DISPERSION THEORY METHODS FOR TRANSITION FORM FACTORS

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Dispersion-theoretical analyses of transition form factors are stepping stones to a model-independent determination of the light-by-light scattering contribution to $(g-2)_\mu$. We extend a calculation of the conversion decays of the lightest isoscalar vector mesons, $\omega/\phi \rightarrow \pi^0 \ell^+ \ell^-$, that builds on a previous dispersive analysis of the $\omega/\phi \rightarrow 3\pi$ partial-wave amplitudes and the pion vector form factor as its sole ingredients. We construct a good parametrization of $e^+e^- \rightarrow 3\pi$ that, together with information on the anomalous process $\gamma\pi \rightarrow \pi\pi$, allows us to obtain a description of the π^0 transition form factor measured in $e^+e^- \rightarrow \pi^0\gamma$.

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1. Introduction: Hadronic Light-by-Light Scattering

The hadronic light-by-light scattering contribution is bound to be the dominant uncertainty in a precision standard-model determination of the anomalous magnetic moment of the muon in the near future. One of its largest contributions is given by pseudoscalar pole terms, whose strength is determined by processes involving a pseudoscalar (π^0, η, η') and two virtual photons, and thus by doubly-virtual transition form factors (see also the discussion on virtual photon–photon scattering in Ref. 1).

Dispersion relations allow for a model-independent description of these transition form factors.² The π^0 transition form factor of fixed isoscalar photon virtuality ($q^2 = M_{\omega/\phi}^2$) can be extracted from $\omega/\phi \rightarrow \pi^0 \ell^+ \ell^-$ ($\ell = e/\mu$), and a dispersive analysis of

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this process has previously been reported.³ An extension of the framework to include arbitrary isoscalar photon virtualities will enable a full-fledged dispersive analysis of the doubly-virtual transition form factor. An intermediate step in complexity is the process $e^+e^- \rightarrow \pi^0\gamma$, which is presented here. For this enterprise the $\omega/\phi \rightarrow 3\pi$ framework discussed in Ref. 2 needs to be extended to include $\gamma^* \rightarrow 3\pi$, which is briefly described in the next section, before we show preliminary results for the $e^+e^- \rightarrow \pi^0\gamma$ cross section in the final section.

2. The $\gamma^* \rightarrow 3\pi$ Amplitude

The once-subtracted dispersive representation of the $\omega/\phi \rightarrow 3\pi$ decay amplitude is given by⁴

$$\begin{aligned}\mathcal{F}(s) &= a_{\omega/\phi}\Omega_1^1(s) \left\{ 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\hat{\mathcal{F}}(s') \sin \delta_1^1(s')}{s'(s' - s)|\Omega_1^1(s')|} \right\}, \\ \hat{\mathcal{F}}(s) &= \frac{3}{2} \int_{-1}^1 dz (1 - z^2) \mathcal{F}(t(s, z)),\end{aligned}\quad (1)$$

where $\Omega_1^1(s)$ is the Omnès function and $\delta_1^1(s)$ the phase shift for the $\pi\pi$ P wave. The main complication when extending Eq. (1) to arbitrary virtualities of the incoming photon arises from the q^2 -dependence of the subtraction constant, i.e. from the replacement $a_{\omega/\phi} \rightarrow a_{e^+e^-}(q^2)$, which leads to the generalization

$$\mathcal{F}(s, q^2) = a_{e^+e^-}(q^2)\Omega_1^1(s) \left\{ 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\hat{\mathcal{F}}(s', q^2) \sin \delta_1^1(s')}{s'(s' - s)|\Omega_1^1(s')|} \right\}. \quad (2)$$

The fundamental difficulty is that $a_{e^+e^-}(q^2)$, which contains three-body resonances such as ω and ϕ , cannot be predicted within the framework itself, but has to be determined by different methods. We choose to use a data-guided approach and represent the subtraction function by the analytically well-behaved function

$$a_{e^+e^-}(q^2) = \alpha + \beta q^2 + \frac{q^4}{\pi} \int_{\text{thr}}^{\infty} ds' \frac{\text{Im}\mathcal{A}(s')}{s'^2(s' - q^2)}, \quad (3)$$

where $\alpha = F_{3\pi}/3 = e/(12\pi^2 F_\pi^3) \times 1.066$ is fixed by the $\gamma \rightarrow 3\pi$ amplitude at $q^2 = 0$,⁵ and

$$\mathcal{A}(q^2) = \frac{c_\omega}{q^2 - M_\omega^2 - iM_\omega\Gamma_\omega(q^2)} + \frac{c_\phi}{q^2 - M_\phi^2 - iM_\phi\Gamma_\phi(q^2)}, \quad (4)$$

where $\Gamma_{\omega/\phi}(q^2)$ is the energy-dependent width of the ω/ϕ . The remaining constants β, c_ω, c_ϕ are then determined from a fit to $e^+e^- \rightarrow 3\pi$ cross-section data,⁶ see Fig. 1 (left). Note that the precise form of the spectral function in Eq. (4) is irrelevant: the only requirement is to have an analytically rigorous representation of the cross section data. Finally, the isoscalar $\gamma^* \rightarrow 3\pi$ partial-wave amplitude is given as $f_1^{\gamma^* \rightarrow 3\pi}(s, q^2) = \mathcal{F}(s, q^2) + \hat{\mathcal{F}}(s, q^2)$.

Fig. 1. Left: Fit of the $e^+e^- \rightarrow 3\pi$ cross section data of Ref. 6. Right: Prediction of the $e^+e^- \rightarrow \pi^0\gamma$ cross section, compared to the data of Refs. 7, 8.

3. $e^+e^- \rightarrow \pi^0\gamma$

With the above definitions we are sufficiently equipped to calculate the full $\gamma^* \rightarrow \pi^0\gamma$ transition form factor according to⁹

$$F_{\pi^0\gamma^*\gamma}(q^2, 0) = F_{\pi\gamma\gamma} + \int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s') F_\pi^{V*}(s')}{12\pi^2 s'^{3/2}} \left\{ f_1^{\gamma^* \rightarrow 3\pi}(s', q^2) - f_1^{\gamma\pi \rightarrow \pi\pi}(s') + \frac{q^2}{s' - q^2} f_1^{\gamma\pi \rightarrow \pi\pi}(s') \right\}, \quad (5)$$

where $q_\pi(s) = (s/4 - M_\pi^2)^{1/2}$, $f_1^{\gamma^* \rightarrow 3\pi}(s, q^2)$ is the isoscalar and $f_1^{\gamma\pi \rightarrow \pi\pi}(s')$ the corresponding isovector partial-wave amplitude (see Refs. 2 and 5 for details), and $F_\pi^V(s)$ is the pion vector form factor. Note that we use a once-subtracted dispersion relation for the isovector transition form factor and fix the full transition form factor at $q^2 = 0$ to the chiral anomaly $F_{\pi\gamma\gamma} = e^2/(4\pi^2 F_\pi)$. Our preliminary results for the $e^+e^- \rightarrow \pi^0\gamma$ cross section are shown in Fig. 1 (right). Note that these results are a prediction solely based on the inputs described above. The error band in the graph is derived from uncertainties in the isoscalar transition form factor. It is combined of uncertainties related to the input parameters, the numerical precision, as well as an additional 10% uncertainty motivated by the fact that a sum rule for the $\omega/\phi \rightarrow \pi^0\gamma$ partial width saturates the experimental value only to about 90% (see the discussion in Ref. 3). More details of this analysis will be presented in Ref. 10.

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