$B \rightarrow K^{(*)}\mu^+\mu^-$ in covariant quark model

A. Liptaj

*Institute of Physics, Slovak Academy of Sciences
Bratislava, Slovakia
andrej.liptaj@savba.sk*

S. Dubnička

*Institute of Physics, Slovak Academy of Sciences
Bratislava, Slovakia*

A. Z. Dubničková

*Faculty of Mathematics, Physics and Informatics
Comenius University, Bratislava, Slovakia*

M. A. Ivanov

*Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
Dubna, Russia*

Published 26 November 2015

The covariant quark model with infrared confinement (CQM) is a well-suited theoretical framework to describe large variety of hadronic processes, including rare decays of heavy mesons. In this text we focus on the reactions $B \rightarrow K^{(*)}\mu^+\mu^-$, which have been recently measured by Refs. 1–4. The measurements include also information about the angular distributions and their significance is given by possible New Physics (NP) effects which are predicted in numerous beyond Standard Model (SM) scenarios. Even with clever choice of experimental observables, a model dependence cannot be fully removed from the theoretical predictions. In this text we present the computation of the $B \rightarrow K^{(*)}$ form factors within the CQM and give results for some of the most commonly used observables (A_{FB} , F_L).

Keywords: Rare decays; heavy mesons; covariant quark model.

PACS Numbers: 12.39.Ki, 13.25.Hw, 13.25.Jx, 14.40.Nd, 13.20.He

This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 3.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

1. Introduction

New high-luminosity machines nowadays in service allow for unprecedented measurements of very rare flavor-changing processes. Especially some heavy meson decays have strong sensitivity to possible NP contributions, where new hypothetical particles enter loops in Feynman diagrams. One of them is the reaction $B \rightarrow K^{(*)}\mu^+\mu^-$, for which, today, even angular distributions have been measured. It is claimed that the SM is confirmed but few tensions at the level of 3σ remain. It is therefore interesting to make an independent theoretical check.

An effort is made to minimize the model dependence of the predictions by a clever choice of observables (asymmetries, ratios). However the dependence cannot be gotten rid of completely and therefore a use of a model is justified. We exploit the CQM with infrared confinement, which relies on a fully relativistic Lagrangian-based approach to hadronic interactions and a non-local quark current definition. It uses standard quantum field theory (QFT) techniques and has a limited number of parameters. Up to now the CQM has provided a nice agreement with experimental data.

2. Covariant Quark Model

The CQM Lagrangian (density) for the quark-meson interaction

$$\mathcal{L}_{\text{int}} = g_M \cdot M(x) \cdot J_M(x) \quad (1)$$

includes a non-local quark current

$$F_M(x, x_1, \dots, x_n) = \delta \left(x - \sum_{i=1}^2 w_i x_i \right) \Phi_H \left(\sum_{i < j} ((x_i - x_j)^2) \right), \quad (2)$$

where the weights are chosen so as to match the barycenter of the quark system with the hadron position $w_i = m_i / \sum_{j=1}^2 m_j$. The function Φ respects translational invariance and for computational convenience is chosen to be a Gaussian (in momentum space) with one free parameter

$$\bar{\Phi}_H(-k^2) = \exp(k^2/\Lambda_H^2). \quad (3)$$

The parameter Λ_H is interpreted as a hadron-size related parameter to be determined for every hadron. In addition, the CQM contains five free parameters: four constituent quark masses $m_q \equiv m_{u,d}, m_s, m_c, m_b$ and one universal infrared cut-off λ . Table 1 summarizes the numerical values settled by fits to the data. Quark-hadron couplings are eliminated as free parameters using so-called compositeness

Table 1. Parameters of the CQM. All values given in GeV.

$m_{u,d}$	m_s	m_c	m_b	λ_{cutoff}	Λ_B	Λ_K	Λ_K^*
0.241	0.428	1.67	5.05	0.181	1.96	1.02	0.75

condition.^{5,6} Unlike in the Lagrangian (1), where quarks and hadrons are elementary, in nature the hadrons are made up of quarks. The compositeness condition states, that the renormalization constant $Z_M^{1/2}$ can be interpreted as the matrix element between the physical state and the corresponding bare state. Requirement $Z_M^{1/2} = 0$ then implies that the physical state does not contain bare state and is properly described as a bound state. The condition can be rewritten as

$$Z_M = 1 - \frac{3g_M^2}{4\pi^2} \tilde{\Pi}'_M(m_M^2) = 0, \quad (4)$$

where $\tilde{\Pi}'_M$ is the derivative of the meson mass operator.

One needs also to address the issue of hadrons heavy enough to decay to constituent quarks. In the CQM the hadron stability is guaranteed by an infrared cutoff which is applied to the integration over the Schwinger parameters. These are introduced by the Schwinger representation of the quark propagator

$$\tilde{S}_q(k) = (m + \hat{k}) \int_0^\infty d\alpha e^{[-\alpha(m^2 - k^2)]} \quad (5)$$

which then leads to an expression with multidimensional improper integral. The latter can be transformed, by inserting unity in the form of a delta function, into an integration over simplex convoluted with only one improper integral. Then the cutoff is introduced

$$\begin{aligned} \Pi &= \int_0^\infty d^n \alpha F(\alpha_1, \dots, \alpha_n) \\ &= \int_0^{\infty \rightarrow \frac{1}{\lambda^2}} dt t^{n-1} \int_0^1 d^n \alpha \delta \left(1 - \sum_{i=1}^n \alpha_i \right) F(t\alpha_1, \dots, t\alpha_n), \end{aligned} \quad (6)$$

where all other ingredients related to the Feynman graph evaluation are implicitly contained in F . As consequence Π becomes smooth function, thresholds in quark loop diagrams and corresponding branch points are removed. A universal cutoff value $\lambda_{cutoff} = 0.181$ is used for all processes, the evaluation of integrals is numerical.

3. Form Factors and Differential Distribution

Depending on the nature of the final state particle (scalar vs. vector) different form factors are introduced.

- $B \rightarrow K\mu^+\mu^-$

$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 O^\mu q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = F_+(q^2) P^\mu + F_-(q^2) q^\mu \quad (7)$$

$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 (\sigma^{\mu\nu} q_\nu) q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = \frac{i}{m_1 + m_2} (q^2 P^\mu - q \cdot P q^\mu) F_T(q^2) \quad (8)$$

• $B \rightarrow K^* \mu^+ \mu^-$

$$\begin{aligned} & \langle V_{[\bar{q}_3, q_2]}(p_2, \epsilon_2) | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle \\ &= \frac{\epsilon_\nu^\dagger}{m_1 + m_2} [-g^{\mu\nu} P \cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) \\ &+ q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2)] \end{aligned} \quad (9)$$

$$\begin{aligned} & \langle V_{[\bar{q}_3, q_2]}(p_2, \epsilon_2) | \bar{q}_2 [\sigma^{\mu\nu} q_\nu (1 + \gamma^5)] q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle \\ &= \epsilon_\nu^\dagger \left[- \left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) P \cdot q a_0(q^2) \right. \\ &\quad \left. + \left(P^\mu P^\nu - q^\mu P^\nu \frac{P \cdot q}{q^2} \right) a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta g(q^2) \right] \end{aligned} \quad (10)$$

Using standard QFT techniques within the CQM, form factor behavior can be determined, as shown on Figs. 1(a)–1(c).

A necessary ingredient to accomplish the computations is an appropriate description of the $b \rightarrow s$ flavour transition. For that purpose we rely on an effective theory with four-fermion interaction which is based on local operators and the Wilson coefficients, their values are taken from the available literature. Invariant amplitudes and

Fig. 1. (a)–(c) Different form factors defined in Eqs. (7)–(10) as predicted by the CQM. (d) Kinematics of the reaction.

subsequently differential decay distributions can be then constructed from form factors using helicity techniques. First, so-called helicity amplitudes are defined which allow for the evaluation of the hadronic and leptonic tensors in their respective (different) frames. Next, the decay rates formulas are constructed, where we also include the case of the cascade-decay of K^* : $B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-$. The kinematics of the reaction is shown in Fig. 1(d), the full four-fold differential distribution as well as the details on the helicity techniques can be found in Ref. 7.

Using the definitions from the latter publication, one can express the usual observables as function of helicity amplitudes:

$$\begin{aligned} \frac{d\Gamma}{dq^2} = \frac{1}{2} & \left(\frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2} + \frac{d\Gamma_L^{11}}{dq^2} + \frac{d\Gamma_L^{22}}{dq^2} \right) \\ & + \frac{1}{2} \frac{d\tilde{\Gamma}_U^{11}}{dq^2} - \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{1}{2} \frac{d\tilde{\Gamma}_L^{11}}{dq^2} - \frac{d\tilde{\Gamma}_L^{22}}{dq^2} + \frac{3}{2} \frac{d\tilde{\Gamma}_S^{22}}{dq^2} \end{aligned} \quad (11)$$

$$A_{FB} = -\frac{3}{2} \frac{\int dq^2 H_P^{12}}{\int dq^2 H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}} \quad (12)$$

$$F_L = \frac{\int dq^2 H_L^{11} + H_L^{22}}{\int dq^2 H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}}. \quad (13)$$

For the meaning of different symbols the reader is advised to consult the Ref. 7. In a similar way the observables usually denoted by P_i (with i a number) can be expressed, covering within the helicity approach the most of the relevant quantities found in literature.

4. Results and Conclusion

Predictions of the CQM for branching fractions and A_{FB} and F_L observables are, together with the experimental values, shown in Tables 2 and 3. In the first case also the results for neutrinos and photon in the final state are included. In the second table we present integrated values over the interval $1 \text{ GeV} < q^2 < 6 \text{ GeV}$, which is the largest interval measured by experiments. The behavior of the unintegrated A_{FB} asymmetry in the whole allowed kinematic region as predicted by the CQM can be seen on Fig. 2. The CQM provides results for all bins of different observables as measured in experiments. However most of the experimental numbers have important errors and the comparison with the CQM is not conclusive.

Table 2. Branching fraction of B decays to $K^{(*)}$ and leptons/photon as measured and as predicted by the CQM.

Reaction	CQM	Experiment	Ref.
$\mathcal{B}[B \rightarrow K^*\mu^+\mu^-]$	1.27×10^{-6}	$(1.05 \pm 0.10) \times 10^{-6}$	8
$\mathcal{B}[B \rightarrow K\mu^+\mu^-]$	7.18×10^{-7}	$(3.4 \pm 0.5) \times 10^{-7}$	8
$\mathcal{B}[B \rightarrow K^*\nu\bar{\nu}]$	1.36×10^{-5}	$< 5.5 \times 10^{-5}$	8
$\mathcal{B}[B \rightarrow K\nu\bar{\nu}]$	0.60×10^{-5}	$< 4.9 \times 10^{-5}$	8
$\mathcal{B}[B \rightarrow K^*\gamma]$	3.74×10^{-5}	$(4.21 \pm 0.18) \times 10^{-5}$	9

Table 3. Observables A_{FB} and F_L : measurements and the CQM predictions.

$\langle A_{FB} \rangle$	$\langle F_L \rangle$	Ref.
$0.26^{+0.27}_{-0.30} \pm 0.07$	$0.67 \pm 0.23 \pm 0.05$	1
$-0.06^{+0.13}_{-0.14} \pm 0.04$	$0.55 \pm 0.10 \pm 0.03$	2
$0.29^{+0.20}_{-0.23} \pm 0.07$	$0.69^{+0.19}_{-0.21} \pm 0.08$	3
0.022	0.75	CQM

Fig. 2. Unintegrated A_{FB} in the CQM.

The CQM describes rather well the experimental numbers shown in Tables 2 and 3, the biggest mismatch being for the $B \rightarrow K\mu^+\mu^-$ decay. The observables A_{FB} and F_L have still important uncertainties (the first one especially) making a clear conclusion difficult to make.

Acknowledgments

The work was partly supported by Slovak Grant Agency for Sciences VEGA, grant No. 1/0158/13 (S. Dubnička, A. Z. Dubníčková, A. Liptaj), by Slovak Research and Development Agency APVV, grant No. APVV-0463-12 (S. Dubnička, A. Z. Dubníčková, A. Liptaj) and by Joint research project of Institute of Physics, SAS and Bogoliubov Laboratory of Theoretical Physics, JINR, No. 01-3-1114 (S. Dubnička, A. Z. Dubníčková, M. A. Ivanov and A. Liptaj).

References

1. J. T. Wei *et al.*, *Phys. Rev. Lett.* **103**, 171801 (2009).
2. R. Aaij *et al.*, *Phys. Rev. Lett.* **108**, 181806 (2012).
3. T. Aaltonen *et al.*, *Phys. Rev. Lett.* **108**, 081807 (2012).
4. J. P. Lees *et al.*, *Phys. Rev. D* **86**, 032012 (2012).
5. A. Salam, *Nuovo Cim.* **25**, 224 (1962).
6. S. Weinberg, *Phys. Rev.* **130**, 776 (1963).
7. A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, *Eur. Phys. J. Direct C* **4**, 18 (2002).
8. K. A. Olive *et al.*, *Chin. Phys. C* **38**, 090001 (2014).
9. Y. Amhis *et al.* (2012).