Scalar and tensor $\pi\pi$ resonances in dispersive analysis

Robert Kamiński

*Henryk Niewodniczański Institute of Nuclear Physics
 Polish Academy of Sciences, Radzikowskiego 152
 31-342, Kraków, Poland
 robert.kaminski@ifj.edu.pl*

Published 26 November 2015

Role of the dispersive method in analysis of the $\pi\pi$ amplitudes in all important partial waves: S , P , D and F is presented. Widely discussed is the role of new (GKPY) and old (Roy) dispersion relations with imposed crossing symmetry condition. Example of a successful application of the GKPY and Roy's equations in test of $\pi\pi$ amplitudes fitted to both experimental data and dispersion relations is presented. Short and purely mathematical proof of the uniqueness and correctness of the dispersive method used to precise determination of $\pi\pi$ interaction parameters is presented.

PACS Numbers: 11.55.Fv, 11.55.-m, 11.80.Et, 13.75.Lb

1. Introduction

New spectacular results of dispersive analyses^{1,2} have been obtained using once subtracted dispersion relation (called GKPY) derived and presented recently³

$$\begin{aligned} \text{Re } t_{\ell}^{I(OUT)}(s) = & \sum_{I'=0}^2 C^{II'} t_0^{I'(IN)}(4m_{\pi}^2) \\ & + \sum_{I'=0}^2 \sum_{\ell'=0}^4 \int_{4m_{\pi}^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s, s') \text{Im } t_{\ell'}^{I'(IN)}(s'). \end{aligned} \quad (1)$$

The output amplitudes $t_{\ell}^{I(OUT)}(s)$ for given spin and isospin ℓ and I are expressed by sum of integrated input amplitudes of all partial waves $t_{\ell'}^{I'(IN)}(s')$ together with corresponding kernels $K_{\ell\ell'}^{II'}(s, s')$. The subtracting terms $C^{II'} t_0^{I'(IN)}(4m_{\pi}^2)$, where $C^{II'}$ is crossing matrix, are in fact combinations of scattering lengths of the S waves with isospin 0 and 2. As has been shown³ such equations guarantee fulfillment of $s - t$ crossing symmetry in the $\pi\pi$ amplitudes.

This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 3.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

2. Dispersive Analysis of the S Wave Amplitude

Previous — twice subtracted dispersion relations — so called the Roy's equations⁴ have been already used in, for example, elimination of long standing up-down ambiguity in the $\pi\pi$ S and P wave amplitudes.^{5,6} Figure 1 shows results for the S wave and new determination of uncertainty using the GKPY dispersion relations. On Fig. 2 presented are positions of the poles corresponding to the $f_0(500)$ resonance taken from Particle Data Tables^{7,8} which, due to new results of dispersive analysis,^{1,2} concentrate around $ImE_{pole} = 280$ MeV and $ReE_{pole} = 450$ MeV. In Table 1 presented are positions of the poles related with three resonances $f_0(500)$, $f_0(980)$ and $\rho(700)$ together with their coupling to the $\pi\pi$ channel calculated using formula

$$g^2 = -16\pi \lim_{s \rightarrow s_{pole}} (s - s_{pole}) t_\ell(s) (2\ell + 1) / (2p)^{2\ell} \quad (2)$$

where $p^2 = s/4 - m_\pi^2$. As is seen, positions of poles and coupling do not depend significantly on whether one uses GKPY or the Roy's equations. This confirms independence of presented results. Important is to notice significantly smaller value of the mass of $\rho(770)$ than that in Particle Data Tables.⁸ In contradiction to common opinion this difference of several MeV is only because of existence of another — complex conjugated pole which is necessary to guarantee unitarity condition and plays minor but important role.

Using dispersive analyses very important is to prove that obtained results (especially positions of poles) are unique. In the case of $f_0(500)$ pole one can do it basing only on purely mathematical rules of trigonometry. As is seen on Fig. 3 in

Fig. 1. Left figure: phase shifts for the scalar-isoscalar amplitude for the up and down solution, right figure: phases as in the left figure but only for the down solution. Ellipse " $\sim 35^\circ$ " indicates uncertainties of results from various experiments, lines and narrow band " $\sim 6^\circ$ " show results and uncertainties from the GKPY equations.

Fig. 2. Positions of poles corresponding to the $f_0(500)$ state. Rectangle "PDG 2010" presents region allowed by Particle Data Group Tables before year 2012. Smaller rectangle "PDG 2012" (presented also in the right figure) shows mostly new values of imaginary and real part of the poles.

Table 1. Poles and residues from Roy and GKPY eqs.

	$\sqrt{s_{\text{pole}}} \text{ (MeV)}$	$ g $
$f_0(500)^{\text{GKPY}}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13} \text{ GeV}$
$f_0(500)^{\text{Roy}}$	$(445 \pm 25) - i(278^{+22}_{-18})$	$3.4 \pm 0.5 \text{ GeV}$
$f_0(980)^{\text{GKPY}}$	$(996 \pm 7) - i(25^{+10}_{-6})$	$2.3 \pm 0.2 \text{ GeV}$
$f_0(980)^{\text{Roy}}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6} \text{ GeV}$
$\rho(770)^{\text{GKPY}}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$
$\rho(770)^{\text{Roy}}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$

the interesting region of $\delta_{\pi\pi}$ the imaginary parts of the amplitudes (inputs ones in dispersion relations (1)) and the real parts (the output ones in the GKPY equations) behave completely differently. For example they have shifted maxima and derivatives what together with smooth behavior of dominant part in Eq. (1) presented on Fig. 4 leads to movement of the outputs (see Fig. 3) for amplitudes before and after fitting (called, respectively, extended and re-fitted amplitudes). This movement results in narrower and lighter $f_0(500)$ ⁸ than was in past.⁷ Full mathematical prove can be found in recent paper analyzing old and new fits to the $\pi\pi$ amplitudes.⁹

On Fig. 4 one can also find information on the role in dispersive analyses played by the left and right cuts used in the integration along contour.³ It is seen that role of the left cut is much smaller than that along the right one what contradicts quite common opinion that in order to fulfill crossing symmetry enough is just to introduce into amplitudes the left cut.

Fig. 3. Upper row: phase shift dependence of the real and imaginary part of the scalar-isoscalar amplitude (left figure) and their derivatives (right figure). Lower row: effective two pion mass dependence of the real parts of the input and output amplitudes before (left figure) and after fitting (right figure).

Fig. 4. Left figure: contour used for the integration, right figure: effective two pion mass dependence of the kernel term for scalar isoscalar input calculated along left (Lhc) and right (Rhc) cuts.

Table 2. Values of the χ^2 before and after fitting to the dispersion relations (DR) for the D wave.

	χ^2_{tot}	χ^2_{Data}	χ^2_{DR}	χ^2/ndf
extended	1129.4	758.1	371.3	4.1
re-fitted	409.8	270.8	139.1	1.5

Table 3. Values of the χ^2 before and after fitting to the dispersion relations (DR) for the F wave.

	χ^2_{tot}	χ^2_{Data}	χ^2_{DR}	χ^2/ndf
extended	1829.4	855.1	974.3	4.5
re-fitted	753.6	489.3	129.4	1.9

3. Dispersive Results for the D - and F -Waves

Using other once subtracted dispersion relations derived few years ago,¹⁰ the same dispersive method has been applied for the higher partial waves i.e. D and F ones. Results show that from 11 tensor resonances proposed in previous analysis without dispersive method¹¹: $f_2(1270)$, $f_2(1430)$, $f_2(1525)$, $f_2(1600)$, $f_2(1730)$, $f_2(1810)$, $f_2(1960)$, $f_2(2000)$, $f_2(2020)$, $f_2(2240)$, $f_2(2410)$ at least the last two ones are not needed.

In case of fit to the F wave the only necessary resonance is $f_3(1690)$ and the other one $f_3(1950)$ does not change χ^2 . Corresponding values of the χ^2 are presented in Tables 2 and 3.

Fits to both waves have been performed independently with parameters of the other — fixed amplitudes taken from original dispersive analysis.³ Number of degree of freedom (ndf) was 275 and 401 for fits to the D and F wave respectively.

4. Conclusions

Dispersive analyses of the $\pi\pi$ interactions appear to be very efficient and precise method for testing amplitudes in all important partial waves: S , P , D and F . They guarantee fulfillment of crossing symmetry condition in amplitudes and correct positions of all poles related with resonances in the complex energy plane.

Worthy is to notice also results of other recent analyses of the position and the nature of the $f_0(500)$ meson.^{12,13} The first one¹² finds very similar position of the pole to that in Table 1 using completely independent approach i.e. calculation of the pion scalar form factor. In the second paper¹³ the authors find that Regge trajectory of the $f_0(500)$ has completely different behavior than that for the well known two-quark state $\rho(770)$ what confirms expectation that this meson is not ordinary $q\bar{q}$ state but rather mixture of for example four quark system and glueball.

Acknowledgments

This work was supported in part by the Polish National Science Center (NCN) Grant No. DEC-2013/09/B/ST2/04382.

References

1. R. Garcia-Martin, R. Kaminski, J. R. Pelaez and J. Ruiz de Elvira, *Phys. Rev. Lett.* **107** (2011) 072001
2. G. Colangelo, J. Gasser and H. Leutwyler, *Nucl. Phys. B* **603**, 125 (2001); B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, *Phys. Rep.* **353**, 207 (2001); I. Caprini, G. Colangelo and H. Leutwyler, *Phys. Rev. Lett.* **96**, 132001 (2006).
3. R. Garcia-Martin, R. Kaminski, J. R. Pelaez, J. Ruiz de Elvira and F. J. Yndurain, *Phys. Rev. D* **83**, 074004 (2011).
4. S. M. Roy, *Phys. Lett.* **36B**, 353 (1971)
5. M. R. Pennington, *Ann. Phys. (N.Y.)* **92**, 164 (1975).
6. R. Kamiński, L. Leśniak and B. Loiseau, *Phys. Lett. B* **551**, 241 (2003).
7. K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
8. J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
9. P. Bydzovsky, R. Kamiński and V. Nazari, *Phys. Rev. D* **90**, 116005 (2014).
10. R. Kamiński, *Phys. Rev. D* **83**, 076008 (2011).
11. Yu. S. Surovtsev, P. Bydzovsky, R. Kamiński and M. Nagy, *Phys. Rev. D* **81**, 016001 (2010); arXiv:1104.0538.
12. Dubnicka, A. Z. Dubnickova, A. Liptaj, *Phys. Rev. D* **90**, 114003 (2014).
13. J. T. Londergan, J. Nebreda, J. R. Pelaez and A. Szczepaniak, *Phys. Lett. B* **729**, 9 (2014).