Nucleon and Δ electromagnetic form factors

Erik Bartoš

*Institute of Physics, Slovak Academy of Sciences
Dúbravská cesta 9, 845 11 Bratislava, Slovakia
erik.bartos@savba.sk*

Stanislav Dubníčka

*Institute of Physics, Slovak Academy of Sciences
Dúbravská cesta 9, 845 11 Bratislava, Slovakia
stanislav.dubnicka@savba.sk*

Anna-Zuzana Dubníčková

*Faculty of Mathematics, Physics and Informatics
Comenius University, Mlynská dolina
842 48 Bratislava, Slovakia
dubnickova@fmph.uniba.sk*

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The unitary and analytic model for nucleons works very well. The model for spin $\frac{1}{2}^+$ baryons can predict static parameters for Λ , Σ^+ , Σ^0 , Σ^- , Ξ , Ξ^- . We prepared the scheme how to construct the model also for Δ nucleon resonance. It was used to describe transition form factor G_M^* and the experimentally measured ratio R_{SM} with the help of relations for magnetic dipole transition and charge quadrupole form factors.

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1. Introduction

The nature of the electromagnetic structure of the nucleons and the mesons has been an ongoing challenge for the physicists in the past decades. The understanding their inner structure seems to be an important step to the description of their non-pointlike nature and their interactions with electrons and photons in the scattering processes. The electromagnetic structure of the nucleons is described by the elastic form factors to whose investigations has been devoted a lot of effort. The new

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measurements using the polarization transfer method at Jefferson Lab¹⁻⁴ revealed us a disagreement with the previous data obtained by Rosenbluth separation technique.⁵ Also the extraction of the charge radius r_p of the proton from hyperfine splitting in muonic hydrogen^{6,7} uncovered the gap between received value and the ones from electron scattering⁸⁻¹⁰ or H spectroscopy results.¹¹ However, one can see the effort to clear up these discrepancies from both sides, the theoretical^{12,13} and the experimental¹⁴ ones.

The obtained theoretical framework can be extended for the prediction of the cross sections of heavier baryons,¹⁵ which will be measured in the near future. With the advent of intense, energetic electron-beam facilities (JLab,¹⁶⁻¹⁸ MAMI,¹⁹⁻²¹ LEGS,²² MIT/Bates²³⁻²⁵) also an investigation of the properties of nucleon resonances plays an important role in fully understanding the structure of the nucleons. $\Delta(1232)$ resonance is the lowest excited state of nucleons, isolated from other nucleon excitations. Since it is a $J = \frac{3}{2}$ state, its electromagnetic structure is described by four multipole form factors: $G_{E0}(Q^2)$ — electric charge, $G_{M1}(Q^2)$ — magnetic dipole, $G_{E2}(Q^2)$ — electric quadrupole and $G_{M3}(Q^2)$ magnetic octupole

$$G_{E0}(Q^2) = \left(1 + \frac{2\tau_\Delta}{3}\right) (F_1^* - \tau_\Delta F_2^*) - \frac{\tau_\Delta}{3} (1 + \tau_\Delta) (F_3^* - \tau_\Delta F_4^*), \quad (1)$$

$$G_{M1}(Q^2) = \left(1 + \frac{4\tau_\Delta}{5}\right) (F_1^* + F_2^*) - \frac{2\tau_\Delta}{5} (1 + \tau_\Delta) (F_3^* + F_4^*), \quad (2)$$

$$G_{E2}(Q^2) = (F_1^* - \tau_\Delta F_2^*) - \frac{1}{2} (1 + \tau_\Delta) (F_3^* - \tau_\Delta F_4^*), \quad (3)$$

$$G_{M3}(Q^2) = (F_1^* + F_2^*) - \frac{1}{2} (1 + \tau_\Delta) (F_3^* + F_4^*), \quad (4)$$

with dimensionless multipole moments at $Q^2 = 0$: $G_{E0}(0) = e_\Delta$, $G_{M1}(0) = \hat{\mu}_\Delta$, $G_{E2}(0) = \hat{Q}_\Delta$, $G_{M3}(0) = \hat{O}_\Delta$ and $\tau_\Delta = Q^2/(4m_\Delta^2)$, $\{F_i^* = F_i^*(Q^2), i = 1, 2, 3, 4\}$ are Poincaré invariant form factors defined by the decomposition of electromagnetic matrix operator.²⁶

While it is relatively straightforward to measure the mass and decay width of Δ -baryon in the reactions such as scattering of electron beams or target photons from nucleons (see Fig. 1), the measurement of its electromagnetic properties is very difficult due to the very small mean lifetime and we know experimentally very little about the elastic form factors of Δ -baryon. At this moment all theoretical predictions (e. g., Ref. 27) can be compared only to lattice-regularized QCD results,²⁸ unfortunately, these results produce very large Δ -baryon masses and the form factors obtained therewith should be interpreted carefully.

2. $\gamma^*N \rightarrow \Delta$ Transition Form Factors

On the other hand, further insight into the structure of nucleons has been achieved through the measurement of the $\gamma^*N \rightarrow \Delta$ transition ($N \rightarrow \Delta$ denotes both

Fig. 1. Inelastic electron-nucleon scattering with π^0 electroproduction and formation of Δ -baryon resonance (left), $\gamma^*N \rightarrow \Delta$ transition vertex (right).

the $p \rightarrow \Delta^+$ and the $n \rightarrow \Delta^0$ transitions), which is described by three transition multipole form factors: $G_{M1}^{N \rightarrow \Delta}(Q^2)$ — magnetic dipole, $G_{E2}^{N \rightarrow \Delta}(Q^2)$ — electric quadrupole, $G_{C2}^{N \rightarrow \Delta}(Q^2)$ — Coulomb quadrupole. The inelastic cross section for electroproduction of the Δ -baryon is given by the formula

$$\left(\frac{d\sigma}{d\Omega} \right)_i = \sigma_M f_r \frac{Q^2}{4m_N m_\Delta} \frac{1}{1 + \tilde{\tau}} \times \left\{ \frac{Q^2 m_N}{9} (G_{C2}^{N \rightarrow \Delta}(Q^2))^2 + \varepsilon \left((G_{M1}^{N \rightarrow \Delta}(Q^2))^2 + \frac{m_N Q^2}{18} (G_{E2}^{N \rightarrow \Delta}(Q^2))^2 \right) \right\} \quad (5)$$

with the Mott cross section σ_M , $\tilde{\tau} = Q^2/(m_\Delta + m_N)^2$, photon transverse polarization $\varepsilon = (1 + 2(m_\Delta^2/m_N^2)(|\mathbf{Q}|^2/Q^2) \tan^2(\vartheta/2))$ and the normalization to the transition magnetic moment $G_{M1}^{N \rightarrow \Delta}(0) = \mu_{N \rightarrow \Delta}$ and quadrupole moment $G_{C2}^{N \rightarrow \Delta}(0) = Q_{N \rightarrow \Delta}$.

The relation between the inelastic $N \rightarrow \Delta$ magnetic dipole $G_{M1}^{N \rightarrow \Delta}(Q^2)$ and elastic neutron magnetic G_M^n form factors is based on the underlying SU(6) spin-flavor symmetry of QCD

$$G_{M1}^{N \rightarrow \Delta}(Q^2) = -\sqrt{2} G_M^n(Q^2), \quad (6)$$

where from their normalization conditions at $Q^2 = 0$ follows

$$\mu_{N \rightarrow \Delta} = -\sqrt{2} \mu_n.$$

Another useful relation was derived in Ref. 29, which connects the inelastic $N \rightarrow \Delta$ charge quadrupole $G_{C2}^{N \rightarrow \Delta}(Q^2)$ and elastic neutron charge G_E^n form factors

$$G_{C2}^{N \rightarrow \Delta}(Q^2) = -\frac{3\sqrt{2}}{Q^2} G_E^n(Q^2). \quad (7)$$

In connection with transition form factors of $\Delta(1232)$ resonance, the following form factor ratios R_{EM} , R_{SM} are often considered (with another definition of dimensionless transition form factors G_E^* , G_C^* , G_M^* used by some authors, see Eq. (9))

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \quad R_{SM} = -\frac{|\mathbf{Q}|}{2m_\Delta} \frac{G_C^*}{G_M^*} \quad (8)$$

and experimentally accessible for large range of the momentum transfer $0 \leq Q^2 \leq 8 \text{ GeV}^2$.^{16–25} Combining the Eqs. (6) and (7) one comes to the relation

$$\frac{C2}{M1} \equiv \frac{|\mathbf{Q}|m_N}{6} \frac{G_{C2}^{N \rightarrow \Delta}(Q^2)}{G_{M1}^{N \rightarrow \Delta}(Q^2)} = \frac{|\mathbf{Q}|m_N}{2Q^2} \frac{G_E^n(Q^2)}{G_M^n(Q^2)} = \frac{|\mathbf{Q}|}{2m_\Delta} \frac{G_C^*}{G_M^*}, \quad (9)$$

which allows to confront the experimental data with theoretical predictions. Such analysis were performed with the results presented in Fig. 2, where our previously

Fig. 2. The measurements of R_{SM} ratio in comparison with theoretical predictions by dipole formulae and our model.

Fig. 3. The measurements of transition form factor G_M^* in comparison with theoretical predictions by dipole formulae and our model.

demonstrated fit U&A model of nucleons¹³ was compared with the result of dipole fit G_D for G_M^n , see Eq. (8) in Ref. 29. The U&A model was also used for the prediction of the behavior of the transition form factor G_M^* in Fig. 3, but the up to now received results differ very little from the dipole formula approach³⁰ and another.

3. Conclusions

We have presented our unitary and analytic model of nucleons which can be easily extended for the description of static parameters of spin $\frac{1}{2}^+$ baryons Λ , Σ^+ , Σ^0 , Σ^- , Ξ , Ξ^- . In framework of our U&A model it would be very interesting to predict the behavior of Δ -baryon elastic form factors. The lack of experimental data (unknown coupling constants $f_{\omega\Delta\Delta}/f_\omega$, $f_{\rho\Delta\Delta}/f_\rho$, $f_{\phi\Delta\Delta}/f_\phi, \dots$) and also not clear lattice-regularized QCD results do not improve the situation, so future experiments are needed.

However, rather rich experimental results for inelastic transition form factors allow us to prepare scheme how to construct the model also for Δ -baryon resonance. It was used to describe transition form factor G_M^* and the experimentally measured ratio R_{SM} with the help of relations for magnetic dipole transition and charge quadrupole form factors. The preliminary results are quite encouraging, mainly with the opportunities of exploration of new Δ -baryon properties, e. g., the extractions of the quadrupole moment $Q_{N \rightarrow \Delta}$ from the data and its relation to neutron charge radius r_n^2 .

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