Pion polarizabilities: Theory vs Experiment

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Published 26 November 2015

The values of charged pion polarizabilities obtained in the framework of chiral perturbation theory at the level of two-loop accuracy are compared with the experimental result recently reported by COMPASS Collaboration. It is found that calculated value for the dipole polarizabilities $(\alpha - \beta)_{\pi^\pm} = (5.7 \pm 1.0) \times 10^{-4} \text{ fm}^3$ fits quite well the experimental result $(\alpha - \beta)_{\pi^\pm} = (4.0 \pm 1.2_{\text{stat}} \pm 1.4_{\text{syst}}) \times 10^{-4} \text{ fm}^3$.

Keywords: Pion polarizabilities; chiral perturbation theory; Primakoff experiment.

PACS Numbers: 11.30.Rd, 12.38.Aw, 12.39.Fe

1. Introduction

Almost ten years ago we have evaluated the amplitude for $\gamma\gamma \rightarrow \pi\pi$ in the framework of chiral perturbation theory (ChPT)¹⁻³ at two-loop order. It was done for neutral pions⁴ as well as for charged ones.⁵ The obtained results were found in agreement with the only previous calculation performed at this accuracy,^{6,7} provided that the same set of low-energy constants (LECs) is used. With updated LECs at order p^4 ,^{8,9} we found⁵ for the dipole polarizabilities the values

$$\begin{aligned}(\alpha - \beta)_{\pi^\pm} &= (5.7 \pm 1.0) \times 10^{-4} \text{ fm}^3, \\(\alpha + \beta)_{\pi^\pm} &= 0.16 \times 10^{-4} \text{ fm}^3.\end{aligned}\tag{1}$$

The MAMI Collaboration¹⁰ has reported in that time the experimental result

$$(\alpha - \beta)_{\pi^\pm} = (11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}) \times 10^{-4} \text{ fm}^3.\tag{2}$$

The index “mod” denotes the uncertainty generated by the theoretical models used to analyze the data. The ChPT calculation was clearly in conflict with MAMI prediction, see also Ref. 11 for a recent discussion.

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The COMPASS collaboration at CERN has recently investigated pion Compton scattering by using Primakoff effect. The pion polarizability has been determined to be¹²

$$(\alpha - \beta)_{\pi^\pm} = (4.0 \pm 1.2_{\text{stat}} \pm 1.4_{\text{syst}}) \times 10^{-4} \text{ fm}^3 \quad (3)$$

under the assumption $(\alpha + \beta)_{\pi^\pm} = 0$. This result is in agreement with the expectation from chiral perturbation theory.

The concept of the polarizability of molecules, atoms and nuclei was applied for the first time to hadrons in Refs. 13–15. By using the general properties of quantum field theory it was shown that an expansion of the Compton scattering amplitude on hadron with spin one half in small photon energy up to the second order contains two structure parameters called as electric and magnetic hadron polarizabilities. The classical sum rule for these quantities has been derived in Ref. 16. Further theoretical investigation of the pion polarizabilities has been pursued since the early 1970s. In current algebra + PCAC approach of Terent'ev,¹⁷ the fundamental low-energy theorem has been proven which allows one to relate the pion polarizability to the ratio $\gamma = h_A(0)/h_V(0)$, of the vector and axial form factors in radiative pion decay $\pi \rightarrow e\nu\gamma$. By using recent precise measurements of the pion weak form factors by PIBETA collaboration¹⁸ one finds $\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = 2.78(10) \times 10^{-4} \text{ fm}^3$.

There were many calculations of the pion polarizabilities by employing various models: the linear σ -model with quarks,¹⁹ the chiral quark model,²⁰ the superconductor quark model,²¹ some chiral models²² and so on.

Almost all of them except Terent'ev approach predicted a value of the electric polarizability within the range

$$4.0 \times 10^{-4} \text{ fm}^3 \leq \alpha_{\pi^\pm} \leq 6.0 \times 10^{-4} \text{ fm}^3 \quad (4)$$

which we call large-valued results. We note that models not based upon a chiral Lagrangian, i.e., dispersion relations and finite-energy sum rules, also obtained the polarizability within this range of values.^{23,24} The pion and kaon polarizabilities have been calculated in the quark confinement model²⁵ in which the emphasis is placed on quark confinement and the composite nature of hadrons. It was found for charged pions $\alpha_{\pi^\pm} \sim 3.6 \times 10^{-4} \text{ fm}^3$ which is smaller than large-valued results but slightly larger than Terent'ev's prediction.

The first correct calculation of the cross section $\gamma\gamma \rightarrow \pi\pi$ within chiral perturbation theory to next-to-leading order (one-loop accuracy) was performed in Ref. 26. It was shown in Ref. 27 that chiral symmetry relates the low-energy constants (LECs) appearing in the $\gamma\gamma \rightarrow \pi\pi$ -amplitude with the axial form factor $h_A(0)$. Thus it was shown explicitly that low-energy Terent'ev's theorem follows from one-loop calculation of the $\gamma\gamma \rightarrow \pi\pi$ process within chiral perturbation theory.

Note that the axial form factor $h_A(0)$ can be expressed through the dispersion integral of the difference of the vector and axial spectral densities.²⁸ By using this sum rule the pion polarizability was estimated in Ref. 29 and found in perfect agreement with chiral perturbation theory.

An actual two-loop ChPT calculation of the $\gamma\gamma \rightarrow \pi\pi$ amplitude was done in Ref. 6 (neutral pions) and Ref. 7 (charged pions). Because the effective Lagrangian at order p^6 was not available at that time, the ultraviolet divergences were evaluated in the $\overline{\text{MS}}$ scheme, then dropped and replaced with a corresponding polynomial in the external momenta. The three new counterterms which enter at this order in the low-energy expansion were estimated with resonance saturation. Whereas such a procedure is legitimate from a technical point of view, it does not make use of the full information provided by chiral symmetry.

Later on, considerable progress has been made in this field, both in theory and experiment. As for theory, the Lagrangian at order p^6 has been constructed,^{30,31} and its divergence structure has been determined.³² This provides an important check on the above calculations: adding the counterterm contributions from the p^6 Lagrangian to the $\overline{\text{MS}}$ amplitude evaluated in Ref. 6 and in Ref. 7 must provide a scale independent result. Also in the theory, improved techniques to evaluate the two-loop diagrams that occur in these amplitudes have been developed.³³ The updated calculation of the $\gamma\gamma \rightarrow \pi\pi$ to two loops was then performed in Ref. 4 (neutral pions) and Ref. 5 (charged pions). The final results for the pion polarizabilities were presented in a rather compact, algebraic form. By using updated values for the LECs one yields the values of pion polarizabilities given in Eq. (1).

The comprehensive review of the modern status of this field maybe found in Refs. 34 and 35.

Finally, one has to mention that the research of pion polarizabilities at Lattice simulation is currently conducted by several groups, see, for instance, Refs. 36–39.

2. Definition of Pion Polarizabilities

Electric (α_H) and magnetic (β_H) polarizabilities characterize the response of hadron to two-photon interactions. These quantities must be considered as fundamental as electromagnetic mean square radii, static magnetic moments, etc. They are defined by the expansion of the Compton scattering amplitude in small photon momenta and energies. Since our interest here is the pion polarizabilities we plot in Fig. 1 the diagram describing the Compton scattering by charged pion. Expanding the Compton scattering amplitudes in small photon momenta and energies,

Fig. 1. A schematic illustration of Compton scattering by pion.

one finds

$$T_{\gamma\pi^+\rightarrow\gamma\pi^+} = \underbrace{-2e^2\vec{\epsilon}_1\cdot\vec{\epsilon}_2}_{\text{Born term}} + 8\pi M_\pi \underbrace{\left\{\alpha_\pi\omega_1\omega_2\vec{\epsilon}_1\cdot\vec{\epsilon}_2 + \beta_\pi(\vec{\epsilon}_1\times\vec{q}_1)\cdot(\vec{\epsilon}_2\times\vec{q}_2)\right\}}_{\text{el-mag polarizabilities}} + \dots \quad (5)$$

It is convenient to use the linear combinations of the electric and magnetic polarizabilities: $(\alpha - \beta)_\pi$ and $(\alpha + \beta)_\pi$ which are obtained from the helicity flip and helicity non-flip amplitudes, respectively.

As follows from the definition, the dipole pion polarizabilities are proportional to $\alpha_\pi(\beta_\pi) \sim \frac{\alpha}{M_\pi} \frac{1}{\Lambda^2} \approx 4 \times 10^{-4} \text{ fm}^3$ where the hadronic scale $\Lambda \sim 4\pi F_\pi \sim 1 \text{ GeV}$ was used. Then a natural choice of units for the polarizabilities is 10^{-4} fm^3 .

3. Effective Lagrangian

We consider effective Lagrangian of QCD with two flavors, in the isospin symmetry limit $m_u = m_d = \hat{m}$. At next-to-next-to-leading order (NNLO), one has²

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6. \quad (6)$$

The subscripts refer to the chiral order. The expression for $\mathcal{L}_2$ is

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + M^2(U + U^\dagger) \rangle, \quad (7)$$

$$D_\mu U = \partial_\mu U - i(QU - UQ)A_\mu, \quad Q = \frac{e}{2} \text{diag}(1, -1),$$

where e is the electric charge, and A_μ denotes the electromagnetic field. The quantity F denotes the pion decay constant in the chiral limit, and M^2 is the leading term in the quark mass expansion of the pion (mass)², $M_\pi^2 = M^2(1 + O(\hat{m}))$. Further, the brackets $\langle \dots \rangle$ denote a trace in flavor space. In Eq. (7), we have retained only the terms relevant for the present application, i.e., we have dropped additional external fields. We choose the unitary 2×2 matrix U in the form

$$U = \sigma + i\pi/F, \quad \sigma^2 + \frac{\pi^2}{F^2} = \mathbf{1}_{2 \times 2}, \quad \pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}. \quad (8)$$

The Lagrangian at NLO has the structure²

$$\mathcal{L}_4 = \sum_{i=1}^7 l_i K_i + \sum_{i=1}^3 h_i \bar{K}_i = \frac{l_1}{4} \langle D_\mu U D^\mu U^\dagger \rangle^2 + \dots, \quad (9)$$

where l_i, h_i denote low-energy couplings, not fixed by chiral symmetry. At NNLO, one has^{30–32}

$$\mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i. \quad (10)$$

As was shown in Ref. 40 the number of operators P_i can be reduced by at least one from 57 to 56. For the explicit expressions of the polynomials $K_i, \bar{K}_i$ and P_i , we

refer the reader to Refs. 2 and 30–32. The vertices relevant for $\gamma\gamma \rightarrow \pi^+\pi^-$ involve $l_1, \dots, l_6$ from $\mathcal{L}_4$ and several c_i 's from $\mathcal{L}_6$, see below.

The couplings l_i and c_i absorb the divergences at order p^4 and p^6 , respectively,

$$\begin{aligned} l_i &= (\mu c)^{d-4} \{l_i^r(\mu, d) + \gamma_i \Lambda\}, \\ c_i &= \frac{(\mu c)^{2(d-4)}}{F^2} \{c_i^r(\mu, d) - \gamma_i^{(2)} \Lambda^2 - (\gamma_i^{(1)} + \gamma_i^{(L)}(\mu, d)) \Lambda\}, \\ \Lambda &= \frac{1}{16\pi^2(d-4)}, \quad \ln c = -\frac{1}{2} \{\ln 4\pi + \Gamma'(1) + 1\}. \end{aligned} \quad (11)$$

The physical couplings are $l_i^r(\mu, 4)$ and $c_i^r(\mu, 4)$, denoted by l_i^r, c_i^r in the following. The coefficients γ_i are given in Ref. 2, and $\gamma_i^{(1,2,L)}$ are tabulated in Ref. 32. We shall use the scale independent quantities $\bar{l}_i$ introduced in Ref. 2,

$$l_i^r = \frac{\gamma_i}{32\pi^2} (\bar{l}_i + l), \quad (12)$$

where the *chiral logarithm* is $l = \ln(M_\pi^2/\mu^2)$. We shall use⁸

$$\bar{l}_1 = -0.4 \pm 0.6, \quad \bar{l}_2 = 4.3 \pm 0.1, \quad \bar{l}_3 = 2.9 \pm 2.4, \quad \bar{l}_4 = 4.4 \pm 0.2, \quad (13)$$

and

$$\bar{l}_\Delta \doteq \bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3 \quad (14)$$

obtained from radiative pion decay to two loop accuracy.^{9,41}

The constants c_i^r occur in the combinations

$$\begin{aligned} a_1^r &= -4096\pi^4 (6c_6^r + c_{29}^r - c_{30}^r - 3c_{34}^r + c_{35}^r + 2c_{46}^r - 4c_{47}^r + c_{50}^r), \\ a_2^r &= 256\pi^4 (8c_{29}^r - 8c_{30}^r + c_{31}^r + c_{32}^r - 2c_{33}^r + 4c_{44}^r + 8c_{50}^r - 4c_{51}^r), \\ b^r &= -128\pi^4 (c_{31}^r + c_{32}^r - 2c_{33}^r - 4c_{44}^r). \end{aligned}$$

As follows from the resonance exchange model⁷

$$(a_1^r, a_2^r, b^r) = (-3.2, 0.7, 0.4) \quad [\text{present work}]. \quad (15)$$

The values of these constants were obtained in the ENJL model⁴² $(a_1^r, a_2^r, b^r) = (-8.7, 5.9, 0.38)$. One can see that only b^r agrees in the two approaches. We shall use $b^r = 0.4 \pm 0.4$. The combinations $(\alpha \pm \beta)_{\pi^\pm}$ are independent of a_2^r and determined precisely by the chiral expansion to two loops, once a_1^r is fixed. We will then simply display this quantity as a function of a_1^r - the result turns out to be rather independent of its exact value.

4. Evaluation of the Diagrams

The lowest-order contributions to the scattering amplitude are described by tree- and one-loop diagrams. These contributions were calculated in Ref. 26. The two-loop diagrams are displayed in the Figs. 2, 4 and 5. The two-loop diagrams in

Fig. 2. A set of two-loop diagrams generated by $\mathcal{L}_2$ and one-loop diagrams generated by $\mathcal{L}_4$.

Fig. 2 may be generated according to the scheme indicated in Fig. 3, where the filled in blob denotes the d -dimensional elastic $\pi\pi$ -scattering amplitude at one-loop accuracy, with two pions off-shell.

The diagrams shown in Fig. 4 may be reduced to tree-diagrams by using Ward identities. They sum up to the expression

$$2Z_\pi g^{\mu\nu} - \left\{ (2p_1 - q_1)^\mu (2p_2 - q_2)^\nu \left[\frac{1}{M_\pi^2 - t} - Z_\pi R(t) \right] + \text{crossed} \right\}, \quad (16)$$

where Z_π is the pion renormalization constant. The function $R(t)$ starts at order $1/F_\pi^4$ and can be obtained from the full pion propagator.

Fig. 3. Construction scheme for the diagrams in Fig. 2.

Fig. 4. A class of one-particle reducible diagrams. The filled in circles summarize self-energy and vertex corrections.

Fig. 5. Acnode and butterfly diagrams.

Fig. 6. The remaining diagrams at order p^6 : one-loop graphs generated by $\mathcal{L}_4$, and counterterm contributions from $\mathcal{L}_6$.

Two further diagrams are displayed in Fig. 5. The first one — called “acnode” in the literature — may again be evaluated by use of a dispersion relation, see Ref. 4. The second one is trivial to evaluate, because it is a product of one-loop diagrams. The remaining diagrams at order p^6 are shown in Fig. 6.

The evaluation of the diagrams was done in the manner described in Refs. 4, 33 and invoking FORM.⁴³ In particular, we have verified that the counterterms from the Lagrangian $\mathcal{L}_6$ ³² remove all ultraviolet divergences, which is a very non-trivial check on our calculation. Furthermore, we have checked that the (ultra-violet finite) amplitude so obtained is scale independent.

5. Chiral Expansion for Pion Polarizabilities

Using the same notation as in,⁷ we find for the *dipole* polarizabilities

$$(\alpha \pm \beta)_{\pi^+} = \frac{\alpha}{16 \pi^2 F_\pi^2 M_\pi} \left\{ c_{1\pm} + \frac{M_\pi^2 d_{1\pm}}{16 \pi^2 F_\pi^2} + O(M_\pi^4) \right\}, \quad (17)$$

where

$$\begin{aligned} c_{1+} &= 0, \quad c_{1-} = \frac{2}{3} \bar{l}_\Delta, \\ d_{1+} &= 8b^r - \frac{4}{9} \left\{ l \left(l + \frac{1}{2} \bar{l}_1 + \frac{3}{2} \bar{l}_2 \right) - \frac{53}{24} l + \frac{1}{2} \bar{l}_1 + \frac{3}{2} \bar{l}_2 + \frac{91}{72} + \Delta_+ \right\}, \\ d_{1-} &= a_1^r + 8b^r - \frac{4}{3} \left\{ l \left(\bar{l}_1 - \bar{l}_2 + \bar{l}_\Delta - \frac{65}{12} \right) - \frac{1}{3} \bar{l}_1 - \frac{1}{3} \bar{l}_2 \right. \\ &\quad \left. + \frac{1}{4} \bar{l}_3 - \bar{l}_\Delta \bar{l}_4 + \frac{187}{108} + \Delta_- \right\}, \end{aligned} \quad (18)$$

with

$$\Delta_+ = \frac{8105}{576} - \frac{135}{64} \pi^2, \quad \Delta_- = \frac{41}{432} - \frac{53}{64} \pi^2. \quad (19)$$

It would be interesting to compare the values of $\Delta_\pm$ given by Eq. (19) with those obtained in Refs.⁷ numerically. One has

$$\Delta_+ = \begin{cases} -6.75 & \text{our} \\ -8.69 & \text{Burgi} \end{cases} \quad \Delta_- = \begin{cases} -8.08 & \text{our} \\ -8.73 & \text{Burgi} \end{cases}$$

The results for polarizabilities evaluated with the central values for the LECs in Eqs. (13)–(15) are shown in Table 1. The uncertainty in the prediction for the polarizability has two sources. First, the low-energy constants are not known precisely. Second, we are dealing here with an expansion in powers of the momenta and of the quark masses up to and including terms of order p^6 . The discussion of estimating uncertainties may be found in our paper.⁵ It was shown that the value for the dipole polarizability $(\alpha - \beta)_{\pi^\pm}$ is rather reliable — there is no sign of any large, uncontrolled correction to the two-loop result. The maximum deviation 1.0 from the central value 5.7 has been used as the final theoretical uncertainty for the dipole polarizability:

$$(\alpha - \beta)_{\pi^\pm} = (5.7 \pm 1.0) \times 10^{-4} \text{ fm}^3. \quad (20)$$

The chiral expansion for the combination $(\alpha + \beta)_{\pi^\pm}$ starts out at order p^6 so we have determined only its leading order term.

6. Experimental Information

There are three types of experiments aiming to measure the pion polarizabilities:

- The scattering of high energy pions off the Coulomb field of heavy nucleus by using Primakoff effect.

Table 1. Central values of polarizabilities in units of 10^{-4} fm^3 .

	to one loop	to two-loops
$(\alpha - \beta)_{\pi^+}$	6.0	5.7
$(\alpha + \beta)_{\pi^+}$	0	0.16

Fig. 7. A schematic images of available experiments.

- The radiative pion photoproduction from the proton.
- The pion pair production in photon-photon collisions.

Schematically, they are shown in Fig. 7. The possibility to measure pion polarizability via Primakoff reaction was proposed in the early 1980s in Ref. 44. The measurement of the pion-photon Compton scattering amplitude by using Primakoff effect was performed in an experiment at Serpukhov,⁴⁵ but the small data sample led to only an imprecise value for polarizability of $\alpha_\pi = (6.8 \pm 1.4_{\text{stat}} \pm 1.2_{\text{syst}}) \times 10^{-4} \text{ fm}^3$. Low statistics made it difficult to evaluate the systematic uncertainty.

COMPASS has now achieved a modern Primakoff experiment, using a 190 GeV pion beam from the Super Proton Synchrotron at CERN directed at a nickel target. It is important that COMPASS was also able to use muon, which is point-like particle, to calibrate the experiment. The Compton $\pi^- \gamma \rightarrow \pi^- \gamma$ scattering is extracted from reaction $\pi^- Ni \rightarrow \pi^- \gamma Ni$ by selecting events from the Coulomb peak at small momentum transfer $Q^2 < 0.0015 \text{ GeV}^2$. From the analysis of a sample of 63,000 events, the collaboration obtained a value of the pion electric polarizability of Ref. 12, $\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3$ under assumption $(\alpha + \beta)_\pi = 0$.

The cross section for the radiative pion photoproduction $\gamma p \rightarrow \gamma \pi^+ n$ has been measured at the Lebedev Institute.⁴⁶ By using an extrapolation to the pion pole in the unphysical region the value of the electric polarizability was obtained $\alpha_{\pi^+} = -\beta_{\pi^+} = (20 \pm 12) \times 10^{-4} \text{ fm}^3$. Similar experiment was performed at the Mainz Microtron MAMI¹⁰ but the pion polarizability have been extracted by a comparison of the data with the predictions of two different models yielding the value $(\alpha - \beta)_{\pi^+} = (11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}) \times 10^{-4} \text{ fm}^3$.

Another possibility to obtain the value for the pion polarizability is to extrapolate the data from the pion pair production in photon-photon collisions $\gamma\gamma \rightarrow \pi\pi$ to the region of the Compton scattering threshold by using crossing symmetry and analyticity. Normally, the procedure involves the construction of the dispersion relations with one or two subtractions. The most recent analysis performed in Ref. 24 produced the value $(\alpha - \beta)_{\pi^\pm} = 13.0^{+2.6}_{-1.9} \times 10^{-4} \text{ fm}^3$ which is close to the MAMI data. There are plenty of previous studies in this direction which give quite broad region for the value of the pion polarizability. The available experimental information is shown in Table 2.

Table 2. Experimental information on $(\alpha - \beta)_{\pi\pm}$, in units of 10^{-4} fm^3 . We indicate the reaction and data used. In⁵²,⁴⁷ and¹² α_π was determined, using as a constraint $\alpha_\pi = -\beta_\pi$. To obtain $(\alpha - \beta)_{\pi\pm}$, we multiplied the results by a factor of 2.

Experiments		$(\alpha - \beta)_{\pi\pm}$
$\pi^- Z \rightarrow \gamma \pi^- Z$	Serpukhov (1983) ⁴⁵	$15.6 \pm 6.4_{\text{stat}} \pm 4.4_{\text{syst}}$
$\gamma p \rightarrow \gamma \pi^+ n$	Lebedev Inst. (1984) ⁴⁶	40 ± 24
D. Babusci <i>et al.</i> (1992) ⁴⁷		
$\gamma\gamma \rightarrow \pi^+ \pi^-$	PLUTO ⁴⁸	$38.2 \pm 9.6 \pm 11.4$
	DM 1 ⁴⁹	34.4 ± 9.2
	DM 2 ⁵⁰	52.6 ± 14.8
	MARK II ⁵¹	4.4 ± 3.2
J. F. Donoghue, B. Holstein (1993) ⁵²		5.4
$\gamma\gamma \rightarrow \pi^+ \pi^-$	MARK II ⁵¹	
A. Kaloshin, V. Serebryakov (1994) ⁵³		5.25 ± 0.95
$\gamma\gamma \rightarrow \pi^+ \pi^-$	MARK II ⁵¹	
	Crystal Ball Coll. ⁵⁴	
$\gamma p \rightarrow \gamma \pi^+ n$	Mainz (2005) ¹⁰	$11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}$
L. Fil'kov, V. Kashevarov (2005) ²⁴		$13.0^{+2.6}_{-1.9}$
$\gamma\gamma \rightarrow \pi^+ \pi^-$	MARK II ⁵¹	
	TPC/2 γ , ⁵⁵ CELLO ⁵⁶	
	VENUS, ⁵⁷ ALEPH, ⁵⁸ BELLE ⁵⁹	
$\pi^- Z \rightarrow \gamma \pi^- Z$	COMPASS (2015) ¹²	$4.0 \pm 1.2_{\text{stat}} \pm 1.4_{\text{syst}}$

Acknowledgments

I would like to thank Juerg Gasser and Mikko Sainio in collaboration with whom the results for pion polarizabilities at two-loop level accuracy were obtained. It has been a pleasure to discuss many points of this subject with L. V. Fil'kov, S. B. Gerasimov, A. G. Olshevski, V. A. Petrunin and M. K. Volkov. I am grateful to Stanislav Dubnička and Zuzana Dubničková for inviting me to give a talk at the conference “Hadron Structure’15”.

References

1. S. Weinberg, *Physica A* **96**, 327 (1979).
2. J. Gasser and H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).
3. J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).
4. J. Gasser, M. A. Ivanov and M. E. Sainio, *Nucl. Phys. B* **728**, 31 (2005) [hep-ph/0506265].
5. J. Gasser, M. A. Ivanov and M. E. Sainio, *Nucl. Phys. B* **745**, 84 (2006) [hep-ph/0602234].
6. S. Bellucci, J. Gasser and M. E. Sainio, *Nucl. Phys. B* **423**, 80 (1994) [Erratum-ibid. B **431**, 413 (1994)] [arXiv:hep-ph/9401206].

7. U. Burgi, *Nucl. Phys. B* **479**, 392 (1996) [arXiv:hep-ph/9602429]; U. Burgi, *Phys. Lett. B* **377**, 147 (1996) [arXiv:hep-ph/9602421].
8. G. Colangelo, J. Gasser and H. Leutwyler, *Nucl. Phys. B* **603**, 125 (2001) [arXiv:hep-ph/0103088].
9. J. Bijnens and P. Talavera, *Nucl. Phys. B* **489**, 387 (1997) [arXiv:hep-ph/9610269].
10. J. Ahrens *et al.*, *Eur. Phys. J. A* **23**, 113 (2005) [arXiv:nucl-ex/0407011].
11. S. Scherer, *Eur. Phys. J. A* **28**, 59 (2006) [hep-ph/0512291].
12. COMPASS Collab. (C. Adolph *et al.*), *Phys. Rev. Lett.* **114**, 062002 (2015) [arXiv:1405.6377 [hep-ex]].
13. A. Klein, *Phys. Rev.* **99**, 998 (1955).
14. A. M. Baldin, *Nucl. Phys.* **18**, 310 (1960).
15. V. A. Petrunin, *JETP* **13**, 804 (1961).
16. V. A. Petrunin, *Nucl. Phys.* **55**, 197 (1964).
17. M. V. Terentev, *Sov. J. Nucl. Phys.* **16**, 87 (1973) [*Yad. Fiz.* **16**, 162 (1972)]; M. V. Terentev, *Sov. J. Nucl. Phys.* **19**, 664 (1974) [*Yad. Fiz.* **19**, 1298 (1974)].
18. M. Bychkov *et al.*, *Phys. Rev. Lett.* **103**, 051802 (2009) [arXiv:0804.1815 [hep-ex]].
19. A. I. L'vov, *Sov. J. Nucl. Phys.* **34**, 289 (1981) [*Yad. Fiz.* **34**, 522 (1981)].
20. M. K. Volkov and D. Ebert, *Sov. J. Nucl. Phys.* **34**, 104 (1981) [*Yad. Fiz.* **34**, 182 (1981)]; *Phys. Lett. B* **101**, 252 (1981).
21. M. K. Volkov and A. A. Osipov, *Sov. J. Nucl. Phys.* **41**, 659 (1985); *Yad. Fiz.* **41**, 1027 (1985).
22. V. Bernard, B. Hiller and W. Weise, *Phys. Lett. B* **205**, 16 (1988).
23. L. V. Fil'kov, I. Guiasu and E. E. Radescu, *Phys. Rev. D* **26**, 3146 (1982).
24. L. V. Fil'kov and V. L. Kashevarov, *Phys. Rev. C* **73**, 035210 (2006) [nucl-th/0512047].
25. M. A. Ivanov and T. Mizutani, *Phys. Rev. D* **45**, 1580 (1992).
26. J. Bijnens and F. Cornet, *Nucl. Phys. B* **296**, 557 (1988).
27. J. F. Donoghue and B. R. Holstein, *Phys. Rev. D* **40**, 2378 (1989).
28. T. Das, V. S. Mathur and S. Okubo, *Phys. Rev. Lett.* **19**, 859 (1967).
29. A. E. Dorokhov and W. Broniowski, *Eur. Phys. J. C* **32**, 79 (2003) [hep-ph/0305037].
30. H. W. Fearing and S. Scherer, *Phys. Rev. D* **53**, 315 (1996) [arXiv:hep-ph/9408346].
31. J. Bijnens, G. Colangelo and G. Ecker, *JHEP* **9902**, 020 (1999) [arXiv:hep-ph/9902437].
32. J. Bijnens, G. Colangelo and G. Ecker, *Ann. Phys.* **280**, 100 (2000) [arXiv:hep-ph/9907333].
33. J. Gasser and M. E. Sainio, *Eur. Phys. J. C* **6**, 297 (1999) [arXiv:hep-ph/9803251].
34. V. A. Petrunin, *Sov. J. Part. Nucl.* **12**, 278 (1981) [*Fiz. Elem. Chast. Atom. Yadra* **12**, 692 (1981)].
35. B. R. Holstein and S. Scherer, *Ann. Rev. Nucl. Part. Sci.* **64**, 51 (2014) [arXiv:1401.0140 [hep-ph]].
36. E. V. Luschevskaya, O. E. Solovjeva, O. A. Kochetkov and O. V. Teryaev, *Nucl. Phys. B* **898**, 627 (2015) [arXiv:1411.4284 [hep-lat]].
37. M. Lujan, A. Alexandru, W. Freeman and F. Lee, *Phys. Rev. D* **89**, 074506 (2014) [arXiv:1402.3025 [hep-lat]].
38. W. Detmold, B. C. Tiburzi and A. Walker-Loud, *Phys. Rev. D* **79**, 094505 (2009) [arXiv:0904.1586 [hep-lat]].
39. J. Hu, F. J. Jiang and B. C. Tiburzi, *Phys. Rev. D* **77**, 014502 (2008) [arXiv:0709.1955 [hep-lat]].
40. C. Haefeli, M. A. Ivanov, M. Schmid and G. Ecker, "On the mesonic Lagrangian of order p^6 in chiral SU(2)," arXiv:0705.0576 [hep-ph].

41. C. Q. Geng, I. L. Ho and T. H. Wu, *Nucl. Phys. B* **684**, 281 (2004) [arXiv:hep-ph/0306165].
42. J. Bijnens and J. Prades, *Nucl. Phys. B* **490**, 239 (1997) [arXiv:hep-ph/9610360].
43. J. A. M. Vermaseren, "New features of FORM," arXiv:math-ph/0010025.
44. A. S. Galperin, G. Mitselmakher, A. G. Olszewski and V. N. Pervushin, *Yad. Fiz.* **32**, 1053 (1980).
45. Y. M. Antipov *et al.*, *Z. Phys. C* **26** (1985) 495; Y. M. Antipov *et al.*, *Phys. Lett. B* **121** (1983) 445.
46. T. A. Aibergenov *et al.*, *Czech. J. Phys. B* **36**, 948 (1986).
47. D. Babusci, S. Bellucci, G. Giordano, G. Matone, A. M. Sandorfi and M. A. Moinester, *Phys. Lett. B* **277**, 158 (1992).
48. PLUTO Collab. (C. Berger *et al.*), *Z. Phys. C* **26**, 199 (1984).
49. DM1-DM2 Collab. (A. Courau *et al.*), *Nucl. Phys. B* **271**, 1 (1986).
50. DM1-DM2 Collab. (Z. Ajaltouni *et al.*), *Phys. Lett. B* **194**, 573 (1987) [Erratum-ibid. *Phys. Lett. B* **197**, 565 (1987)].
51. MARK-II Collab. (J. Boyer *et al.*), *Phys. Rev. D* **42**, 1350 (1990).
52. J. F. Donoghue and B. R. Holstein, *Phys. Rev. D* **48**, 137 (1993) [arXiv:hep-ph/9302203].
53. A. E. Kaloshin and V. V. Serebryakov, *Z. Phys. C* **64**, 689 (1994) [arXiv:hep-ph/9306224].
54. Crystal Ball Collab. (H. Marsiske *et al.*), *Phys. Rev. D* **41**, 3324 (1990).
55. TPC/Two-Gamma Collab. (H. Aihara *et al.*), *Phys. Rev. Lett.* **57**, 404 (1986).
56. CELLO Collab. (H. J. Behrend *et al.*), *Z. Phys. C* **56**, 381 (1992).
57. VENUS Collab. (F. Yabuki *et al.*), *J. Phys. Soc. Jap.* **64**, 435 (1995).
58. ALEPH Collab. (A. Heister *et al.*), *Phys. Lett. B* **569**, 140 (2003).
59. BELLE Collab. (H. Nakazawa *et al.*), *Phys. Lett. B* **615**, 39 (2005) [arXiv:hep-ex/0412058].