Role of Spin in $NN \rightarrow NN\pi$

Vadim Baru

*Institut für theoretische Physik II, Fakultät für Physik und Astronomie
Ruhr-Universität Bochum, 44780 Bochum, Germany
and
Institute for Theoretical and Experimental Physics,
117218, B. Cheremushkinskaya 25, Moscow, Russia
vadimb@tp2.rub.de*

Published 29 February 2016

The recent measurements of the reactions $pp \rightarrow pp\pi^0$ and $pn \rightarrow pp\pi^-$ by the ANKE collaboration at COSY are analyzed with the focus on the p-wave pion production amplitudes. These amplitudes are known to provide an important connection between $NN \rightarrow NN\pi$ and other low-energy few-nucleon reactions. The results of the recent partial wave analysis of the ANKE data are discussed and compared with the theoretical predictions.

Keywords: Chiral EFT; pion production; NN and πN interactions; partial wave analysis.

PACS numbers: 13.75.-n, 12.39.Fe, 25.10.+s, 24.70.+s

1. Introduction

There is a close correlation between pion production and a variety of other low-energy reactions, see e.g. Refs. 1, 2 and also Ref. 3 for a recent review article. It is related to the fact that one low-energy constant (LEC) parameterizing the short range few-nucleon physics in chiral effective field theory (EFT) contributes simultaneously to $NN \rightarrow NN\pi$, three-nucleon force and many other low-energy reactions involving weak and electromagnetic currents. To pin down this important quantity from $NN \rightarrow NN\pi$ two things are necessary:

- the pion production operator should be calculated within a well defined theoretical framework with a controlled theoretical uncertainty
- in order to disentangle the relevant partial wave from other sometimes numerically larger effects, the experimental database should include spin observables with good control over statistical and systematical uncertainties.

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While it is now understood that chiral EFT is an appropriate theoretical framework suitable for systematic calculations,^{4,5} the results for $NN \rightarrow NN\pi$ might converge slower than in standard ChPT due to the relatively large expansion parameter in the production process⁶⁻⁸ ($\chi = \sqrt{m_\pi/m_N}$ with $m_\pi(m_N)$ being the pion (nucleon) mass). On the other hand, the recent progress in the calculation of the s- and p-wave pion production amplitudes yielded a fairly good understanding of total and differential cross sections in the final two-nucleon spin triplet channels^{2,9} as well as some spin observables.¹ These results served as a pre-requisite for several important applications such as i) an extraction of the quark-mass induced contribution to the proton-neutron mass difference¹⁰ from charge symmetry breaking¹¹ in $pn \rightarrow d\pi^0$ (see also Ref.12 for an early work) and ii) an extraction of the πN scattering lengths from pionic atoms data.^{13,14}

A large step towards an extraction of the relevant LEC from data was provided by the recent polarisation measurements of $pp \rightarrow pp\pi^0$ and $pn \rightarrow pp\pi^-$ by the ANKE collaboration at the COSY accelerator. These experimental results allowed one to carry out the amplitude analysis which is published in Refs. 15-17 and will be discussed in this contribution.

2. Amplitude Analysis of the ANKE Data

The idea of using $NN \rightarrow NN\pi$ to extract the LEC connecting different few-nucleon reactions at low energies was suggested in Ref. 1. In Ref. 2 a combined theoretical analysis of different pion production channels, namely $pn \rightarrow pp\pi^-$, $pp \rightarrow pn\pi^+$, and $pp \rightarrow d\pi^+$, was carried out using the p-wave pion production amplitudes calculated up to next-to-next-to-leading order (NNLO) in chiral EFT. Although a combined description of data in different channels appeared possible, it was understood that the channel $pn \rightarrow pp\pi^-$ exhibits a much larger sensitivity to the LEC than all the other channels.

In the framework of the experimental program devoted to the study of pion production at the COSY accelerator in Jülich, the ANKE collaboration has carried out a combined measurement of the differential cross section and the analysing power of two reaction channels $pn \rightarrow \{pp\}_s\pi^-$ and $pp \rightarrow \{pp\}_s\pi^0$ ^{15,16} at the energy $T_{\text{lab}} = 353$ MeV. The LEC of interest contributes to the production of p-wave pions in $NN \rightarrow NN\pi$ while connecting S-wave nucleons. It is therefore crucial that the ANKE measurements selected the events with the very small diproton excitation energy, $E_{pp} < 3$ MeV, which means that the final pp -pair is primarily in the S-wave (see the label $\{ \}_s$).

Below, we follow Refs. 15-17 to discuss how to extract the partial wave amplitudes from these data individually. As long as the final pp -system is purely in an S-wave, the most general structure of the reaction amplitude can be written as

$$M = A \mathbf{S} \cdot \hat{\mathbf{p}} + B \mathbf{S} \cdot \hat{\mathbf{q}}, \quad (1)$$

where $\hat{\mathbf{p}}$ is the unit vector of the initial nucleon momentum in the overall center of mass system (cms), $\hat{\mathbf{q}}$ is the unit vector of the pion momentum and $\hat{\mathbf{p}} \cdot \hat{\mathbf{q}} = \cos \theta_\pi$.

Here $\mathbf{S} = \chi_2^T \frac{\sigma_2}{\sqrt{2}} \sigma \chi_1$ denotes the normalized spin structure corresponding to the initial spin-triplet state and $\chi_{1,2}$ stand for the spinors of the initial nucleons.

It was pointed out in Refs. 15, 16, 17 that the data do not support polynomial terms proportional to $\cos^4 \theta_\pi$ and higher. This fact suggests that the partial waves higher than d-waves for the pion can safely be ignored. Thus, up-to-and-including pion d-waves the expressions for A and B for $pn \rightarrow \{pp\}_s \pi^-$ read

$$\begin{aligned} A &= M_s^P + M_p^D \cos \theta_\pi - \frac{1}{3} M_d^P + M_d^F \left(\cos^2 \theta_\pi - \frac{1}{5} \right), \\ B &= M_p^S - \frac{1}{3} M_p^D + \left(M_d^P - \frac{2}{5} M_d^F \right) \cos \theta_\pi. \end{aligned} \quad (2)$$

where the superscript in the amplitudes M_i^L refers to the partial wave of the initial nucleons and the subscript corresponds to the pion partial wave in the overall cms. Altogether five partial waves are included in the analysis: one s-wave (M_s^P), two p-wave (M_p^S and M_p^D) and two d-wave (M_d^P and M_d^F) amplitudes. These amplitudes correspond to the transitions ${}^3P_0 \rightarrow {}^1S_0 s$, ${}^3S_1 \rightarrow {}^1S_0 p$, ${}^3D_1 \rightarrow {}^1S_0 p$, ${}^3P_2 \rightarrow {}^1S_0 d$, and ${}^3F_2 \rightarrow {}^1S_0 d$, respectively.

Since pion p-wave amplitudes do not contribute to $pp \rightarrow \{pp\}_s \pi^0$ one has to omit them in Eq.(2) to arrive at the expressions in this channel

$$\begin{aligned} A &= M_s^P - \frac{1}{3} M_d^P + M_d^F \left(\cos^2 \theta_\pi - \frac{1}{5} \right), \\ B &= \left(M_d^P - \frac{2}{5} M_d^F \right) \cos \theta_\pi. \end{aligned} \quad (3)$$

The observables in terms of these amplitude can be written as

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |A|^2 + |B|^2 + 2\text{Re}[AB^*] \cos \theta_\pi, \\ A_y \frac{d\sigma}{d\Omega} &= 2\text{Im}[AB^*] \sin \theta_\pi. \end{aligned} \quad (4)$$

By matching Eqs. (2)-(4) with the measured coefficients in the expansion in powers of $\cos \theta_\pi$ for the differential cross section and the analyzing power in $pn \rightarrow \{pp\}_s \pi^-$ and $pp \rightarrow \{pp\}_s \pi^0$ one gets two systems of linear equations to determine the partial waves amplitudes. In particular, in the channel $pp \rightarrow \{pp\}_s \pi^0$ there are three observable quantities (two from the differential cross section and one from the analysing power) to pin down three complex-valued amplitudes, see Eq. (3). In order to reduce the number of parameters, one can still use information about the phases of the production amplitudes provided by the Watson theorem. In particular, for the uncoupled partial wave corresponding to the 3P_0 initial state the Watson theorem relates the phase of the transition amplitude to that of the initial state NN interaction (ISI), i.e. $M_s^P = |M_s^P| e^{i\delta_{3P_0}}$. Strictly speaking, the Watson theorem can only be applied to the uncoupled partial waves. However, given the negligibly small value of the mixing parameter between ${}^3P_2 - {}^3F_2$ partial waves at 353 MeV,¹⁸ the phases of these amplitudes were also fixed in Refs. 15, 16, 17 using

the Watson theorem. It should be stressed that no assumption about the phases for the coupled $^3S_1 - ^3D_1$ partial waves was made. Using this approach, the fit to the combined $pp \rightarrow \{pp\}_s \pi^0$ and $pn \rightarrow \{pp\}_s \pi^-$ data sets has been performed in Refs. 15, 16, 17, with the results for s- and d-waves from the global fit to be in good agreement with the values extracted from the analysis of the π^0 data only. This consistency check suggests that the systematic errors are indeed under control. Unfortunately, the amplitude analysis did not yield the unique solution for the p-wave amplitudes which are most interesting in this study (especially the amplitude $^3S_1 \rightarrow ^1S_0 p$ in which the contact term contributes).¹⁷ In addition to the global minimum, several local minima with only slightly larger χ^2 were found yielding completely different amplitudes for the p-waves, c.f. solutions 1, 2 and 3 in Table 1. The data on the differential cross section and the analysing power only do not allow one to discriminate between the different solutions, see the upper panels in Fig.1. The transverse spin-correlation parameter A_{xx} (see the lower left panel of Fig.1) also did not allow to distinguish between different solutions. As argued in Ref. 17, see also Fig.1 lower right panel, the predictions for the double polarization measurement of A_{xz} in $pn \rightarrow \{pp\}_s \pi^-$ look radically different. Therefore A_{xz} could be useful in order to lift the ambiguity with the different χ^2 solutions.

Although experimental data allow several solutions, theory can be helpful to decide which of them is to be preferred. In particular, the phases of the p-wave amplitudes corresponding to different solutions in Table 1 turned out to differ significantly from each other, as can be seen from the last column of this Table. On the other hand, in spite of the sizeable coupling between the 3S_1 and 3D_1 partial waves, one would naively assume that the phases of the p-wave pion-production amplitudes should not differ drastically from the elastic NN phases. The values in the last column of Table 1 should be compared with nucleon-nucleon phase-shift analysis values of $(\tan \delta_{^3S_1}, \tan \delta_{^3D_1}) = (0.03, -0.46)$,¹⁸ and to the values from the theoretical analysis of the p-wave pion production amplitudes of $(0.04, -0.61)$.² This comparison reveals a clear preference against the solution 1 and possibly in favour of solution 2, as pointed out in Ref.17. Furthermore, the chiral EFT calculation²

Table 1. Values of the real and imaginary parts of the p-wave amplitudes M_p^S and M_p^D deduced from fits to the ANKE $pp \rightarrow \{pp\}_s \pi^0$ and $np \rightarrow \{pp\}_s \pi^-$ measurements at 353 MeV. Also shown are the ratios of the imaginary to real parts of the p-wave amplitudes that have been freely fitted (with no constraints from the Watson theorem).

Solution, χ^2/ndf	Amplitude	Real	Imaginary	Im/Re
Solution 1 $\chi^2/ndf = 101/82$	$^3S_1 \rightarrow ^1S_0 p$	-37.5 ± 1.7	16.5 ± 1.9	-0.44 ± 0.06
	$^3D_1 \rightarrow ^1S_0 p$	-93.1 ± 6.5	122.7 ± 4.4	-1.32 ± 0.11
Solution 2 $\chi^2/ndf = 103/82$	$^3S_1 \rightarrow ^1S_0 p$	-63.7 ± 2.5	-1.3 ± 1.6	0.02 ± 0.03
	$^3D_1 \rightarrow ^1S_0 p$	-109.9 ± 4.2	52.9 ± 3.2	-0.48 ± 0.03
Solution 3 $\chi^2/ndf = 106/82$	$^3S_1 \rightarrow ^1S_0 p$	-25.4 ± 1.9	-7.3 ± 1.5	0.20 ± 0.07
	$^3D_1 \rightarrow ^1S_0 p$	-172.2 ± 5.6	92.0 ± 6.2	-0.53 ± 0.04

Fig. 1. Predictions of the amplitude analysis for the $pn \rightarrow \{pp\}_s \pi^-$ reaction at 353 MeV with the $E_{pp} < 3$ MeV cut. Also shown are the ANKE experimental data with statistical errors. The full, long-dashed, and short-dashed lines correspond to solutions 1, 2, and 3, as noted in Table 1. a) Differential cross-section taken from Ref. 15, 16, b) A_y^p data from Ref. 17, c) A_{xx} data from Ref. 17, d) A_{xz} , for which there are yet no experimental data. The figure is taken from Ref. 17.

at NNLO yields $M_s^P = -53.7 - 2.6i$ which provides further indications in favour of solution 2.

3. Summary

As discussed in this contribution, p-wave pion production opens an attractive possibility to extract the strength of the $(\bar{N}N)^2\pi$ contact operator which provides bridging between different low-energy reactions. We discussed the progress in this direction related to the recent measurements at COSY. In particular, the combined measurements of $pp \rightarrow \{pp\}_s \pi^0$ and $np \rightarrow \{pp\}_s \pi^-$ allowed one to extract self-consistently the s- and d-wave pion production amplitudes. However, the analysis of data revealed that several equivalent solutions exist for the p-wave amplitudes. The measurement of A_{xz} at COSY could be very helpful in resolving this discrepancy but it seems very unlikely due to its complexity. On the other hand, we argued that some reasonable constraints from theory can be used to make a clear preference between different solutions.

Given that the loop contributions to the pion production amplitudes are potentially important, as a next step, the chiral EFT calculation should be extended to a one-loop order, along the lines of Refs. 19, 20. Furthermore, thank to the recent advances in the chiral theory of nuclear forces²¹ the use of chiral NN wave functions even at the energies around the pion production threshold becomes now possible. The consistent treatment of the production operator and nuclear wave functions should be of importance for the future extraction of the LEC from data.

Acknowledgments

I would like to thank E. Epelbaum, A. Filin, J. Haidenbauer, C. Hanhart, H. Krebs, A. Kudryavtsev, V. Lensky, U.-G. Meißner, F. Myhrer and all the members of the experimental collaboration ANKE at COSY for a fruitful and enjoyable collaboration. I thank the organisers for the invitation to give this talk. This work is supported in part by the EU HadronPhysics3 project Study of strongly interacting matter, the European Research Council (ERC-2010-StG 22259218 NuclearEFT).

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