STATUS OF HADRONIC LIGHT-BY-LIGHT SCATTERING IN THE MUON $g - 2$

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We give an update on the status of the hadronic light-by-light scattering contribution to the muon $g - 2$. We review recent work, list some open problems and give an outlook on how to better control the uncertainty of this contribution. We think the estimate $a_\mu^{\text{HLbL}} = (116 \pm 40) \times 10^{-11}$ still gives a fair description of the current situation.

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The anomalous magnetic moment of the muon has served over many years as an important test of the Standard Model. Unfortunately, the theoretical uncertainties from hadronic vacuum polarization (HVP) and hadronic light-by-light scattering (HLbL) make it difficult to interpret the current discrepancy between theory and experiment of three to four standard deviations as a clear sign of New Physics.^{1,2}

The HLbL contribution to the $g - 2$ involves the Green function of four electromagnetic currents, connected to off-shell photons, see Ref. 2 for details. In contrast to the HVP contribution, it cannot be directly related to experimental data and therefore hadronic models have been used to estimate HLbL. The dependence on several momenta also leads to a mixing of long and short distances and makes it difficult to avoid a double counting of quark-gluon and hadronic contributions.

A selection of estimates for HLbL is presented in Table 1. Note that only Refs. 3, 4 are full calculations, using, as much as possible, one model for all the contributions. The compilations^{2,7,8} are based on these full calculations, with revised or newly

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Table 1. Summary of selected estimates for the different contributions to $a_\mu^{\text{HLbL}} \times 10^{11}$. For comparison, the last line shows some results when no form factors are used.

π, K -loops	π^0, η, η'	axial-vectors	scalars	quark-loop	Total	Reference
-19(13)	85(13)	2.5(1.0)	-6.8(2.0)	21(3)	83(32)	3
-4.5(8.1)	82.7(6.4)	1.7(1.7)	-	9.7(11.1)	89.6(15.4)	4
-	83(12)	-	-	-	80(40)	5
0(10)	114(10)	22(5)	-	0	136(25)	6
-	-	-	-	-	110(40)	7
-19(19)	114(13)	15(10)	-7(7)	2.3 [c-quark]	105(26)	8
-19(13)	99(16)	22(5)	-7(2)	21(3)	116(40)	2
-	81(2)	-	-	107(2)	188(4)	9
-(11 - 71)	-	-	-	-	-	10
-20(5)	-	-	-	-	-	11
-45	$+\infty$	-	-	60	-	undressed

calculated values for some of the contributions. More estimates, mostly for the pseudoscalar contribution, can be found in Ref. 12. Until 2010, a consensus had been reached about the central value $a_\mu^{\text{HLbL}} \approx 110 \times 10^{-11}$, but there was a discussion on the error estimate, $\pm 26 \times 10^{-11}$ in Ref. 8, and, more conservatively, $\pm 40 \times 10^{-11}$ in Ref. 2. In view of the precision goal of future $g - 2$ experiments¹³ at Fermilab and J-PARC with $\delta a_\mu = 16 \times 10^{-11}$ and the continued progress in improving HVP, HLbL might soon become the main uncertainty in the theory prediction.^{1,2}

In the last few years, several works have appeared which yield much larger (absolute) values for some of the contributions, see Table 1. In Ref. 9 the quark-loop was studied using a Dyson-Schwinger equation approach. In contrast to Refs. 3, 4, no damping compared to the bare constituent quark-loop result was seen, when a dressing was included. The large size of the quark-loop contribution in Ref. 9 was questioned in the papers,¹⁴ using various quark-models and approaches. The pion-loop contribution was analyzed in Ref. 10. The authors stressed the importance of the pion-polarizability effect and the role of the axial-vector resonance a_1 , which are not included in the models used in Refs. 3, 4. The issue was taken up in Ref. 11 where several models for the pion-loop were studied. The inclusion of the a_1 resonance was attempted, but no finite result for $g - 2$ could be achieved. With a cutoff of 1 GeV, a result close to the earlier estimate in Ref. 3 was obtained.

In the future, maybe lattice QCD will provide a reliable calculation of HLbL.¹⁵ In the meantime, only a close collaboration between theory and experiment can lead to a better controlled estimate for HLbL. On the theory side, the hadronic models can be improved by short-distance constraints^{2,5,6} from perturbative QCD and dispersion relations can be used to connect the theory with experimental data, e.g. in $\gamma\gamma \rightarrow \pi\pi$.¹⁶ On the experimental side, informations from various processes of hadrons interacting with photons at low and intermediate momenta, $|q| \leq 2$ GeV, can help to constrain the models. Important experiments include more precise measurements of the (transition) form factors of light pseudoscalars with possibly two off-shell photons in the process $e^+e^- \rightarrow e^+e^-P$ ($P = \pi^0, \eta, \eta'$) and the two-photon

decay widths of these mesons. This could further reduce the error of the dominant pseudoscalar exchange contribution.¹⁷ For the pion-loop contribution, in addition to studying $\gamma\gamma \rightarrow \pi\pi$, measurements of the pion-polarizability in various processes can help to improve the models.¹⁰ Also more information on the a_1 would be useful.

If the recent results for the quark-loop and pion-loop are taken at face value, one obtains $a_\mu^{\text{HLbL}} = (64 - 202) \times 10^{-11}$. While the new approaches raise some important issues and point to potential shortcomings in the previously used models, these estimates are also still preliminary. The estimate $a_\mu^{\text{HLbL}} = (116 \pm 40) \times 10^{-11}$ from Ref. 2 therefore still gives a fair description of the current situation.

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