Predictions for Transverse-Momentum Dependence in Electron-Positron Annihilation

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We calculate the transverse momentum dependence of back-to-back production of one hadron and one jet in electron-positron annihilation, i.e., $e^+e^- \rightarrow h \text{ jet } X$, at $Q^2 = 100 \text{ GeV}^2$. We use the parameters of the transverse-momentum-dependent (TMD) fragmentation functions we recently extracted from HERMES data at 2.4 GeV^2 . We apply TMD evolution according to two different approaches and using different parameters for the so-called nonperturbative part of TMD evolution. We discuss the differences in our predictions and how experimental measurements could discriminate between different implementations of TMD evolution and different choices of nonperturbative parameters.

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Information about transverse-momentum-dependent (TMD) parton distribution and fragmentation functions (FFs) can be obtained through the study of semi-inclusive DIS, as discussed in the contribution of M. Radici to these proceedings. However, to pin down separately the characteristics of distribution and fragmentation functions, it is essential to have information from different processes.

In this discussion, we take into account the process of back-to-back production of one hadron and one jet in electron-positron annihilation, i.e., $e^+e^- \rightarrow h \text{ jet } X$.

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This is a theoretically very clean process to study the features of fragmentation functions, but in practice it is hampered by the uncertainties induced in the jet reconstruction algorithms. Nevertheless, jet+hadron process is interesting to make some qualitative statements of principle. A phenomenologically more useful process would be $e^+e^- \rightarrow h_1 h_2 X$, which has however some extra complications and will be presented in a forthcoming publication (see also Ref. [1] for preliminary results).

We consider an electron-positron pair annihilating into a virtual photon with four-momentum q , so that $q^2 = Q^2 > 0$. We observe in the end a jet with momentum P_j and a hadron h , with momentum P_h and mass M_h . We introduce the light-cone momentum fraction

$$z = \frac{P^-}{q^-} \approx \frac{P_j \cdot P_h}{P_j \cdot q}. \quad (1)$$

We introduce the component of P_h transverse to the direction of P_j and we denote it with P_{hT} . We consider here only the cross section integrated over the azimuthal angle of P_{hT} . We introduce the rapidity variable y , which is connected to the angle θ between the electron-positron axis and the jet direction in the lepton center-of-mass frame, $y = (1 + \cos \theta)/2$.

In the region where $|P_{hT}^2| \ll Q^2$, we can apply the TMD formalism and write the cross section for the process under consideration as

$$\begin{aligned} \frac{d\sigma}{dz dy dP_{hT}^2} &= \frac{12\pi^2 \alpha^2}{Q^2} (y^2 - y + 1/2) \sum_q e_q^2 D_1^q(z, \mathbf{P}_{hT}^2; Q^2) \\ &= \frac{6\pi \alpha^2}{Q^2} (y^2 - y + 1/2) \sum_q e_q^2 \int db_T b_T J_0(b_T P_{hT}/z) \tilde{D}_1^q(z, b_T^2; Q^2). \end{aligned} \quad (2)$$

The first line of the above expression coincides with Eq. (96) of Ref. [2], integrated over the azimuthal angle of the hadron. In the second line, we introduced the Fourier transform of the TMD FFs.

TMD distributions generally depend on two scales, ζ and μ , and they satisfy evolution equations with respect to both of them (see Refs. [3, 4]). The evolution with respect to ζ is determined by a process-independent soft factor, whereas the evolution in μ is determined by renormalization group equations. In this work, for simplicity we consider $\zeta = \mu = Q$.

In spite of the fact that the evolution equations are known, the evolution kernel cannot be computed exactly at large b_T . For this reason, it is necessary to introduce a universal (i.e., independent of the process, of the initial quark, and of the final hadron) nonperturbative part of the evolution equations.

To write explicitly the evolution of the TMD FF, we follow the approach of Ref. [3] and use the so-called $\mathbf{b}_T^*$ prescription to separate perturbative and

nonperturbative regions. $\mathbf{b}_T^*$ is defined as

$$\mathbf{b}_T^* = \frac{\mathbf{b}_T}{\sqrt{1 + \mathbf{b}_T^2/b_{\max}^2}}. \quad (3)$$

The parameter $b_{\max}$ represents the value where we stop trusting perturbative QCD (pQCD).

We introduce also $\mu_b = 2e^{-\gamma_E}/b^*$. The formula for the Fourier transform of the TMD FFs at a scale Q has different expressions in different evolution schemes (see, e.g., the discussion in Ref. [5]). Here, we consider the approach of Collins (which we name “new CSS”) [3] and a modified version of Echevarría, Idilbi, Scimemi (which we name “Hybrid”) [4].

In the two approaches, the Fourier transform of the TMD FFs at a scale Q , up to the next-to-leading-log level (NLL), can be written as

$$\begin{aligned} \tilde{D}_1^{q \rightarrow h}(z, \mathbf{b}_T^2; Q^2) &\stackrel{\text{New=CSS}}{=} \exp \left\{ - \int_{\mu_b}^Q \frac{d\bar{\mu}}{\bar{\mu}} \left(\Gamma_{\text{cusp}} \ln \frac{Q^2}{\bar{\mu}^2} + \gamma^V \right) \right\} \\ &\times \left(\frac{Q^2}{Q_0^2} \right)^{-\frac{1}{4}g_2\mathbf{b}_T^2} D_1^{q \rightarrow h}(z; \mu_b^2) e^{-\frac{1}{2}g_1\mathbf{b}_T^2}, \end{aligned} \quad (4)$$

$$\begin{aligned} \tilde{D}_1^{q \rightarrow h}(z, \mathbf{b}_T^2; Q^2) &\stackrel{\text{Hybrid}}{=} \exp \left\{ - \int_{Q_0}^Q \frac{d\bar{\mu}}{\bar{\mu}} \left(\Gamma_{\text{cusp}} \ln \frac{Q^2}{\bar{\mu}^2} + \gamma^V \right) \right\} \\ &\times \left(\frac{Q^2}{Q_0^2} \right)^{-\frac{1}{4}g_2\mathbf{b}_T^2} D_1^{q \rightarrow h}(z; Q_0^2) e^{-\frac{1}{2}g_1\mathbf{b}_T^2}. \end{aligned} \quad (5)$$

The collinear fragmentation functions $D_1^{q \rightarrow h}(z; \mu^2)$ are taken from the parametrization of Ref. [6].

At $Q = Q_0$, the expression in the Hybrid approach, Eq. (5), reduces exactly to the ansatz we adopted in fitting the HERMES multiplicities at 2.4 GeV². This is not the case in the new CSS approach, Eq. (4), mainly due to the fact that the collinear FF are evaluated at μ_b instead of Q_0 . However, for choices of $b_{\max} \approx 2e^{-\gamma_E}/Q_0$, also the expressions in the new CSS approach are approximately equal to our starting ansatz in the region $P_{hT} \ll Q$, where HERMES data are.

The large- $\mathbf{b}_T$ region in Eq. (4) and (5) is inspired to the model in Ref. [7], to the BLNY model (see Ref. [8]) and to Ref. [9]. In the present context the parameter g_1 is related to the flavor and kinematic dependent Gaussian widths of Ref. [9]:

$$g_1 \equiv \frac{\langle \mathbf{P}_{\perp, a \rightarrow h}^2 \rangle(z)}{2}. \quad (6)$$

Other parametrizations are available for the large- $\mathbf{b}_T$ region (see, e.g., Refs. [10, 11, 12]) and multiple parameter sets could describe presently available data. Considering our choice of functional form, we do not know which values of the parameters $\{b_{\max}, g_2\}$ are the best ones in order to reproduce the transverse momentum spectrum of e^+e^- annihilation into hadrons. The same holds for the flavor-dependent

configuration	$b_{\max}$ [GeV ⁻¹]	g_2	Ref.
A	0.5	0.68	[8]
B	1.0	0.41	—
C	1.5	0.18	[10]

widths of the Gaussian FFs. From the SIDIS point of view there are 200 equivalent sets of values. This study is exactly aimed at underlying the sensitivity of e^+e^- multiplicities to the non-perturbative parameters (concerning flavor structure and evolution).

The parameters $b_{\max}$ and g_2 are anti-correlated. This is because the first one selects the $\mathbf{b}_T$ value where we do not trust pQCD any more and the second one shapes the effects of gluon radiation for $\mathbf{b}_T > b_{\max}$. So, lowering $b_{\max}$ results in a larger $\mathbf{b}_T$ -range where the evolution needs to be parametrized and, eventually, in a higher value for the g_2 parameter. In Tab. 1 we summarize the three different configurations of values for $b_{\max}$ and g_2 explored in this study.

Concerning the nonperturbative parameters of the TMD FFs, three distinct favored process and one class of unfavored processes have been distinguished, assuming charge conjugation and isospin symmetry. This results in four different Gaussian widths:

$$\begin{aligned}
\langle \mathbf{P}_{\perp, u \rightarrow \pi^+}^2 \rangle &= \langle \mathbf{P}_{\perp, \bar{d} \rightarrow \pi^+}^2 \rangle = \langle \mathbf{P}_{\perp, \bar{u} \rightarrow \pi^-}^2 \rangle = \langle \mathbf{P}_{\perp, d \rightarrow \pi^-}^2 \rangle \equiv \langle \mathbf{P}_{\perp, \text{fav}}^2 \rangle, \\
\langle \mathbf{P}_{\perp, u \rightarrow K^+}^2 \rangle &= \langle \mathbf{P}_{\perp, \bar{u} \rightarrow K^-}^2 \rangle \equiv \langle \mathbf{P}_{\perp, uK}^2 \rangle, \\
\langle \mathbf{P}_{\perp, \bar{s} \rightarrow K^+}^2 \rangle &= \langle \mathbf{P}_{\perp, s \rightarrow K^-}^2 \rangle \equiv \langle \mathbf{P}_{\perp, sK}^2 \rangle, \\
\langle \mathbf{P}_{\perp, \text{all others}}^2 \rangle &\equiv \langle \mathbf{P}_{\perp, \text{unf}}^2 \rangle.
\end{aligned} \tag{7}$$

Each width is also z -dependent:

$$\langle \mathbf{P}_{\perp, a \rightarrow h}^2 \rangle(z) = \langle \hat{\mathbf{P}}_{\perp, a \rightarrow h}^2 \rangle \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}, \tag{8}$$

where

$$\langle \hat{\mathbf{P}}_{\perp, a \rightarrow h}^2 \rangle \equiv \langle \mathbf{P}_{\perp, a \rightarrow h}^2 \rangle(\hat{z}) \quad \text{and} \quad \hat{z} = 0.5. \tag{9}$$

The kinematic parameters β , γ and δ are flavor-independent, contrary to the normalizations. For each parameter there are 200 sets of values available (see Ref. [9]) and in this study we exploit the first 100.

For each predictions, we build the 68% confidence-level bands by computing the observable with the 100 sets of parameters and rejecting, point by point, the largest and the lowest 16% of the predicted values.

In Fig. 1 we show the predicted cross section for $e^+e^- \rightarrow \pi^+ \text{ jet } X$, at $Q^2 = 100$ GeV² and at $z = 0.7$. We restrict our attention to the range $P_{h_T}^2 \leq 1.2$ GeV², where we believe the formalism should be applicable (even without the addition of NLO fixed-order contributions, which are essential at higher P_{h_T}). The three bands correspond to three different sets of values for the nonperturbative evolution

Fig. 1. Transverse-momentum-dependent cross section for the process $e^+e^- \rightarrow \pi^+ \text{ jet } X$, at $Q^2 = 100 \text{ GeV}^2$. The results are obtained starting from 100 nonperturbative inputs at 2.4 GeV^2 , to which TMD evolution is applied using: (a) the “New CSS” approach, (b) the “Hybrid” approach. Each band represents the 68% confidence-level range obtained from the 100 inputs. The three bands correspond to three different sets of values for the nonperturbative evolution parameters b_{max} and g_2 (Tab. 1).

Fig. 2. Transverse-momentum-dependent cross section for the process $e^+e^- \rightarrow h \text{ jet } X$, at $Q^2 = 100 \text{ GeV}^2$. The results are obtained starting from 100 nonperturbative inputs at 2.4 GeV^2 , to which TMD evolution is applied using: (a) the “New CSS” approach, (b) the “Hybrid” approach, with the set of parameters C of Tab. 1. Each band represents the 68% confidence-level range obtained from the 100 inputs. The three bands correspond to three different values of z .

parameters b_{max} and g_2 , described in Tab. 1. In Fig. 1a, the “New CSS” approach has been used. In Fig. 1b, the “Hybrid” approach was used. A measurement of this cross section would be extremely useful to discriminate between different values of b_{max} and g_2 . In both approaches, differences are less pronounced at lower values of z , indicating that higher values of z are particularly interesting in this respect.

Secondly, we note that the two formalisms give different predictions, even if the starting TMD FFs at 2.4 GeV^2 are essentially the same. In principle, the two approaches are supposed to be equivalent in the region where the TMD formalism is applicable. This could still be the case if the two formalisms are used with different

Fig. 3. Ratio between the cross section for π^+ and K^+ , normalized to its value at $P_{h_T} = 0$. This quantity is related to flavor differences in the TMD FFs. (a) “Hybrid” approach, (b) “New CSS” approach, starting from 100 flavor-independent nonperturbative inputs at 2.4 GeV^2 . (c) “Hybrid” approach, (d) “New CSS” approach, starting from 100 flavor-dependent nonperturbative inputs at 2.4 GeV^2 . Each band represents the 68% confidence-level range obtained from the 100 inputs. The three bands correspond to three different values of z .

sets of nonperturbative parameters b_{max} and g_2 . Configuration B in the “New CSS” approach is closer to configuration C in the “Hybrid” approach, therefore it seems that the “Hybrid” approach requires higher values of b_{max} , i.e., a wider extension of the perturbative contribution.

In Fig. 2, we show the predictions for the cross sections at three different values of z . We observe once again some qualitative differences between the two approaches, especially at low z . Since the predictions at low z are not very sensitive to the values of parameters b_{max} and g_2 , it seems more difficult to reconcile the two formalisms at low z . However, it could still be possible that a different z dependence of the nonperturbative input parameters in Eq. (8) is required in the two approaches.

Finally, we come to observables which may be sensitive to flavor differences in the TMD FFs. We though in particular to discuss the ratio between π^+ and K^+ , normalized to its value at $P_{h_T} = 0$. If the nonperturbative parameters were all the

same (flavor-independent scenario), we would expect a flat behavior of the ratio. This is indeed the case for the “Hybrid” approach, as shown in Fig. 3a. However, in the “New CSS” approach this expectation is violated: even with no flavor dependence in the nonperturbative parameters at some low scale, the formalism generates flavor-dependent TMDs at higher scales. This is due to the fact that the transverse-momentum-dependence in this approach is also influenced by the behavior of the collinear FFs, through the dependence on μ_b in Eq. (4). This effect is quite striking at low z and for $b_{\max} = 1.5 \text{ GeV}^{-1}$, see Fig. 3b. The effect is reduced if the value of $b_{\max}$ is reduced.

When the nonperturbative parameters in Eq. (7) are different, the ratio between pions and kaons is not expected to be flat. This effect is evident in the difference between Fig. 3c (flavor dependent) and Fig. 3c (flavor independent) in the “Hybrid” approach. In the “New CSS” approach, the effect is masked, but still creates significant differences between Fig. 3d (flavor dependent) and Fig. 3b.

In conclusion, the measurement of transverse-momentum-dependent cross sections in electron-positron annihilation can be extremely useful to gain insight into several issues related to TMD FFs: better knowledge of nonperturbative parameters and their flavor dependence, better knowledge of the applicability of different approaches to TMD evolution, better knowledge of the nonperturbative parameters involved in the implementation of TMD evolution. The discussion presented here applies to the process $e^+e^- \rightarrow h \text{ jet } X$, which represents a theoretically clean ideal case. Realistic measurements should be performed in $e^+e^- \rightarrow h_1 h_2 X$, but we expect the outcomes to be similar, i.e., we expect similar sensitivity to nonperturbative parameters and to the difference between TMD evolution approaches.

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