TMD Evolution for Collins Asymmetries in e^+e^- Annihilation and SIDIS

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In this talk, we present our recent work about QCD evolution effect on the transverse momentum dependent(TMD) observables for collins asymmetries in e^+e^- annihilation to di-hadron and semi-inclusive deep inelastic scattering(SIDIS) processes, by taking into account TMD evolution effect is studied under CSS formalism. Our result shows the evolution effect will bring a significant correction to TMD distributions extracted from experiment data.

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1. Introduction

Transverse momentum dependent(TMD) parton distributions have always been a hot topic for both experiment and theory sides in recent years. They play an important role in the study of the strong interaction physics and the fundamental properties of nucleon. Among the TMD distributions, the collins transverse momentum fragmentation function¹ has attracted great interests from nuclear physics community. There have been strong evidence of collins effect in the SIDIS and e^+e^- to

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di-hadron processes experiments observed by the HERMES, COMPASS, JLab, Belle and barbar collaborations. Especially, measuring the Collins azimuthal asymmetries in SIDIS is an important channel to investigate the quark transversity distribution which describes transverse polarized parton density in one unpolarized hadron. Through the transversity distribution we can obtain the Nucleon tensor charge which is one of the fundamental properties of the proton and its determination is among the main goals of existing and future experimental facilities.²⁻⁷

According to the universality of TMD distributions, we are allowed to simulate different processes with the same TMD distribution. However, the collins asymmetries in SIDS processes at HERMES, COMPASS and JLab were measured with Q^2 from 1 to 6 GeV², while the energy Q^2 in the processes of $e^+e^- \rightarrow hh$ at Belle and Barbar is fixed at 110 GeV². Since the TMD distributions depend on Q^2 , in order to apply the same distributions in all these processes, we must evolve the different Q^2 scale to the same point. In this work, we apply CSS formalism^{8,9} to calculate the energy Q^2 evolution effect, in the next section, I will present this detail about this subject.

2. Collins-Soper-Sterman Evolution

TMD factorization is applicable in the processes with two different scales observed such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions. For example, under the TMD factorization formalism, a SIDIS cross section can be factorized into four parts as:

$$\sigma(q_t, Q^2) = \sigma_0 \int e^{ib \cdot q_t} F_{uu}(Q^2, b^2) d^2b \quad (1)$$

with structure function

$$F_{uu}(Q^2, b^2) = f(x, b, Q^2, \mu) d(z, b, Q^2, \mu) H(\mu, Q^2) S(\mu, b) \quad (2)$$

Where, f and d are TMD distribution, while H and S are hard and soft part respectively, Q^2 is invariant mass of virtual photon, q_t is transverse momentum of final lepton pair, when we consider the kinematical regime $q_t \ll Q^2$, a large logarithms of $\ln Q^2 b^2$ in b space appear at every order of perturbative expansion as the form of $\alpha_s^n \ln^{2n} Q^2 b^2$ or $\alpha_s^n \ln^n Q^2 b^2$, when $\ln Q^2 b^2 > 1/\alpha_s$, the convergence of the conventional perturbative expansion is impaired. In order to make a reliable prediction, these large logs have to be resummed. After evolving Q^2 in $F_{uu}(Q^2, b^2)$ to the point $1/b$, these large logarithms will be resummed into an evolution factor automatically, and the Q^2 dependence in $F_{uu}(Q^2, b^2)$ satisfies the CSS evolution equation:

$$\frac{\partial}{\partial \ln Q^2} F_{uu}(Q^2, b^2) = (K(b, \mu) + G(Q^2, \mu)) F_{uu}(Q^2, b^2), \quad (3)$$

here the K and G are the associated soft and hard part in the evolution kernel, after solving above equation, we can obtain:

$$F_{uu}(Q^2, b^2) = F_{uu}(1/b^2, b^2) e^{-S(Q^2, 1/b, C_1, C_2)}, \quad (4)$$

All the large logarithms are resummed into the Sudakov form factor S , which can be written as

$$S(Q, 1/b) = \int_{C_1/b}^{C_2 Q} \frac{d\mu}{\mu} \left[\ln \left(\frac{C_2 Q^2}{\mu^2} \right) A(\mu) + B(\mu) \right], \quad (5)$$

where A and B can be calculated order by order in perturbation theory, and reads

$$A(\mu) = \sum_1^n \frac{\alpha_s(\mu)}{\pi} A^{(n)} \quad B(\mu) = \sum_1^n \frac{\alpha_s(\mu)}{\pi} B^{(n)} \quad (6)$$

in our calculation, we only take into account the $A^{(1)}$, $A^{(2)}$ and $B^{(1)}$ for next-to-leading-logarithm(NLL') accuracy with

$$A^{(1)} = C_F, \quad A^{(2)} = \frac{C_F}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right], \quad B^{(1)} = -\frac{3}{2} C_F. \quad (7)$$

The Sudakov form factor S has universality for SIDIS, $e^+e^- \rightarrow hh$ and Drell-Yan processes, however, when we transfer the cross section from b space back to q_t space by a Fourier transformation, the transformation have to enter into large b region so that the low limit (C_1/b) of integration in Sudakov form factor can be smaller than Λ_{QCD} . Therefore, a cut-off for b have to be made for the low limit of the integral, a typical method to do this is to make an exchange

$$b \Rightarrow b_* = b / \sqrt{1 + b^2/b_{max}^2} \quad (8)$$

with $b_{max} < 1/\Lambda_{QCD}$ in S . According to above definition, obversely the b_* will never be larger than b_{max} , which make $1/b_*$ will always larger than Λ_{QCD} . In order to compensate the cut-off effect, we have to introduce additional non-perturbative form factor.

$$S_{sud} \Rightarrow S_{pert}(Q, b_*) + S_{NP}(Q, b) \quad (9)$$

In this work, we follow the parameterizations of Ref. [10],

$$S_{NP}^{SIDIS}(Q, b) = g_2 \ln \left(\frac{b}{b_*} \right) \ln \left(\frac{Q}{Q_0} \right) + \left(\frac{g_1}{2} + g_3 \left(\frac{x_B}{x_0} \right)^\lambda + \frac{g_h}{z_h^2} \right) b^2, \quad (10)$$

$$S_{NP}^{SIDIS \text{ collins}}(Q, b) = g_2 \ln \left(\frac{b}{b_*} \right) \ln \left(\frac{Q}{Q_0} \right) + \left(\frac{g_1}{2} + g_3 \left(\frac{x_B}{x_0} \right)^\lambda + \frac{g_h - g_c}{z_h^2} \right) b^2, \quad (11)$$

$$S_{NP}^{e^+e^-}(Q, b) = g_2 \ln \left(\frac{b}{b_*} \right) \ln \left(\frac{Q}{Q_0} \right) + \left(\frac{g_h}{z_{h1}^2} + \frac{g_h}{z_{h2}^2} \right) b^2, \quad (12)$$

$$S_{NP}^{e^+e^- \text{ collins}}(Q, b) = g_2 \ln \left(\frac{b}{b_*} \right) \ln \left(\frac{Q}{Q_0} \right) + \left(\frac{g_h - g_c}{z_{h1}^2} + \frac{g_h - g_c}{z_{h2}^2} \right) b^2, \quad (13)$$

Where, g_1 , g_2 , g_3 and g_h are free parameters extracted from unpolarized Drell-Yan and SIDIS processes. With help of the above non-perturbative and perturbative

Sudakov form factors, the structure functions for SIDIS processes can be written as:

$$F_{UU}(Q; P_{h\perp}) = \frac{1}{z_h^2} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp} \cdot \vec{b}/z_h} \tilde{F}_{UU}(Q; b), \quad (14)$$

$$F_{UT}^{\sin(\phi_h + \phi_s)}(Q; P_{h\perp}) = \frac{1}{z_h^2} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp} \cdot \vec{b}/z_h} \hat{P}_{h\perp}^\alpha \tilde{F}_{\text{collins}}^\alpha(Q; b), \quad (15)$$

with

$$\tilde{F}_{UU}^\alpha(Q; b) = e^{-S_{\text{pert}}(Q^2, b_*) - S_{NP}^{SIDIS}(Q, b)} \tilde{F}_{UU}(b_*), \quad (16)$$

$$\tilde{F}_{\text{collins}}^\alpha(Q; b) = \left(\frac{-ib_\perp^\alpha}{2} \right) e^{-S_{\text{pert}}(Q^2, b_*) - S_{NP}^{SIDIS}(Q, b)} \tilde{F}_{\text{collins}}(b_*), \quad (17)$$

and for $e^-e^+ \rightarrow hh$ processes, the structure functions become:

$$Z_{uu}(Q; P_{h\perp}) = \frac{1}{z_{h1}^2} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp} \cdot \vec{b}/z_{h1}} \tilde{Z}_{UU}(Q; b), \quad (18)$$

$$Z_{\text{collins}}^{\alpha\beta}(Q; P_{h\perp}) = \frac{1}{z_{h1}^2} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp} \cdot \vec{b}/z_{h1}} (2\hat{P}_{h\perp}^\alpha \hat{P}_{h\perp}^\beta - g_\perp^{\alpha\beta}) \tilde{Z}_{\text{collins}}^{\alpha\beta}(Q; b), \quad (19)$$

with

$$\tilde{Z}_{uu}(Q; b) = e^{-S_{\text{pert}}(Q^2, b_*) - S_{NP}^{e^+e^-}(Q, b)} \tilde{Z}_{uu}(b_*), \quad (20)$$

$$\tilde{Z}_{\text{collins}}^{\alpha\beta}(Q; b) = \left(\frac{-ib_\perp^\alpha}{2z_1} \right) \left(\frac{-ib_\perp^\beta}{2z_2} \right) e^{-S_{\text{pert}}(Q^2, b_*) - S_{NP}^{e^+e^-}(Q, b)} \tilde{Z}_{\text{collins}}(b_*). \quad (21)$$

In the CSS formalism, we do not use the TMD distribution directly, instead we match the TMD distributions to collinear distributions,

$$f_1^q(x, b^2; C_1/b) = C_{q \leftarrow i}^{f_1} \otimes f_1^i(x, C_1/b) \quad (22)$$

$$h_1^q(x, b^2; C_1/b) = C_{q \leftarrow i}^{h_1} \otimes h_1^i(x, C_1/b) \quad (23)$$

$$D_{h/q}(z, b^2; C_1/b) = C_{i \leftarrow q}^{D_1} \otimes D_{h/i}(z, C_1/b) \quad (24)$$

$$H_{1h/q}^\perp(z, b^2; C_1/b) = C_{i \leftarrow q}^{\text{collins}} \otimes \hat{H}_{h/i}^{(3)}(z, C_1/b). \quad (25)$$

Terms in the right sides of above equations are TMD distributions at $Q = C_1/b$, while ones in the left sides are collinear distributions convoluted by Wilson coefficients C with the renormalization scale $\mu = C_1/b$.

3. Global fitting

Based on the CSS formalism, we can perform a global fitting of existing experimental data to extract the Collins function and quark transversity. Firstly, we need to parameterize the distributions at an initial scale Q_0 . For example, in order to

satisfy the Soffer bound,^{11,12} we parameterize the quark transversity distributions as:

$$h_1^q(x, Q_0) = N_q^h x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{a_q + b_q}}{a_q^{a_q} b_q^{b_q}} \frac{1}{2} (f_1(x, Q_0) + g_1(x, Q_0)) , \quad (26)$$

with $|N_q^h| \leq 1$ for up and down quarks $q = u, d$, respectively, where f_1 are the unpolarized CT10 NLO quark distributions¹³ and g_1 are the DSSV helicity NLO distributions.¹⁴ In present work, we ignore the contribution from sea quark. Similarly, the favored and unfavored Collins fragmentation functions are parameterized as

$$\hat{H}_{fav}^{(3)}(z, Q_0) = N_u^c z^{\alpha_u} (1-z)^{\beta_u} D_{\pi^+/u}(z, Q_0) , \quad (27)$$

$$\hat{H}_{unf}^{(3)}(z, Q_0) = N_d^c z^{\alpha_d} (1-z)^{\beta_d} D_{\pi^+/d}(z, Q_0) , \quad (28)$$

where D is normal unpolarized fragmentation function. Since we apply CSS formalism, we have to evolve the scale Q_0 in both transversity and Collins fragmentation functions to the same point $\mu_b = C_1/b$ by the DGLAP evolution equations.

In total we have 13 parameters in our global fit: $N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_u, \beta_d, g_c$. In the fit, we include all existing SIDIS data (140 points), all points in x_B, z_h , and $P_{h\perp}$ where formalism is valid (we limit $P_{h\perp} < 0.8$ GeV) for $\pi^\pm$ pion production from HERMES [15, 16] COMPASS [17] and JLab HALL A [18]. For the Collins asymmetries in e^+e^- annihilation experiments we have 122 data points, measurements as function of z_{h1}, z_{h2} , and $P_{h\perp}$ (we limit $P_{h\perp}/z_{h1} < 3.5$ GeV) from BELLE¹⁹ and BABAR²⁰ collaborations. We use MINUIT minimization package to perform the fit. The resulting parameters are presented in Table 1. The total $\chi^2 = 221.5$, $n_{d.o.f.} = 249$, and $\chi^2/n_{d.o.f.} = 0.89$. The extracted transversity and Collins fragmentation function are plotted in Fig. 1 at two different scales $Q^2 = 10$ and 1000 GeV².

We also show an example of description of experimental data, namely $P_{h\perp}$ dependence of asymmetry in e^+e^- from BABAR²⁰ collaboration in Fig. 2. The solid line is our fitting result with Sudakov factor including $A^{(1)}, A^{(2)}$ and $B^{(1)}(\text{NLL}')$, after removing the $A^{(2)}(\text{LL})$, we will get the dash line. Since the difference between solid and dash line is obversely, the contribution from $A^{(2)}$ can not be ignore. When the whole Sudakov factor are removed, the asymmetry will become the dot line, it is

Table 1. Best values of fitted parameters of the transversity quark distributions for u and d quarks, as well as Collins fragmentation functions.

N_u^h	=	0.891,	a_u	=	0.67,	b_u	=	0.04,
N_d^h	=	-0.999,	a_d	=	1.47,	b_d	=	3.94,
N_u^c	=	0.256,	α_u	=	1.50,	β_u	=	0.00,
N_d^c	=	-0.198,	α_d	=	0.32,	β_d	=	0.00,
g_c	=	0.024	(GeV ²)					

$$\chi_{min}^2 = 221.5, \quad \chi_{min}^2/n_{d.o.f.} = 0.89$$

Fig. 1. Extracted transversity distribution and Collins fragmentation function at two different scales $Q^2 = 10 \text{ GeV}^2$ (solid lines) and $Q^2 = 1000 \text{ GeV}^2$ (dashed lines).

Fig. 2. Collins asymmetries measured by BABAR²⁰ collaboration as a function of $P_{h\perp}$ in production of unlike sign “U” over like sign “L” pion pairs at $Q^2 = 110 \text{ GeV}^2$. The solid line corresponds to the full NLL' calculation, the dashed line to the LL calculation, and the dotted to the calculation without TMD evolution. Calculations are performed with parameters from Table 1.

much larger than solid one by a factor about 3, it means that the energy evolution effect is so important at $Q^2 = 110 \text{ GeV}^2$.

4. Summary

In this talk, We present our recent work on a global analysis of the Collins azimuthal asymmetries in e^+e^- annihilation and SIDIS processes, by taking into account the appropriate TMD evolution effects under CSS formalism. The collins function and

quark transversity are extracted from existing data, which can be used to make a prediction for future experiment. Our result shows the TMD evolution effect is very important to the globe fitting, especially for experiment data sets within a large regime of Q^2 .

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