in both dimensions, so that

$$y_{\pm} = \Delta\chi \frac{i \pm j}{2} \quad (19)$$

According to "Eq. 16", the functional form of $\Delta\nu_s(y_+, y_-)$ is a sum of two, concave and convex, parabolas:

$$\Delta\nu_s(y_+, y_-) \sim (y_{\pm} - a_{\pm})^2 \quad (20)$$

and the extremum, the saddle point, is located at

$$a_+ = -\sin(\pi Q_s) \frac{c_3 + c_3^*}{1 + \cos(\pi Q_s)} \quad (21)$$

$$a_- = -\sin(\pi Q_s) \frac{c_3 - c_3^*}{1 - \cos(\pi Q_s)} \quad (22)$$

where c_3 and c_3^* are the longitudinal projections of spin closed orbit at the points 1 and 2 of the ring, as defined in "Eq. 11" and "Eq. 12".

The procedure of the spin tune measurements with different strength of the spin kicks in static spin rotators can be referred as "spin tune mapping". As a matter of fact, it provides a partial determination of the stable spin axis at the point of the applied spin kick, which in turn, gives a hint on the magnitude of the imperfection spin kicks in the ring.

To recover a saddle point in the spin tune map with two solenoids, the function "(16)" was fit to data points $\Delta\nu_s(y_+, y_-)$. Fig. 3 shows one of the maps that has been measured during September 2014 run. The saddle point is located at $a_+ = -0.00111077 \pm 6.1 \cdot 10^{-8} \text{ rad}$ and $a_- = -0.00244326 \pm 2.05 \cdot 10^{-7} \text{ rad}$, which defines $c_3 = -0.00299124 \pm 1.8 \cdot 10^{-7}$ and $c_3^* = -0.00163653 \pm 7.1 \cdot 10^{-8}$. The results for $a_{\pm}$ are the preliminary ones, more scrutiny of the impact of possible misalignment of the solenoid axes is in order.

The method also enables to measure c_1 and c_1^* if the static Wien filters operating as the MDM rotators with the radial rotation axis are used alongside the solenoids.⁶ Besides the construction of the spin tune maps with two solenoids, part of September

Fig. 3. Spin tune map. Each white dot represent single measurement of $\Delta\nu_s(y_+, y_-)$, the surface is a fit to the dataset. Error bars are smaller than the size of the symbols. Note that weak parabolic dependence in y_- has negative curvature.

2014 run have used the method in the studies of the systematic effects coming from steering magnets.⁷

2. Summary and Outlook

The spin-tune map emerges as a tool to determine the spin closed orbit with an unprecedented accuracy. In the present experiment only the longitudinal projection of the stable spin axis has been measured, but adding a static Wien filter next to the solenoid would enable to determine the radial projection of the stable spin axis as well. We anticipate that the spin tune map technique would prove most useful in a calibration of various devices to be employed in the high precision EDM searches at all magnetic rings.

References

1. F. Rathmann *et al.*, *J. Phys.: Conf. Ser.* **012011**, 447 (2013).
2. A. Lehrach, B. Lorentz, W. Morse, N. Nikolaev and F. Rathmann, *arXiv:1201.5773*.
3. S. Y. Lee, *Spin Dynamics and Snakes in Synchrotrons*. (World Scientific, 1997).
4. D. Eversmann *et al.*, Precise Spin Tune Measurement in a Storage Ring, in print.
5. E. Stephenson, Polarimetry for Stored Polarized Hadron Beams, *Proc. of SPIN'14*, (2015).
6. S. Mey, A Novel E×B Spin Manipulator at COSY, *Proc. of SPIN'14*, (2015).
7. S. Checkmenev, Estimation of Systematic Errors for Deuteron Electric Dipole Moment at COSY, *Proc. of SPIN'14*, (2015).