Higher Twist Effects in e^+e^- Annihilation at High Energies

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In the two papers published recently,^{1,2} we apply collinear expansion to both inclusive ($e^+e^- \rightarrow h + X$) and semi-inclusive ($e^+e^- \rightarrow h + \bar{q} + X$) hadron production in e^+e^- annihilation to derive a formalism suitable for a systematic study of leading as well as higher twist contributions to fragmentation functions at the tree level. We carry out the calculations for hadrons with spin-0, spin-1/2 as well as spin-1. This proceeding is mainly a summary of these two papers.

Keywords: Collinear expansion; fragmentation function; higher twist; azimuthal asymmetry; spin asymmetry.

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1. Introduction

The e^+e^- annihilation process is most suitable to study fragmentation functions among all different high energy reactions, as there is no hadron involved in the initial state. The one dimensional fragmentation functions can be studied via inclusive process, while, the 3D information can only be extracted from semi-inclusive process.

Higher twist terms may have very important contributions to azimuthal asymmetries and spin asymmetries,³ which are usually measured in experiments to study the properties of fragmentation functions and parton distribution functions. Collinear expansion, first developed in 1980s, seems to be the unique method to calculate leading twist and higher twist contributions in a systematic way.

This method was applied to inclusive DIS process⁴ to get the cross section up to twist-4 level at first. It was summarized as four steps,⁵ and was applied to SIDIS to get the form of azimuthal asymmetries up to twist-3⁵ and twist-4.⁶ Recently,

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we applied collinear expansion to inclusive e^+e^- annihilation process¹ and semi-inclusive process.²

2. Collinear Expansion

Collinear expansion was inspired by the collinear approximation, which is stated as follows.

- We only keep the collinear component of the quark momentum, $k_i \approx p/z_i$.
- We only keep the plus component of the gluon field, $A^\mu \approx A^+ \bar{n}^\mu$.

This approximation is reasonable, if we are only care about the leading twist contributions, since other components are power suppressed compared to the plus component. We apply this approximation to the following diagrams, and we have,

$$\tilde{W}_{\mu\nu}^{\text{approx}} = \frac{1}{2p^+} \text{Tr} \left[\Gamma_\mu \not{n} \Gamma_\nu \hat{\Xi}^{\text{approx}}(z_B, p, S, k'_\perp) \right], \quad (1)$$

where, $\Gamma_\mu = \gamma_\mu (c_V^q - c_A^q \gamma_5)$ is the vertex for weak interaction. And the soft matrix Ξ^{approx} is a little different for inclusive process and semi-inclusive process. For the inclusive process, this soft matrix does not dependent on $k'_\perp$,

$$\hat{\Xi}_{\text{in}}^{\text{approx}}(z, p, S) = \hat{\mathcal{F}}(\xi^-) \sum_X \langle 0 | \mathcal{L}^\dagger(0^-, \infty) \psi(0) | hX \rangle \langle hX | \bar{\psi}(\xi^-) \mathcal{L}(\xi^-, \infty) | 0 \rangle. \quad (2)$$

While, for the semi-inclusive process,

$$\hat{\Xi}_{\text{si}}^{\text{approx}}(z, k'_\perp, p, S) = \hat{\mathcal{F}}(\xi^-, \vec{\xi}_\perp) \sum_X \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | hX \rangle \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) | 0 \rangle. \quad (3)$$

$\hat{\mathcal{F}}$ is just the Fourier Transformation operator. We see, these soft matrices are automatically gauge invariant.

If we are going to the twist-3 level, these transverse components can not be simply neglected any more. So, we need the collinear expansion to take them back into account by the following steps.^{1,2}

Fig. 1. The first few Feynman diagrams as examples of the diagram series with exchange of j gluon(s).

- Make a Taylor expansion to the hard part, $H^i(k) = H^i(k = p/z) + \frac{\partial H}{\partial k} \Big|_{k=p/z} \times (k - p/z) + \dots$.
- Decompose the gluon fields in this way, $A^\rho(y) = A^+(y)\bar{n}^\rho + (A^\rho(y) - A^+(y)\bar{n}^\rho)$.

Reducing them with Ward identities, we get the hadronic tensor twist by twist,

$$W_{\mu\nu} = \tilde{W}_{\mu\nu}^{(0)} + \tilde{W}_{\mu\nu}^{(1L)} + (\tilde{W}_{\mu\nu}^{(1L)})^* + \dots, \quad (4)$$

where,

$$\tilde{W}_{\mu\nu}^{(0)} = \frac{1}{2} \text{Tr} \left[\hat{h}_{\mu\nu}^{(0)} \hat{\Xi}^{(0)} \right], \quad \tilde{W}_{\mu\nu}^{(1L)} = -\frac{1}{4p \cdot q} \text{Tr} \left[\hat{h}_{\mu\nu}^{(1)\rho} \omega_\rho^{\rho'} \hat{\Xi}_{\rho'}^{(1)} \right]. \quad (5)$$

$\tilde{W}^{(0)}$ is, actually, $\tilde{W}_{\mu\nu}^{\text{approx}}$, and $\hat{\Xi}^{(0)} = \hat{\Xi}^{\text{approx}}$. $\tilde{W}_{\mu\nu}^{(1L)}$ is the new hadronic tensor, whose leading contribution is at twist-3. The new quark gluon correlators are defined as,

$$\hat{\Xi}_\rho^{(1)} \text{ in} = \hat{\mathcal{F}}(\xi^-) \sum_X \langle 0 | \mathcal{L}^\dagger(0, \infty) D_\rho(0) \psi(0) | hX \rangle \langle hX | \bar{\psi}(\xi^-) \mathcal{L}(\xi^-, \infty) | 0 \rangle, \quad (6)$$

$$\hat{\Xi}_\rho^{(1)} \text{ si} = \hat{\mathcal{F}}(\xi^-, \vec{\xi}_\perp) \sum_X \langle 0 | \mathcal{L}^\dagger(0, \infty) D_\rho(0) \psi(0) | hX \rangle \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) | 0 \rangle. \quad (7)$$

These Ξ 's defined above are all 4 by 4 matrices which can always be decomposed in terms of Gamma matrices. In this case, only γ^α and $\gamma_5 \gamma^\alpha$ that will contribute.

$$\hat{\Xi}^{(i)} = \Xi_\alpha^{(i)} \gamma^\alpha + \tilde{\Xi}_\alpha^{(i)} \gamma_5 \gamma^\alpha, \quad (8)$$

where, parity invariant constrains the possible structures of them, since $\Xi_\alpha^{(i)}$ is a vector and $\tilde{\Xi}_\alpha^{(i)}$ is an axis vector. We will discuss the details of this decomposition in the following two sections for inclusive process and semi-inclusive process separately.

3. Inclusive Process $e^+e^- \rightarrow h + X$

These Ξ 's are made up by p , n and S for the inclusive process. S here refers to the spin parameters of produced hadrons. For spin-1/2 hadrons, we only need the spin vector S^μ , while for vector mesons, we need also a spin tensor $T^{\mu\nu}$, which can be decomposed in terms of S_{LL} , S_{LT}^μ and $S_{TT}^{\mu\nu}$.

3.1. Spin Independent Part and Spin Vector Dependent Part

$$z \tilde{\Xi}_{\text{in}}^{(0)\alpha} = p^\alpha [D_1(z) + S_{LL} D_{1LL}(z)] + M [\epsilon_\perp^{\alpha\gamma} S_{\perp\gamma} D_T(z) + S_{LT}^\alpha D_{LT}(z)], \quad (9)$$

$$z \tilde{\Xi}_{\text{in}}^{(0)\alpha} = \lambda_h p^\alpha \Delta D_{1L}(z) + M S_\perp^\alpha \Delta D_T(z) + M \epsilon_\perp^{\alpha\gamma} S_{LT,\gamma} \Delta D_{LT}(z), \quad (10)$$

$$z \tilde{\Xi}_{\text{in}}^{(1)\rho\alpha} = p^\alpha \left[M \epsilon_\perp^{\rho\gamma} S_{\perp\gamma} \xi_{\perp S}^{(1)}(z) + M S_{LT}^\rho \xi_{LTS}^{(1)}(z) \right], \quad (11)$$

$$z \tilde{\Xi}_{\text{in}}^{(1)\rho\alpha} = i p^\alpha \left[M S_\perp^\rho \tilde{\xi}_{\perp S}^{(1)}(z) + i M \epsilon_\perp^{\rho\gamma} S_{LT,\gamma} \tilde{\xi}_{LTS}^{(1)}(z) \right]. \quad (12)$$

For spin-0 hadrons, only the spin independent terms that will contribute,

$$\frac{d^2\sigma^{unp}}{dzdy} = \sum_{q,c} \frac{2\pi\alpha^2}{Q^2} \chi T_0^q(y) D_1^q(z). \quad (13)$$

For spin-1/2 particles, both the spin independent part and spin vector dependent part contribute. The spin vector dependent part is given by,

$$\begin{aligned} \frac{d\sigma^{Vpol}}{dzdy} = \frac{2\pi\alpha^2}{Q^2} \chi \Big\{ \lambda_h T_1(y) \Delta D_{1L}(z) + \frac{4M}{zQ^2} [T_2(y) D_T(z) \epsilon_{\perp}^{l_{\perp} S_{\perp}} \\ + T_3(y) \Delta D_T(z) l_{\perp} \cdot S_{\perp}] \Big\}. \end{aligned} \quad (14)$$

Here, the summation over quark flavor and color is not written out explicitly. We see, the first term is at leading twist and the last two terms are at twist-3. So, it is obvious that at leading twist, produced hadrons will be longitudinally polarized.

$$P_{Lh} = \frac{\sum_q T_1^q(y) \Delta D_{1L}^{q \rightarrow h}(z)}{\sum_q T_0^q(y) D_1^{q \rightarrow h}(z)} = \frac{\sum_q P_q(y) T_0^q(y) \Delta D_{1L}^{q \rightarrow h}(z)}{\sum_q T_0^q(y) D_1^{q \rightarrow h}(z)}. \quad (15)$$

It is just proportional to the polarization of quark, $P_q(y) = T_1^q(y)/T_0^q(y)$, times a spin transfer, $\Delta D_{1L}^{q \rightarrow h}(z)$.

There are also two transverse polarizations at twist-3. One is perpendicular to the leptonic plane, $\epsilon_{\perp}^{l_{\perp} S_{\perp}}$, while the other one lies in the leptonic plane, $l_{\perp} \cdot S_{\perp}$.

$$P_{hy} = \frac{4M}{zQ} \frac{\sum_q \tilde{T}_2^q(y) D_T^{q \rightarrow h}(z)}{\sum_q T_0^q(y) D_1^{q \rightarrow h}(z)}, \quad P_{hx} = -\frac{4M}{zQ} \frac{\sum_q \tilde{T}_3^q(y) \Delta D_T^{q \rightarrow h}(z)}{\sum_q T_0^q(y) D_1^{q \rightarrow h}(z)}. \quad (16)$$

The first one is T-odd. Hence this is NO corresponding term in inclusive DIS process. The last one is P-odd, which will disappear in the electromagnetic process.

3.2. Spin Tensor Dependent Part

$$z\tilde{\Xi}_{in}^{(0)\alpha} = p^{\alpha} S_{LL} D_{1LL}(z) + M S_{LT}^{\alpha} D_{LT}(z), \quad z\tilde{\Xi}_{in}^{(0)\alpha} = M \epsilon_{\perp}^{\alpha\gamma} S_{LT,\gamma} \Delta D_{LT}(z), \quad (17)$$

$$z\tilde{\Xi}_{in}^{(1)\rho\alpha} = p^{\alpha} M S_{LT}^{\rho} \xi_{LTS}^{(1)}(z), \quad z\tilde{\Xi}_{in}^{(1)\rho\alpha} = i M p^{\alpha} \epsilon_{\perp}^{\rho\gamma} S_{LT,\gamma} \tilde{\xi}_{LTS}^{(1)}(z). \quad (18)$$

For vector mesons, the cross section contains contributions from spin independent, vector polarization dependent and tensor polarization dependent parts.

$$\begin{aligned} \frac{d\sigma^{Tpol}}{dzdy} = \frac{2\pi\alpha^2}{Q^2} \chi \Big\{ T_0(y) S_{LL} D_{1LL}(z) + \frac{4M}{zQ^2} [T_2(y) l_{\perp} \cdot S_{LT} D_{LT}(z) \\ + T_3(y) \epsilon_{\perp}^{l_{\perp} S_{LT}} \Delta D_{LT}(z)] \Big\}. \end{aligned} \quad (19)$$

The most interesting spin parameter of vector mesons is spin alignment (ρ_{00}). This leading twist effect is verified by LEP experiment. We predict that this tensor polarization can also be easily measured by BES experiment since it is P-even.

$$\rho_{00}^{weak} = \frac{1}{3} - \frac{1}{3} \frac{\sum_q T_0^q(y) D_{1LL}^{q \rightarrow h}(z)}{\sum_q T_0^q(y) D_1^{q \rightarrow h}(z)}, \quad \rho_{00}^{em} = \frac{1}{3} - \frac{1}{3} \frac{\sum_q e_q^2 D_{1LL}^{q \rightarrow h}(z)}{\sum_q e_q^2 D_1^{q \rightarrow h}(z)}. \quad (20)$$

4. Semi-Inclusive Process $e^+e^- \rightarrow h + \bar{q} + X$

For the semi-inclusive process, these Ξ 's are also functions of the transverse momentum $k'_\perp$. So, we have much more new structures.

4.1. Spin Independent Part and Azimuthal Asymmetries

$$z\Xi_\alpha^{(0)} = p_\alpha \hat{D}_1 + k_{\perp\alpha} \hat{D}^\perp, \quad z\tilde{\Xi}_\alpha^{(0)} = \epsilon_{\perp\alpha k_\perp} \Delta \hat{D}^\perp, \quad (21)$$

$$z\Xi_{\rho\alpha}^{(1)} = p_\alpha k_{\perp\rho} \xi_\perp^{(1)}, \quad z\tilde{\Xi}_{\rho\alpha}^{(1)} = ip_\alpha \epsilon_{\perp\rho k_\perp} \tilde{\xi}_\perp^{(1)}. \quad (22)$$

And the corresponding cross section is,

$$\frac{d\sigma^{(si,unp)}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2} \left\{ T_0^q(y) \hat{D}_1 + \frac{4}{zQ^2} [T_2^q(y) l_\perp \cdot k'_\perp \hat{D}^\perp + T_3^q(y) \epsilon_\perp^{l_\perp k'_\perp} \Delta \hat{D}^\perp] \right\}. \quad (23)$$

We see immediately that there are two azimuthal asymmetries at twist-3,

$$A_{unp}^{\cos\varphi} = -\frac{2|\vec{k}'_\perp|}{zQ} \frac{\sum_q \tilde{T}_2^q(y) \hat{D}^{\perp q \rightarrow h}}{\sum_q T_0^q(y) \hat{D}_1^{q \rightarrow h}}, \quad A_{unp}^{\sin\varphi} = \frac{2|\vec{k}'_\perp|}{zQ} \frac{\sum_q \tilde{T}_3^q(y) \Delta \hat{D}^{\perp q \rightarrow h}}{\sum_q T_0^q(y) \hat{D}_1^{q \rightarrow h}}. \quad (24)$$

The second one is P-odd, so it will disappear in electromagnetic process.

4.2. Spin Vector Dependent Part

$$z\Xi_\alpha^{(0)} = p_\alpha \frac{\epsilon_\perp^{k_\perp S_\perp}}{M} \hat{D}_{1T}^\perp + k_{\perp\alpha} \frac{\epsilon_\perp^{k_\perp S_\perp}}{M} \hat{D}_T^\perp + \lambda_h \epsilon_{\perp\alpha k_\perp} \hat{D}_L^\perp + M \epsilon_{\perp\alpha S_\perp} \hat{D}_T, \quad (25)$$

$$z\tilde{\Xi}_\alpha^{(0)} = p_\alpha \left[\lambda_h \Delta \hat{D}_{1L} + \frac{k_\perp \cdot S_\perp}{M} \Delta \hat{D}_{1T}^\perp \right] + \frac{\epsilon_\perp^{k_\perp S_\perp}}{M} \epsilon_{\perp\alpha k_\perp} \Delta \hat{D}_T^\perp + \lambda_h k_{\perp\alpha} \Delta \hat{D}_L^\perp + M S_{\perp\alpha} \Delta \hat{D}_T, \quad (26)$$

$$z\Xi_{\rho\alpha}^{(1)} = p_\alpha \left[M \epsilon_{\perp\rho S_\perp} \xi_T^{(1)} + k_{\perp\rho} \frac{\epsilon_\perp^{k_\perp S_\perp}}{M} \xi_T^{(1)\perp} + \lambda_h \epsilon_{\perp\rho k_\perp} \xi_L^{(1)\perp} \right], \quad (27)$$

$$z\tilde{\Xi}_{\rho\alpha}^{(1)} = ip_\alpha \left[M S_{\perp\rho} \tilde{\xi}_T^{(1)} + \frac{\epsilon_\perp^{k_\perp S_\perp}}{M} \epsilon_{\perp\rho k_\perp} \tilde{\xi}_T^{(1)\perp} + \lambda_h k_{\perp\rho} \tilde{\xi}_L^{(1)\perp} \right]. \quad (28)$$

The cross section is then given by,

$$\begin{aligned} \frac{d\sigma^{(si,Vpol)}}{dydzd^2k'_\perp} = & \frac{\alpha^2\chi}{2\pi Q^2} \left\{ T_0^q(y) \frac{\epsilon_\perp^{k'_\perp S_\perp}}{M} \hat{D}_{1T}^\perp + T_1^q(y) [\lambda_h \Delta \hat{D}_{1L} + \frac{k'_\perp \cdot S_\perp}{M} \Delta \hat{D}_{1T}^\perp] \right. \\ & + \frac{4\lambda_h}{zQ^2} [T_2^q(y) \epsilon_\perp^{l_\perp k'_\perp} \hat{D}_L^\perp + T_3^q(y) l_\perp \cdot k'_\perp \Delta \hat{D}_L^\perp] + \frac{4\epsilon_\perp^{k'_\perp S_\perp}}{zMQ^2} [T_2^q(y) l_\perp \cdot k'_\perp \hat{D}_T^\perp \\ & \left. + T_3^q(y) \epsilon_\perp^{l_\perp k'_\perp} \Delta \hat{D}_T^\perp] + \frac{4M}{zQ^2} [T_2^q(y) \epsilon_\perp^{l_\perp S_\perp} \hat{D}_T + T_3^q(y) l_\perp \cdot S_\perp \Delta \hat{D}_T] \right\}. \quad (29) \end{aligned}$$

We see, that there is a leading twist polarization in the longitudinal direction,

$$P_{Lh}^{(0)} = \frac{\sum_q T_0^q(y) P_q(y) \Delta \hat{D}_{1L}}{\sum_q T_0^q(y) \hat{D}_1}. \quad (30)$$

The most suitable directions to study transverse polarization and corresponding fragmentation functions are those in and transverse to the production plane. At leading twist,

$$P_{hn}^{(0)} = -\frac{|\vec{k}'_{\perp}|}{M} \frac{\sum_q T_0^q(y) \hat{D}_{1T}^{\perp}}{\sum_q T_0^q(y) \hat{D}_1}, \quad P_{ht}^{(0)} = -\frac{|\vec{k}'_{\perp}|}{M} \frac{\sum_q P_q(y) T_0^q(y) \Delta \hat{D}_{1T}^{\perp}}{\sum_q T_0^q(y) \hat{D}_1}. \quad (31)$$

Those twist-3 terms gives us corrections² which are suppressed by M/Q .

4.3. Spin Alignment

The Lorentz structures for vector mesons are much more complicated, since there are much more spin parameters. We would not show you all those structures in this proceeding, for the limitation of length. Please find the details in [2] if you are interested in this topic. Here we only show you results concerning spin alignment.

$$z\tilde{\Xi}_{\alpha LL}^{(0)} = p_{\alpha} S_{LL} \hat{D}_{1LL} + k_{\perp\alpha} S_{LL} \hat{D}_{LL}^{\perp}, \quad z\tilde{\Xi}_{\alpha LL}^{(0)} = \epsilon_{\perp\alpha k_{\perp}} S_{LL} \Delta \hat{D}_{LL}^{\perp}, \quad (32)$$

$$z\tilde{\Xi}_{\rho\alpha LL}^{(1)} = p_{\alpha} k_{\perp\rho} S_{LL} \xi_{LL}^{\perp}, \quad z\tilde{\Xi}_{\rho\alpha LL}^{(1)} = ip^{\alpha} \epsilon_{\perp\rho k_{\perp}} S_{LL} \tilde{\xi}_{LL}^{\perp}. \quad (33)$$

The corresponding cross section is given by,

$$\frac{d\sigma^{(si,LL)}}{dydzd^2k'_{\perp}} = \frac{\alpha^2\chi}{2\pi Q^2} S_{LL} \left\{ T_0^q(y) \hat{D}_{1LL} + \frac{4}{zQ^2} [T_2^q(y) l_{\perp} \cdot k'_{\perp} \hat{D}_{LL}^{\perp} + T_3^q(y) \epsilon_{\perp}^{l_{\perp} k'_{\perp}} \Delta \hat{D}_{LL}^{\perp}] \right\}. \quad (34)$$

And then, we get the spin alignment up to twist-3,

$$S_{LL} = \frac{\sum_q T_0^q(y) \hat{D}_{1LL}}{2 \sum_q T_0^q(y) \hat{D}_1} \left[1 + \frac{M}{Q} \hat{\Delta} \right] - \frac{\sum_q 2 [\tilde{T}_2^q(y) k'_x \hat{D}_{LL}^{\perp} - \tilde{T}_3^q(y) k'_y \Delta \hat{D}_{LL}^{\perp}]}{zQ \sum_q T_0^q(y) \hat{D}_1}. \quad (35)$$

5. Summary

In inclusive process, there is a leading twist longitudinal polarization for spin- $\frac{1}{2}$ hadrons and also spin alignment ($\rho_{00} \neq \frac{1}{3}$) for vector mesons. At twist-3, there are transverse polarizations for spin- $\frac{1}{2}$ hadrons in and transverse to the leptonic plane.

In semi-inclusive process, for spin-0 hadrons, there are two azimuthal asymmetries at twist-3. For spin- $\frac{1}{2}$ hadrons, there is a longitudinal polarization and also transverse polarizations in and transverse to the production plane at leading twist.

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