Solution of the NLO BFKL Equation and $\gamma^*\gamma^*$ Cross-Section at NLO

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The analytic NLO cross section of the $\gamma^*\gamma^*$ scattering process at high-energy is calculated. The result is obtained factorizing the scattering amplitude in rapidity space using the high-energy Operator Product Expansion in terms of composite Wilson line operators. The next-to-leading logarithmic resummation is included employing the solution of the NLO BFKL equation which obtained from the linearization of the evolution in rapidity space of the composite Wilson line operators.

Keywords: Perturbative QCD; NLO QCD.

1. Introduction

The next-to-leading (NLO) cross section for $\gamma^*\gamma^*$ scattering is obtained by calculating the photon impact factor at NLO and employing the next-to-leading logarithm resummation at high energy. The high-energy Operator Product Expansion^{1,2} in terms of composite Wilson-line operators is a natural frame work where the NLO cross-section can be calculated. Figure 1 provides a diagrammatic representation of the factorization of the $\gamma^*\gamma^*$ scattering cross section using the high-energy OPE.

Let us review the logic of the OPE at high-energy before performing the actual calculation of the $\gamma^*\gamma^*$ scattering cross section at NLO. To this end, we consider the forward scattering amplitude of two virtual photons with four-momenta q_1 and q_2 and polarizations λ_1 and λ_2 represented by four electromagnetic currents. At high energy the correlation function of four electromagnetic currents can be factorized as follow

$$\mathcal{A}_{\lambda_1\lambda_2}(q_1, q_2) = -i\varepsilon_{\rho_1}^{\lambda_1*}(q_1)\varepsilon_{\sigma_1}^{\lambda_1}(q_1)\varepsilon_{\rho_2}^{\lambda_2*}(q_2)\varepsilon_{\sigma_2}^{\lambda_2}(q_2) \int d^2z_{\perp} \mathcal{N}^{-1} \int \mathcal{D}A e^{iS(A)} \det(i\nabla)$$

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Fig. 1. The upper panel contains a diagrammatic representation of the $\gamma^*\gamma^*$ scattering amplitude factorized into the impact factors, small- x evolution, and the (energy-independent) daughter dipole-dipole scattering. This structure of the factorization is suggested by the high-energy OPE applied to the correlation function of four electromagnetic currents.

$$\begin{aligned} & \times \int dz^+ \int d^4x e^{iq_1 \cdot x} \langle T j^{\rho_1}(x^+ + z^+, x^-, \vec{x}_\perp + \vec{z}_\perp), j^{\sigma_1}(z^+, 0, \vec{z}_\perp) \rangle_A \\ & \times \int dz^- \int d^4y e^{iq_2 \cdot y} \langle T j^{\rho_2}(y^+, y^- + z^-, \vec{y}_\perp) j^{\sigma_2}(0, z^-, 0_\perp) \rangle_A, \quad (1) \end{aligned}$$

Here $\varepsilon_\rho^\lambda(q)$ are the gluon polarization vectors, $x^\pm = (x^0 \pm x^3)/\sqrt{2}$, and j^σ are the electromagnetic currents; $S(A)$ is the part of the QCD action which depends only on the gluon field A_μ , $\det(i\nabla)$ is the fermionic determinant with ∇ the covariant derivative, angle brackets $\langle \dots \rangle_A$ denote the expectation value in the background gluon field A_μ , and the normalization factor $\mathcal{N}$ is

$$\mathcal{N} = \int \mathcal{D}A e^{iS(A)} \det(i\nabla). \quad (2)$$

The factor $\left(\sum_f e_f^2\right)^2$ (where e_f labels the electromagnetic charge of the quark with flavor f) will be omitted throughout the calculation and it will be reinstated in the expressions for the cross sections.

At high-energy the $\gamma^*\gamma^*$ forward scattering amplitude can be factorized as in Eq. (1) because a typical dominant high-energy diagram that is shown in Fig. 2a can be obtained from Eq. (1). On the other hand diagrams like those in Fig. 2b are

Fig. 2. Left panel depicts a Feynman diagram that can be obtained from Eq. (1) which contributes to the high-energy asymptotics of $\gamma^*\gamma^*$ scattering. The right panel shows an example of the “box” diagram that can not be generated from Eq. (1): box diagrams are suppressed at high energies by a power of energy and, therefore, can be neglected. However, as we will see in the numerical results section, such types of diagrams become relevant, and therefore not anymore negligible at low rapidity where energy is not high enough to suppress such contributions.

suppressed by a power of center-of-mass energy squared s compared to the leading graphs in Fig. 2a, and can be neglected in the high-energy asymptotics.

Each term in the angle brackets $\langle \dots \rangle_A$ in Eq. (1) is evaluated using the OPE at high energy⁷ and the background field technique. The coefficient functions of the OPE of two electromagnetic currents at high energy is the photon impact factor (PIF) as originally defined in Ref. [3]. The PIF is convoluted with a matrix element of an operator made out of Wilson lines along the light cone.

Since we are interested in the NLO $\gamma^*\gamma^*$ cross section, we need the LO and NLO PIF. The calculation of NLO PIF has been performed in Refs. [4, 5]. We will only employ the results of that work in our calculation and refer the reader to the original papers^{4,5} for the details of the PIF calculation. At NLO the PIF contains also terms that are rapidity-dependent (or energy-dependent). As explained in Ref. [2], when the high-energy OPE is performed in terms of composite Wilson-line operators, by redefining the operator at hand all the energy dependent terms can be *shifted* into the matrix elements of Wilson lines, thus leaving the NLO impact factor energy-scale invariant. This is what has been done in the PIF calculation of Refs. [4, 5]. In addition, the composite Wilson lines operator obtained by the shifting procedure of Refs. [4, 5] renders the impact factor conformally invariant also at NLO: the energy-dependent terms clearly would have broken conformal invariance. The high-energy OPE in terms of composite Wilson line operator provides an operatorial and systematic procedure to factorize scattering amplitudes into coefficient functions which are energy-scale invariant and matrix elements of composite Wilson line operators that encode the energy dependence of the amplitude.

In $\gamma^*\gamma^*$ scattering at high energy, each of the two virtual photons can be treated at the same time as the projectile and as the target due to the symmetry of the process. We will work in a frame in which the photons collide head-on, such that

their four-momenta are

$$q_1^\mu = \left(q_1^+, -\frac{Q_1^2}{2q_1^+}, 0_\perp \right), \quad q_2^\mu = \left(-\frac{Q_2^2}{2q_2^-}, q_2^-, 0_\perp \right) \quad (3)$$

with q_1^+ and q_2^- very large. We will work in a gauge which reduces to the $A^+ = 0$ light-cone gauge near the $x^- = 0$ light cone, and to the $A^- = 0$ light-cone gauge near the $x^+ = 0$ light cone, that is, it reduces to the light-cone gauges for each of the photons: examples of such gauges include the Coulomb $\vec{\nabla} \cdot \vec{A} = 0$ gauge and the $A^0 = 0$ temporal gauge. In such gauge, each virtual photon, long before the interaction, splits into a quark–antiquark pair which then traverses the gluonic field produced by the target (the other $q\bar{q}$ pair): this interaction is described by a color dipole made of two fundamental Wilson lines scattering on another fundamental dipole. In DIS the small- x evolution of Wilson lines is described by the Balitsky-JIMWLK non-linear equation;^{1,6–9} in the case of a fundamental dipole in the large- N_c limit the evolution equation corresponds to the Balitsky-Kovchegov equation^{1,10–12} and its linearization corresponds to the BFKL equation.^{3,13,14} While it is not presently clear which evolution equation describes Wilson-line correlators in $\gamma^*\gamma^*$ scattering at high energy, it is clear that in the linear regime the evolution of a dipole operator is given by the BFKL equation.

We are interested in the $\gamma^*\gamma^*$ scattering cross section in the linear case where the resummation in the leading logarithmic approximation in the Regge limit is achieved by the linearization of the evolution equation for the color Wilson line operator defined as

$$U^\eta(\vec{x}_\perp, \vec{y}_\perp) = 1 - \frac{1}{N_c} \text{tr} \{ U^\eta(\vec{x}_\perp) U^{\eta\dagger}(\vec{y}_\perp) \}, \quad (4)$$

where $U^\eta(\vec{x}_\perp)$ is the Wilson line operator in the fundamental representation, which, for a projectile moving along the x^+ light cone is defined by

$$U^\eta(\vec{x}_\perp) = \text{P exp} \left\{ i g \int_{-\infty}^{\infty} dx^+ A^-(x^+, \vec{x}_\perp) \right\}, \quad (5)$$

and x and y are the points of interaction of the quark and anti-quark pair with the gluonic external field of the target; in the center-of-mass frame the gluonic external field of the target reduces to a shock wave due to Lorentz contraction and time dilation. The Wilson line operator depends on the rapidity η , which may be included in Eq. (5) either by changing the slope of the Wilson line to also include the x^- direction or by a rigid cut-off imposed on the longitudinal momenta of the gluons when calculating matrix elements of the Wilson line operators. In the DIS case the NLO evolution of the color dipole (4) with respect to rapidity η has been calculated in Ref. [15], where its linearization was proven to coincide with the known result for the NLO BFKL equation.^{16,17}

At NLO the high-energy OPE is performed in terms of the composite operator $[U^a(\vec{x}_\perp, \vec{y}_\perp)]^{\text{comp}}$ which differs from the one in Eq. (4) by the addition of a counter

term that restores conformal symmetry present in the leading order impact factor.^{2,4,5} Note also, that the evolution parameter for the composite operator is not the rapidity parameter η from Eq. (4), but is the new parameter a which ensures that the evolution equation of the composite operator $[\mathcal{U}^a(\vec{x}_\perp, \vec{y}_\perp)]^{\text{comp}}$ with respect to a is the same as the one for $\mathcal{U}^\eta(\vec{x}_\perp, \vec{y}_\perp)$ with respect to η at LO. At NLO the evolution equation for $[\mathcal{U}^a(\vec{x}_\perp, \vec{y}_\perp)]^{\text{comp}}$ with respect to a has a conformal kernel in $\mathcal{N} = 4$ SYM; in QCD the NLO evolution kernel for $[\mathcal{U}^a(\vec{x}_\perp, \vec{y}_\perp)]^{\text{comp}}$ consists of a conformally-invariant piece and a running-coupling term.

For DIS the momentum-space high-energy OPE of two electromagnetic currents (in the linearized approximation, that is, in absence of nonlinear saturation effects) is^{4,5}

$$\begin{aligned} & \int dz^+ d^2z \int d^4x e^{iq \cdot x} \langle T j^\rho(x^+ + z^+, x^-, \vec{x}_\perp + \vec{z}_\perp) j^\sigma(z^+, 0, \vec{z}_\perp) \rangle_A \\ &= \sqrt{\frac{s}{2}} \int \frac{d^2k}{(2\pi)^2 k^2} I^{\rho\sigma}(q, k) \int d^2z k^2 \langle \mathcal{U}^\eta(\vec{k}_\perp, \vec{z}_\perp) \rangle_A \end{aligned} \quad (6)$$

where $I^{\rho\sigma}(q, k)$ is the photon impact factor and the Wilson-line operator in the matrix element is (see Ref. 18 for details)

$$\mathcal{U}^\eta(\vec{k}_\perp, \vec{z}_\perp) = \int d^2x e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} \mathcal{U}^\eta(\vec{x}_\perp + \vec{z}_\perp, \vec{z}_\perp). \quad (7)$$

Note that in Eq. (6) the composite operator $[\mathcal{U}^a(\vec{k}_\perp, \vec{z}_\perp)]^{\text{comp}}$ has to be used instead of $\mathcal{U}^\eta(\vec{k}_\perp, \vec{z}_\perp)$ at NLO and beyond. The background field in the averaging $\langle \dots \rangle_A$ is due to the target, which could be a proton or a nucleus.

The impact factor $I^{\rho\sigma}(q, k)$ is known up to NLO [4, 5]. At the LO and NLO the impact factor can be written as a Mellin transform

$$I_{\text{LO+NLO}}^{\rho\sigma}(q, k) = \int_{-\infty}^{\infty} d\nu \left(\frac{k^2}{Q^2} \right)^{\frac{1}{2}-i\nu} \tilde{I}_{\text{LO+NLO}}^{\rho\sigma}(q, \nu) \quad (8)$$

with^{4,5}

$$\begin{aligned} \tilde{I}_{\text{LO+NLO}}^{\rho\sigma}(q, \nu) &= \frac{N_c}{64} \frac{(\pi\nu)^{-1} \sinh \pi\nu}{(1+\nu^2) \cosh^2 \pi\nu} \left\{ \left(\frac{9}{4} + \nu^2 \right) \left[1 + \frac{\alpha_\mu}{\pi} + \frac{\bar{\alpha}_\mu}{2} \mathcal{F}_1(\nu) \right] P_1^{\rho\sigma} \right. \\ &\quad \left. + \left(\frac{11}{4} + 3\nu^2 \right) \left[1 + \frac{\alpha_\mu}{\pi} + \frac{\bar{\alpha}_\mu}{2} \mathcal{F}_2(\nu) \right] P_2^{\rho\sigma} \right\}, \end{aligned} \quad (9)$$

where the functions $\mathcal{F}_i(\nu)$ with $i = 1, 2, 3$ are defined in,⁵ $k = |\vec{k}_\perp|$, and

$$P_1^{\rho\sigma} = g^{\rho\sigma} - \frac{q^\rho q^\sigma}{q^2}, P_2^{\rho\sigma} = \frac{1}{q^2} \left(q^\rho - \frac{p_2^\rho q^2}{q \cdot p_2} \right) \left(q^\sigma - \frac{p_2^\sigma q^2}{q \cdot p_2} \right) \quad (10)$$

with $p_2^\mu = (0, p_2^-, 0_\perp)$.

As was shown in Ref. [5], linearization of the NLO evolution equation for the operator

$$\mathcal{L}^a(k) \equiv \frac{k^2 [\mathcal{U}^a(\vec{k}_\perp)]^{\text{comp}}}{\alpha_s(k^2)} \quad (11)$$

coincides with the NLO BFKL equation obtained in Ref.[16]. Therefore, the eigenfunctions $H_\nu(k)$ of the NLO BFKL equation obtained in Ref. [19] are the eigenfunctions of the kernel at NLO of the linearized equation for $\mathcal{L}^a(k)$.

Considering only the $n = 0$ contribution which dominates in high energy scattering, we use the completeness relation $H_\nu(k)$ function for $n = 0$,

$$\int_{-\infty}^{\infty} \frac{d\nu}{2\pi} H_\nu(k) H_{-\nu}(k') = \delta(k^2 - k'^2) \quad (12)$$

to rewrite Eq. (1) as

$$\begin{aligned} \mathcal{A}_{\lambda_1, \lambda_2}(q_1, q_2) = & -i \frac{s}{2} \varepsilon_{\rho_1}^{\lambda_1 *} (q_1) \varepsilon_{\sigma_1}^{\lambda_1} (q_1) \varepsilon_{\rho_2}^{\lambda_2 *} (q_2) \varepsilon_{\sigma_2}^{\lambda_2} (q_2) \int \frac{d\nu_1}{2\pi} \frac{d\nu_2}{2\pi} I^{\rho_1 \sigma_1}(q_1, \nu_1) \\ & \times I^{\rho_2 \sigma_2}(q_2, \nu_2) \frac{1}{S_\perp} \left\langle \int_0^\infty dk_1'^2 H_{-\nu_1}(k_1') \mathcal{L}^{a_1}(k_1') \right. \\ & \left. \times \int_0^\infty dk_2'^2 H_{-\nu_2}(k_2') \mathcal{L}^{a_2}(k_2') \right\rangle \end{aligned} \quad (13)$$

with

$$I^{\rho\sigma}(q, \nu) = \int \frac{d^2 k}{(2\pi)^2 k^2} I^{\rho\sigma}(q, k) \alpha_s(k^2) H_\nu(k) \quad (14)$$

a projection of the impact factor on the LO+NLO BFKL eigenfunctions.

Since, as we have just mentioned, the functions $H_\nu(k)$ are the eigenfunctions of the evolution equation for $\mathcal{L}^a(k)$ [5], the linearized evolution of $\mathcal{L}^a(k)$ for $n = 0$ can be written as

$$2a \frac{d}{da} \mathcal{L}^a(\nu) = \Delta(\nu) \mathcal{L}^a(\nu) \quad (15)$$

where $\Delta(\nu) = \bar{\alpha}_\mu \chi_0(0, \nu) + \bar{\alpha}_\mu^2 \chi_1(0, \nu)$ and

$$\mathcal{L}^a(\nu) \equiv \int dk^2 H_{-\nu}(k) \mathcal{L}^a(k). \quad (16)$$

We thus rewrite Eq. (13) as

$$\begin{aligned} \mathcal{A}_{\lambda_1, \lambda_2}(q_1, q_2) = & -i \frac{s}{2} \varepsilon_{\rho_1}^{\lambda_1 *} (q_1) \varepsilon_{\sigma_1}^{\lambda_1} (q_1) \varepsilon_{\rho_2}^{\lambda_2 *} (q_2) \varepsilon_{\sigma_2}^{\lambda_2} (q_2) \int \frac{d\nu_1}{2\pi} \frac{d\nu_2}{2\pi} \\ & \times \frac{1}{S_\perp} \langle \mathcal{L}^{a_0}(\nu_1) \mathcal{L}^{a_0}(\nu_2) \rangle I^{\rho_1 \sigma_1}(q_1, \nu_1) \\ & \times I^{\rho_2 \sigma_2}(q_2, \nu_2) e^{\frac{1}{2} \Delta(\nu_1) \ln \frac{a_1}{a_0} + \frac{1}{2} \Delta(\nu_2) \ln(a_0 a_2)}. \end{aligned} \quad (17)$$

Here $(1/2) \ln a_0$ is an arbitrary rapidity for the LO+NLO dipole-dipole scattering while $(1/2) \ln(a_1 a_2) = \ln \frac{s}{Q_1 Q_2} = \eta_1 - \eta_2$. Note that the expression (17) for the

forward amplitude is quite general, and is likely to be valid at any order in α_s within the linear evolution approximation.

We see that the eigenfunctions of LO+NLO BFKL allowed us to write down the evolution effects explicitly in Eq. (17). To complete the calculation, we now need to find the projections $I^{\rho\sigma}(q, \nu)$ of the impact factors onto the eigenfunctions $H_\nu(k)$ (see Eq. (14)) and to construct the correlator $\langle \mathcal{L}^{\eta_0}(\nu_1) \mathcal{L}^{\eta_0}(\nu_2) \rangle$ at the LO+NLO level. The details of the calculation can be found in Ref. [18]. The result for the dipole–dipole forward scattering amplitude is

$$\langle \mathcal{L}^{a_0}(\nu_1) \mathcal{L}^{a_0}(\nu_2) \rangle / S_\perp = -4 (2\pi)^4 \frac{N_c^2 - 1}{N_c^2} [1 + \bar{\alpha}_\mu F(\nu_1)] \delta(\nu_1 + \nu_2) \quad (18)$$

where the NLO correction to the eikonal dipole-dipole scattering was calculated in Ref. [20] for $\mathcal{N} = 4$ SYM, which we modified to work for QCD obtaining

$$\text{Re}[F(\nu)] = \chi_0(\nu) \left(2\gamma_E - \frac{4}{4\nu^2 + 1} \right) + \frac{67}{18} - \frac{\pi^2}{6} - \frac{5N_f}{9N_c}. \quad (19)$$

The calculation of the LO+NLO impact factor projection can be found in Ref. [18]. The result is

$$\begin{aligned} I^{\rho\sigma}(q, \nu) = & \frac{\alpha_\mu}{2} \left\{ 1 + \bar{\alpha}_\mu \beta_2 \left[i \frac{\chi_0(\nu)}{2\chi'_0(\nu)} (i\partial_\nu + \ln \mu^2)^2 \right. \right. \\ & \left. \left. - \left(\partial_\nu \frac{\chi_0(\nu)}{2\chi'_0(\nu)} \right) (i\partial_\nu + \ln \mu^2) + i\partial_\nu + \ln \mu^2 \right] \right\} \\ & \times Q^{-1+2i\nu} \tilde{I}_{\text{LO+NLO}}^{\rho\sigma}(q, \nu) \end{aligned} \quad (20)$$

where $\tilde{I}_{\text{LO+NLO}}^{\rho\sigma}(q, \nu)$ is the Mellin transform of the LO+NLO impact factor defined in Eq. (8) and $\partial_\nu \equiv \partial/\partial\nu$.

Substituting Eqs. (18) and (20) into Eq. (17) we obtain for transversely polarized photons the cross section

$$\begin{aligned} \sigma_{\text{LO+NLO}}^{\gamma^*\gamma^*}(TT) = & \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm 1} 2\pi^2 \varepsilon_{\rho_1}^{\lambda_1*}(q_1) \varepsilon_{\sigma_1}^{\lambda_1}(q_1) \varepsilon_{\rho_2}^{\lambda_2*}(q_2) \varepsilon_{\sigma_2}^{\lambda_2}(q_2) \frac{N_c^2 - 1}{N_c^2} \left(\sum_f e_f^2 \right)^2 \\ & \times \frac{\alpha_s(Q_1^2) \alpha_s(Q_2^2)}{Q_1 Q_2} \int d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \\ & \times \left(\frac{s}{Q_1 Q_2} \right)^{\bar{\alpha}_s(Q_1 Q_2) \chi_0(\nu) + \bar{\alpha}_s^2(Q_1 Q_2) \chi_1(\nu)} \tilde{I}_{\text{LO+NLO}}^{\rho_1 \sigma_1}(q_1, \nu) \\ & \times \tilde{I}_{\text{LO+NLO}}^{\rho_2 \sigma_2}(q_2, -\nu) \left[1 + \bar{\alpha}_s(Q_1 Q_2) \text{Re}[F(\nu)] \right], \end{aligned} \quad (21)$$

while for the longitudinally polarized gluons we get

$$\begin{aligned}
\sigma_{\text{LO+NLO}}^{\gamma^*\gamma^*}(LL) &= 2\pi^2 \varepsilon_{\rho_1}^{L*}(q_1) \varepsilon_{\sigma_1}^L(q_1) \varepsilon_{\rho_2}^{L*}(q_2) \varepsilon_{\sigma_2}^L(q_2) \frac{N_c^2 - 1}{N_c^2} \\
&\times \left(\sum_f e_f^2 \right)^2 \frac{\alpha_s(Q_1^2) \alpha_s(Q_2^2)}{Q_1 Q_2} \int d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \\
&\times \left(\frac{s}{Q_1 Q_2} \right)^{\bar{\alpha}_s(Q_1 Q_2) \chi_0(\nu) + \bar{\alpha}_s^2(Q_1 Q_2) \chi_1(\nu)} \tilde{I}_{\text{LO+NLO}}^{\rho_1 \sigma_1}(q_1, \nu) \\
&\times \tilde{I}_{\text{LO+NLO}}^{\rho_2 \sigma_2}(q_2, -\nu) \left[1 + \bar{\alpha}_s(Q_1 Q_2) \text{Re}[F(\nu)] \right], \quad (22)
\end{aligned}$$

where we have used $\frac{1}{2} \ln(a_1 a_2) = \ln \frac{s}{Q_1 Q_2}$ along with the fact that $F(\nu) = F(-\nu)$. The latter condition, while satisfied by $F(\nu)$ in Eq. (19), is also true in general, since it follows from the symmetry of the dipole-dipole scattering under the interchange of the dipoles. In Eqs. (21) and (22), $\tilde{I}_{\text{LO+NLO}}^{\rho\sigma}(q, \nu)$ are given by (9) with $\bar{\alpha}_\mu \rightarrow \bar{\alpha}_s(Q_1 Q_2)$ and $\text{Re}[F(\nu)]$ is given by Eq. (19) [20]. Note that $\tilde{I}_{\text{NLO}}^{\rho\sigma}(q, \nu) \neq \tilde{I}_{\text{NLO}}^{\rho\sigma}(q, -\nu)$. In Eq. (21) the transverse polarizations are $\varepsilon^{\lambda, \mu} = (0, 0, \bar{\varepsilon}_\perp^\lambda)$ with $\bar{\varepsilon}_\perp^\lambda = (-1/\sqrt{2})(\lambda, i)$. The longitudinal polarizations in Eq. (22) are $\varepsilon^{L, \mu}(q) = (q^+/Q, Q/2q^+, 0_\perp)$. We have also used the notation $\chi_0(\nu) = \chi_0(0, \nu)$ and $\chi_1(\nu) = \chi_1(0, \nu)$.

1.1. Numerical result of the $\gamma^*\gamma^*$ cross-section

The NLO cross section for $\gamma^*\gamma^*$ scattering, Eqs. (21) and (22), are analytic expression that can be easily evaluated numerically. Let us plot then the $\gamma^*\gamma^*$ cross section as a function of rapidity for virtual photons with momenta $Q_1 = Q_2 = 1$ GeV in Fig. 3. The left panel of Fig. 3 depicts the TT cross section, while the right panel contains the LL cross section. The one-loop coupling constant in result (22) runs with the momenta of the virtual photons. In Fig. 3 we use $N_f = 3$ and $\Lambda_{\text{QCD}} = 250$ MeV; hence $\alpha_s(1\text{GeV}^2) \approx 0.50$. Fitting the curves in Fig. 3 with $e^{\Delta Y}$ we make an estimate of the effective intercept; we get $\Delta \approx 0.78$ for the TT cross section and $\Delta \approx 0.75$ for the LL cross section. Compared to the LO BFKL intercept $\Delta_{\text{LO}} \approx \frac{4\alpha_s N_c}{\pi} \ln 2 \approx 1.33$ for $\alpha_s \approx 0.50$ we conclude that the NLO BFKL evolution indeed tends to significantly reduce the intercept compared to the LO result. However, as is clear from Fig. 3, the LO+NLO cross section keeps growing with rapidity even for rather large values of the coupling $\alpha_s \approx 0.50$.

In Fig. 4 we plot the TT (left panel) and LL (right panel) cross sections for $Q_1 = Q_2 = 5$ GeV (dashed line) and $Q_1 = Q_2 = 10$ GeV (solid line). In Fig. 4 we use $N_f = 5$ such that $\alpha_s((5\text{GeV})^2) = 0.27$ and $\alpha_s((10\text{GeV})^2) = 0.22$ respectively. The estimates of the intercepts are as follows: $\Delta \approx 0.34$ for the TT cross section and $\Delta \approx 0.32$ for the LL cross section for $Q_1 = Q_2 = 5$ GeV and $\Delta \approx 0.27$ for both the

Fig. 3. NLO $\gamma^*\gamma^*$ cross section with $Q_1 = Q_2 = 1$ GeV plotted as a function of rapidity Y . Virtual photons have transverse polarizations in the left panel and longitudinal polarizations in the right panel.

Fig. 4. NLO $\gamma^*\gamma^*$ cross section with $Q_1 = Q_2 = 5$ GeV (dashed line) and $Q_1 = Q_2 = 10$ GeV (solid line) plotted as functions of rapidity Y . Again the virtual photons have transverse polarizations in the left panel and longitudinal polarizations in the right panel.

TT and LL cross sections at $Q_1 = Q_2 = 10$ GeV. Comparing this to $\Delta_{LO} \approx 0.73$ for $Q_1 = Q_2 = 5$ GeV and $\Delta_{LO} \approx 0.59$ for $Q_1 = Q_2 = 10$ GeV we again see a reduction of the intercept at NLO, though again it remains positive corresponding to a cross section growing with Y .

Notice that, at small rapidities, $Y \approx 0$, there are corrections to the dipole–dipole cross section which are inversely proportional to s and are given by the diagrams like that in Fig. 2b along with the subleading contributions to the diagram in Fig. 2a. Such corrections were neglected in deriving Eq. (18) [20]. However, such corrections are likely to become important near $Y \approx 0$, modifying the dipole–dipole cross section and possibly making it positive again even for the range of Q_1, Q_2 considered here. Hence the negativity of the cross sections in Figs. 3 and 4 at small Y is not necessarily a sign of the failure of the formalism, but rather an indication that energy-suppressed corrections need to be added in that region. Since the NLO cross section (21) and (22) are likely to dominate over the energy-suppressed corrections at rapidities of the order $Y \approx 2 \div 3$, we conclude that our NLO result gives a reliable prediction for the $\gamma^*\gamma^*$ cross section only for sufficiently large values of Y , e.g. for $Y \gtrsim 2$ in the kinematics of the plots in Figs. 3 and 4.

At very high rapidity ($Y = Y_U$) the unitarity corrections due to nonlinear evolution will become important; hence the validity region of Eqs. (21) and (22) in rapidity is also limited from above, $Y < Y_U$. At asymptotically small values of α_s a simple parametric estimate (see e.g. 12) shows that unitarity corrections become important at $Y \approx Y_U \sim (1/\alpha_s) \ln(1/\alpha_s)$, which is lower than the rapidity at which NLO corrections become large, $Y \sim 1/\alpha_s^2$ (obtained by requiring that $\alpha_s^2 Y \sim 1$). However, for realistic values of α_s the situation is not so clear-cut, and it is possible that the NLO corrections discussed here would delay the onset of saturation effects.

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References

1. I. Balitsky, *Nucl.Phys.* **B463**, 99 (1996).
2. I. Balitsky and G. A. Chirilli, *Nucl.Phys.* **B822**, 45 (2009).
3. I. Balitsky and L. Lipatov, *Sov.J.Nucl.Phys.* **28**, 822 (1978).
4. I. Balitsky and G. A. Chirilli, *Phys.Rev.* **D83**, p. 031502 (2011).
5. I. Balitsky and G. A. Chirilli, *Phys.Rev.* **D87**, p. 014013 (2013).
6. J. Jalilian-Marian, A. Kovner and H. Weigert, *Phys.Rev.* **D59**, p. 014015 (1998).
7. J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *Phys.Rev.* **D59**, p. 014014 (1998).
8. E. Iancu, A. Leonidov and L. D. McLerran, *Phys.Lett.* **B510**, 133 (2001).
9. E. Iancu, A. Leonidov and L. D. McLerran, *Nucl.Phys.* **A692**, 583 (2001).
10. I. Balitsky, *Phys.Rev.* **D60**, p. 014020 (1999).
11. Y. V. Kovchegov, *Phys.Rev.* **D60**, p. 034008 (1999).
12. Y. V. Kovchegov, *Phys.Rev.* **D61**, p. 074018 (2000).
13. E. Kuraev, L. Lipatov and V. S. Fadin, *Sov.Phys.JETP* **45**, 199 (1977).
14. E. A. Kuraev, L. N. Lipatov and V. S. Fadin, *Sov.Phys.JETP* **44**, 443 (1976).
15. I. Balitsky and G. A. Chirilli, *Phys.Rev.* **D77**, p. 014019 (2008).
16. V. S. Fadin and L. Lipatov, *Phys.Lett.* **B429**, 127 (1998).
17. M. Ciafaloni and G. Camici, *Phys.Lett.* **B430**, 349 (1998).
18. G. A. Chirilli and Y. V. Kovchegov, *JHEP* **1405**, p. 099 (2014).
19. G. A. Chirilli and Y. V. Kovchegov, *JHEP* **1306**, p. 055 (2013).
20. I. Balitsky and G. A. Chirilli, *Phys.Lett.* **B687**, 204 (2010).