Quark Angular Momentum and Gravitational Form Factors

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Published 29 February 2016

We investigate the quark angular momentum in a light-cone spectator model. The canonical angular momentum sum rule is satisfied in both the scalar and the axial-vector cases. Then we perform a calculation of the gravitational form factors which are expected to be related to the angular momentum of each constituent in this no gauge field model. We find this relation is satisfied for the total angular momentum as a direct conclusion of the momentum fraction sum rule and the anomalous gravitomagnetic moment sum rule, but for each constituent it is violated.

Keywords: Angular momentum; gravitational form factor; light-cone.

1. Introduction

The spin structure of hadrons is one of the most active frontiers in particle physics. Since the observation^{1,2} that the quark spin only contributes a small fraction (about 30% in recent analyses³⁻⁵) to proton spin, the “proton spin crisis” has puzzled the physics community for more than 25 years. As a relativistic effect, even if the proton in the rest frame is an S-wave system as described in the naive quark model, the orbital angular momentum (OAM) is nonnegligible in the infinite momentum frame or the light-cone formalism where the parton model is well established.^{6,7} Besides, the gluon can also contribute a large fraction.⁸

Although the angular momentum of an isolated system is well defined, it is of great challenge and interest to decompose the total angular momentum to the spin

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and orbit of each constituent and relate each part to physical observables. However the definition of the OAM in a gauge field theory is still under debates. Nowadays all the decomposition versions are usually classified into two groups, the canonical one and the kinetic (or mechanical) one.^{9,10}

In this talk, instead of focusing on the controversy on which one is more physical, we calculate quark OAM in a light-cone spectator model where the canonical OAM and the kinetic OAM are expected to be the same. We find that even in this no gauge field model where the two kinds of quark angular momentum are identified at the operator level, half the sum of two gravitational form factors are not equal to quark angular momentum.¹¹

2. Light-Cone Wave Functions

In the spectator model, the proton is viewed as a struck quark and a spectator. This kind of model has been applied to investigate many physical observables, such as the structure functions, form factors, transverse momentum dependent parton distributions and generalized parton distributions. The results are usually comparable with the experiments.

Constrained by the quantum numbers of the quark and the proton, the spectator is either a scalar or an axial-vector. Thus we may effectively introduce quark-spectator-proton vertex as

$$\mathcal{L}_I = g_s \bar{\Psi} \phi \psi + g_v \bar{\Psi} \gamma^\mu \gamma_5 A_\mu \psi + h.c., \quad (1)$$

where ψ , Ψ , ϕ and A_μ are the operators of quark, proton, scalar diquark and axial-vector diquark fields. Some suitable form factors can be included in the couplings $g_{s/v}$. With the Dirac structures, we can write down the light-cone wave functions (LCWFs) as

$$\psi_{\lambda}^{S\Lambda}(x, \mathbf{k}_\perp) = \frac{\bar{u}(k, \lambda)}{\sqrt{2k^+}} \mathbf{1} \frac{u(P, \Lambda)}{\sqrt{2P^+}} \phi^S(x, \mathbf{k}_\perp), \quad (2)$$

$$\psi_{\lambda\lambda'}^{V\Lambda}(x, \mathbf{k}_\perp) = \frac{\bar{u}(k, \lambda)}{\sqrt{2k^+}} \epsilon_\mu^*(p, \lambda') \gamma^\mu \gamma_5 \frac{u(P, \Lambda)}{\sqrt{2P^+}} \phi^V(x, \mathbf{k}_\perp), \quad (3)$$

where λ , λ' and Λ are the light-cone helicities of the quark, axial-vector spectator and proton, and the superscripts S and V denote the type of the spectator. They are normalized as

$$\int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \sum_{\lambda, \lambda'} |\psi^{S/V\Lambda}_{\lambda\lambda'}(x, \mathbf{k}_\perp)|^2 = \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} N_{S/V}^2 |\phi^{S/V}(x, \mathbf{k}_\perp)|^2 = 1, \quad (4)$$

where $\phi^{S/V}$ is a spin-average wave function and $N_{S/V}$ is the normalization factor of spin states. Practically, a dipole-like choice for the wave function $\phi^{S/V}$ is usually

adopted,^{12–15} but the conclusions in this study are essentially determined by the spin structures and therefore has no dependence of the choice of the form of the spin-average wave function. Here we specify a choice of the spin-average wave function

$$\phi^{S/V}(x, \mathbf{k}_\perp) = \frac{g_{S/V}}{x\sqrt{1-x}} \left(M^2 - \frac{m^2 + \mathbf{k}_\perp^2}{x} - \frac{M_D^2 + \mathbf{k}_\perp^2}{1-x} \right)^{-2}, \quad (5)$$

to have numerical results. This choice respects the Lorentz invariance¹⁶ and the polynomiality property of the generalized parton distribution moments.¹⁴ Technically it makes the integrals over the transverse momenta analytically integrable. The values of the parameters are fitted to the electromagnetic form factors of the proton and neutron.¹⁵

3. Canonical Angular Momentums

The Fock state bases are eigenstates of the total OAM and the helicity of each constituent. Thus they are directly obtained from the LCWFs. Their values are

$$S_q^S = 0.383, \quad (6)$$

$$L^S = L_q^S + L_D^S = 0.117, \quad (7)$$

in the scalar case, and

$$S_q^V = -0.135, \quad (8)$$

$$S_D^V = 0.621 \quad (9)$$

$$L^V = L_q^V + L_D^V = 0.014, \quad (10)$$

in the axial-vector case. The angular momentum sum rule is satisfied in both the scalar and the axial-vector cases

$$J^S = S_q^S + L_q^S + L_D^S = \frac{1}{2}, \quad (11)$$

$$J^V = S_q^V + L_q^V + S_D^V + L_D^V = \frac{1}{2}. \quad (12)$$

The OAM carried by the quark can be evaluated with the intrinsic OAM operator,¹⁷ and for the quark-spectator system it can be calculated via^{18,19}

$$-i(1-x)\mathbf{k}_\perp \times \frac{\partial}{\partial \mathbf{k}_\perp}. \quad (13)$$

The numerical values are

$$L_q^S = 0.089, \quad (14)$$

$$L_q^V = 0.017. \quad (15)$$

Including the quark helicities, one may have quark total angular momentums as

$$J_q^S = S_q^S + L_q^S = 0.472, \quad (16)$$

$$J_q^V = S_q^V + L_q^V = -0.118. \quad (17)$$

Although the angular momentums here are evaluated via the canonical definition, they are equal to the kinetic ones since the difference between the canonical and kinetic OAM operators has no contribution in this no gluon model. Therefore they are expected to be related to the gravitational form factors as²⁰

$$J_q = S_q + L_q = \frac{1}{2}[A_q(0) + B_q(0)]. \quad (18)$$

4. Gravitational Form Factors

Similar to the Dirac and Pauli electromagnetic form factors, the two gravitational form factors $A(Q^2)$ and $B(Q^2)$ can be calculated from the spin-conserving and spin-flip matrix elements of the energy-momentum tensor as

$$\langle P + q, \uparrow | T^{++}(0) | P, \uparrow \rangle = 2(P^+)^2 A(Q^2), \quad (19)$$

$$\langle P + q, \uparrow | T^{++}(0) | P, \downarrow \rangle = 2(P^+)^2 \frac{-(q^1 - iq^2)}{2M} B(Q^2), \quad (20)$$

where $q^2 = -Q^2$ is transferred momentum square. With the effective Lagrangian (1), the contributions from the quark and the spectator are separated as

$$T_q^{++} = \frac{i}{2} [\bar{\psi} \gamma^+ \partial^+ \psi - (\partial^+ \bar{\psi}) \gamma^+ \psi], \quad (21)$$

$$T_D^{++} = \partial^+ \phi \partial^+ \phi - F^{+\rho} \partial^+ A_\rho, \quad (22)$$

where the quark part is exactly the same as that in QCD with the light-cone gauge $A_g^+ = 0$.

Then the quark gravitational form factors are expressed in terms of the LCWFs as

$$A_q^S(Q^2) = \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \sum_\lambda x \psi_{\lambda}^{S\dagger*}(x, \mathbf{k}'_\perp) \psi_{\lambda}^{S\dagger}(x, \mathbf{k}_\perp), \quad (23)$$

$$B_q^S(Q^2) = -\frac{2M}{q^1 - iq^2} \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \sum_\lambda x \psi_{\lambda}^{S\dagger*}(x, \mathbf{k}'_\perp) \psi_{\lambda}^{S\dagger}(x, \mathbf{k}_\perp), \quad (24)$$

for the scalar case, and

$$A_q^V(Q^2) = \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \sum_{\lambda, \lambda'} x \psi_{\lambda\lambda'}^{V\dagger*}(x, \mathbf{k}'_\perp) \psi_{\lambda\lambda'}^{V\dagger}(x, \mathbf{k}_\perp), \quad (25)$$

$$B_q^V(Q^2) = -\frac{2M}{q^1 - iq^2} \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \sum_{\lambda, \lambda'} x \psi_{\lambda\lambda'}^{V\dagger*}(x, \mathbf{k}'_\perp) \psi_{\lambda\lambda'}^{V\dagger}(x, \mathbf{k}_\perp), \quad (26)$$

Fig. 1. Quark gravitational form factors. The curves are our model results. The data are taken from two lattice QCD groups.

for the axial-vector case, where $\mathbf{k}'_{\perp} = \mathbf{k}_{\perp} + (1-x)\mathbf{q}_{\perp}$ is quark intrinsic transverse momentum in the final state.

At $Q^2 = 0$ limit, their values are

$$A_q^S(0) = 0.290, \quad B_q^S(0) = 0.422, \quad (27)$$

$$A_q^V(0) = 0.294, \quad B_q^V(0) = -0.370. \quad (28)$$

Together with the contribution from the spectator, one may find the momentum fraction sum rule

$$A^{S/V}(0) = A_q^{S/V}(0) + A_D^{S/V}(0) = 1, \quad (29)$$

and the anomalous gravitomagnetic moment sum rule²¹

$$B^{S/V}(0) = B_q^{S/V}(0) + B_D^{S/V}(0) = 0, \quad (30)$$

are satisfied in both cases. As a direct result, the total angular momentum is equal to half the sum of these two form factors

$$J = \frac{1}{2}[A(0) + B(0)] = \frac{1}{2}. \quad (31)$$

However this relation is violated when it is decomposed to each constituent

$$\frac{1}{2}[A_q^S(0) + B_q^S(0)] \neq J_q^S, \quad (32)$$

$$\frac{1}{2}[A_q^V(0) + B_q^V(0)] \neq J_q^V. \quad (33)$$

5. Conclusions

We perform a calculation of quark angular momentum and gravitational form factors in a light-cone spectator model. The sum rules of angular momentum, longitudinal momentum fraction and anomalous gravitomagnetic moment are all satisfied in both the scalar and the axial-vector cases. As a direct result, the total angular momentum of the proton is equal to half the sum of the gravitational form factors $A(0)$ and $B(0)$, which can be measured through the deeply virtual Compton scattering processes.

However, in such a no gluon field model the canonical angular momentums evaluated here coincide with the kinetic angular momentums which are usually believed equal to half the sum of the two gravitational form factors for each constituent. Our calculation shows that this relation is only valid in total but violated when decomposed to each constituent. Recently a perturbative QED example leads to the same conclusion.²³ Therefore more scrutinies are required on issues related to angular momentums.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grants No. 11035003, No. 11120101004 and No. 11475006).

Appendix: $A(Q^2)$ and $B(Q^2)$ with None-Zero Q^2 Transferred

Here we display the numerical results of quark gravitational form factors at isospin symmetric limit.²²

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