Nonperturbative study of the jet quenching parameter from AdS/CFT

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This talk presents our derivation of the jet quenching parameter of $\mathcal{N} = 4$ super symmetric Yang-Mills theory to the sub-leading term in the large 't Hooft coupling λ at a nonzero temperature by including the world-sheet fluctuations around the classical world sheet from AdS/CFT. The strong coupling expansion corresponds to the semi-classical expansion of the string-sigma model, the gravity dual of the Wilson loop operator, with the sub-leading term expressed in terms of functional determinants of fluctuations. The contribution of these determinants are evaluated numerically. We find the jet quenching parameter is reduced due to world sheet fluctuations by a factor $(1 - 1.97\lambda^{-1/2})$. Some insights have been discussed by the comparison between experimental data and our result and other theoretical calculations.

Keywords: Jet quenching parameter; strongly coupled quark-gluon plasma; AdS/CFT.

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1. Introduction

The experimental program at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory has led to strong evidences that a new state of matter is in the deconfinement phase of QCD, but behaves very differently from a

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free gas of partons. The name “strongly coupled quark-gluon plasma (SQGP)” was given to this matter because of several striking experimental discoveries. Firstly, the measured elliptic flow data can be well described by hydrodynamical models only with very low shear viscosity, suggesting that SQGP resembles a nearly perfect liquid. Secondly, due to the interaction with the medium, high energy partons traversing the medium are strongly quenched. This phenomenon is usually characterized by the so-called jet quenching parameter (or transport coefficient) $\hat{q}$, which describes the average transverse momentum square transferred from the traversing parton, per unit mean free path.^{1,2} There are still some model-dependent uncertainties on the extracted values data of the transport coefficient from heavy-ion collisions, ranging from $1 \rightarrow 25 \text{ GeV}^2/\text{fm}$,^{3,4} which is considerably larger than those from perturbative QCD estimation,⁵ higher values demand additional non-perturbative mechanisms. Therefore, it is of great importance to get further information on the possible values of $\hat{q}$ in the strong coupling limit. The conventional nonperturbative first principle tool namely lattice QCD, cannot be applied for this purpose, which requires real-time dynamics.

A prominent implication of the AdS/CFT duality⁶⁻⁹ is the correspondence between the type IIB superstring theory formulated on $\text{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) in four dimensions. In particular, the supergravity limit of the string theory corresponds to the leading behavior of SYM at large N_c and large 't Hooft coupling $\lambda \equiv g_{\text{YM}}^2 N_c = \frac{L^4}{\alpha'^2}$ with L the AdS radius and α' the reciprocal of the string tension. This relation thereby provides a powerful tool to explore the strongly coupled QGP created in RHIC in spite of its underlying dynamics, QCD, is different from $\mathcal{N} = 4$ SYM. It is expected that some of the properties of the latter is universal for all strongly interacting system. Computationally, AdS/CFT correspondence translates the quantum field theory of supersymmetric Yang-Mills theories at strong 't Hooft coupling and large number of colors N_c into a classical field theory in a gravitational background. The AdS/CFT correspondence has been successfully applied into the RHIC physics, including the thermodynamical the transport properties of sQGP. One of the celebrated finding is the universal value of the ratio between the shear viscosity and the entropy density, $\eta/s = \frac{1}{4\pi}$, for quantum field theories admitting a holographic description. It was further conjectured that $\eta/s \geq \frac{1}{4\pi}$ for all matters. See¹⁰ for a review on the subject and the references therein. The RHIC data of the viscosity ratio is compatible with this lower bound. In addition, the entropy density of a $\mathcal{N} = 4$ SYM plasma normalized by its Stefan-Boltzmann limit equals to $3/4$,¹¹ which is close to the corresponding quantity extracted from lattice QCD.

Motivated by these similarities, Liu, Rajagopal and Wiedemann (LRW) computed the jet-quenching parameter of a $\mathcal{N} = 4$ SYM plasma with the aid of AdS/CFT correspondence. The Eikonal approximation relates the jet quenching parameter with the expectation value of an adjoint Wilson loop $W^A[\mathcal{C}]$ with $\mathcal{C}$ a rectangular contour of size $L \times L_-$, where the sides with length L_- run along the

light-cone and the limit $L_- \rightarrow \infty$ is taken in the end.¹² Under the dipole approximation, which is valid for small transverse separation L , the jet-quenching parameter $\hat{q}$ defined in Ref. 1 is extracted from the asymptotic expression for $TL \ll 1$

$$\langle W^A[C] \rangle \approx W_0 \exp \left[-\frac{1}{4\sqrt{2}} \hat{q} L_- L^2 \right] \quad (1)$$

with T the temperature and the prefactor W_0 independent of L . The correspondence principle relates $\langle W^A[C] \rangle$ to the quantum effective action of a type IIB superstring in $AdS_5 \times S^5$ with a black hole. In the strong coupling limit, the action is proportional to the minimum area of the string world sheet in the AdS_5 bulk spanned by the C at the boundary. They find that¹³

$$\hat{q}_{SYM}^{(0)} = \frac{\pi^{3/2} \Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} \sqrt{\lambda} T^3. \quad (2)$$

Interestingly the magnitude of $\hat{q}_{SYM}^{(0)}$ turns out to be closer to the value extracted from RHIC data^{14,15} than pQCD result for the typical value of the 't Hooft coupling, $\lambda \simeq 6\pi$, of QCD. This proposal has attracted lots of interest. Alternately, the energy loss and jet quenching problem was also studied from a drag force on the brane by several authors.¹⁶⁻²⁰ Their extracted jet quenching parameter is also proportional to $\sqrt{\lambda} T^3$ but with a smaller coefficient (by 20 percent).

The LRW formula is strictly valid only when the 't Hooft coupling constant goes to infinity and large N_c limit. However, with a finite value of λ and the $N_c = 3$, an understanding of how these computations are affected by finite λ corrections may be essential for more precise theoretical predictions. There are two sources of contributions to finite λ corrections: 1) The string theory corrections of the AdS-Schwarzschild background where the world sheet underlying the jet quenching Wilson loop is embedded. 2) The fluctuation of the world sheet itself. The former has been calculated in Ref. 21 and it amounts to multiply $\hat{q}_{SYM}^{(0)}$ by a factor $1 - 1.765\lambda^{-3/2}$. The parallel correction within the framework of the drag force has also been figured out in Ref. 17, and has been found to enhance the jet quenching parameter. The latter contribution, that from the fluctuations, however, is of the order $O(\lambda^{-1/2})$ and thereby dominates the former. But it is much harder to evaluate. We had recently studied this contribution and obtained the corrected jet-quenching parameter²²

$$\hat{q}_{SYM}(\lambda) = \hat{q}_{SYM}^{(0)} [1 + \kappa \lambda^{-1/2} + O(\lambda^{-1})], \quad (3)$$

with the value of κ

$$\kappa \simeq -1.97. \quad (4)$$

2. Sub-Leading Order Correction on the Jet Quenching Parameter

In this section, we will expand the Nambu-Goto action around the word sheet underlying the order jet quenching in the quadratic order in the fluctuation coordinates and then carry it out.

It follows from the AdS/CFT correspondence that the expectation value of a Wilson loop of the $\mathcal{N} = 4$ SYM in the fundamental representation of the gauge group is related to the path integral of a type IIB superstring in a 10-dimensional spacetime that is asymptotically $AdS_5 \times S^5$, i.e.

$$\langle W[C] \rangle = \text{const.} \int [dX][d\theta] e^{iS[X, \theta]} \equiv e^{iS_{\text{eff.}}[C]}, \quad (5)$$

where X 's and θ 's are the bosonic coordinates and fermionic coordinates, and the superstring action

$$S[X, \theta] = S_B[X] + S_F[\theta], \quad (6)$$

with $S_B[X]$ and $S_F[\theta]$ the bosonic and fermionic parts.^{23,24} The semiclassical approximation of the effective action $S_{\text{eff.}}$ reads

$$S_{\text{eff.}}[C] = \sqrt{\lambda} \left[S_0[C] + \frac{S_1[C]}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right) \right], \quad (7)$$

where $\sqrt{\lambda}S_0[C] \equiv S[\bar{X}, 0]$ with $\bar{X}$ the classical solution and $S_1[C]$ stands for the one-loop correction.

The bosonic part of (6) $S_B[X]$ is the Nambu-Goto action of a 2d world sheet embedded in the target spacetime, given by

$$S_B[X] = \frac{1}{2\pi\alpha'} \int d\tau \int dr \sqrt{-g}, \quad (8)$$

where g is the determinant of the induced 2d metric

$$g_{\alpha\beta} = G_{\mu\nu}(X) \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X^\nu}{\partial \sigma^j}, \quad (9)$$

with $\sigma^\alpha (\alpha = 0, 1)$ the world sheet parametrization.

The fermionic part of (6), $S_F[\theta]$, takes the form²³⁻²⁵

$$S_F[\theta] = \frac{1}{2\pi\alpha'} \int d^2\sigma (2\sqrt{-g} g^{\alpha\beta} \bar{\theta} \rho_\alpha D_\beta \theta - i\epsilon^{\alpha\beta} \bar{\theta} \rho_i \rho_j \theta), \quad (10)$$

where θ and $\bar{\theta}$ are two 16×1 Majorana-Weyl spinors in the target spacetime and

$$\rho_\alpha \equiv \eta_\alpha^\mu E_\mu^a \Gamma_a, \quad \eta_\alpha^\mu = \frac{\partial X^\mu}{\partial \sigma^\alpha}, \quad (11)$$

with $\Gamma_a (a = 0, 1, 2, 3, 4)$ the 16×16 Gamma matrices satisfying

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}, \quad (12)$$

and $\eta = \text{diag}(-1, 1, 1, 1, 1)$. The higher powers of θ has been ignored and the bosonic coordinates in $S_F[\theta]$ has been approximated by the classical solution. The Majorana condition relates θ and $\bar{\theta}$ via

$$\bar{\theta} = \tilde{\theta} C \times C, \quad (13)$$

with C the 4×4 charge conjugation matrix. The κ -symmetry of the original fermionic action up to the order θ^2 in the Schwarzschild $AdS_5 \times S^5$ background has been

employed to reduce the fermionic degrees of freedom by half. We refer the interested readers to²⁶ for the detailed reduction for the present application. It is a straightforward generalization of the κ -symmetry in the $AdS_5 \times S^5$ background.^{27–29}

At last, we have the explicit form of the fluctuation action

$$\Delta S[\xi, \eta, \zeta, \delta, \chi] \equiv \frac{1}{2\pi\alpha'} (\hat{S}_{\xi\eta} + \hat{S}_{\zeta} + \hat{S}_{\delta}) + iS_F[\chi]. \quad (14)$$

The partition function of the fluctuations is defined to be the path integral of

$$\mathcal{Z} = \text{const.} \int [d\xi][d\eta][d\zeta][d\delta][d\chi] e^{i\Delta S[\xi, \eta, \zeta, \delta, \chi]}, \quad (15)$$

and the effective action up to one-loop order reads

$$S_{\text{eff.}} = S[\bar{X}, 0] - i \ln \mathcal{Z}. \quad (16)$$

Next, we transform the effective action (16) into the form suitable for the numerical calculations. We have

$$Z(\omega) = \left| \frac{\det \mathcal{D}_+(\omega) \det \mathcal{D}_-(\omega)}{\det^{\frac{1}{2}} D_{\xi\eta}(\omega) \det^{\frac{1}{2}} D_1(\omega) \det^{\frac{5}{2}} D_0(\omega)} \right|. \quad (17)$$

In the limit $L_- \rightarrow \infty$,

$$\sum_{\omega} (\dots) \rightarrow L_- ' \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} (\dots), \quad (18)$$

with $L_- ' = \frac{r_h}{\sqrt{1+\epsilon^2}} L_-$ the length of the domain of t . It follows that

$$z = -\frac{T}{2\sqrt{1+\epsilon^2}} \int_{-\infty}^{\infty} d\omega \ln Z(\omega). \quad (19)$$

Since $Z(\omega)$ is an even function of ϵ , we have the Taylor expansion

$$\ln Z(\omega) = a(\omega) + b(\omega)\epsilon^2 + O(\epsilon^4), \quad (20)$$

and

$$z = -\frac{T}{2} \int_{-\infty}^{\infty} d\omega a(\omega) + \epsilon^2 \frac{T}{2} \int_{-\infty}^{\infty} d\omega \left[\frac{1}{2} a(\omega) - b(\omega) \right] + O(\epsilon^4). \quad (21)$$

It is the second integral that contributes to the correction to the jet quenching parameter.

Hence, the strong coupling expansion of the jet quenching parameter can be written as

$$\hat{q}_{SYM} = \hat{q}_{SYM}^{(0)} [1 + \kappa \lambda^{-1/2} + O(\lambda^{-1})], \quad (22)$$

where

$$\kappa = \frac{8\sqrt{\pi}}{\Gamma^2(\frac{1}{4})} \int_{-\infty}^{\infty} d\omega \left[\frac{1}{2} a(\omega) - b(\omega) \right]. \quad (23)$$

Applying the numerical calculation, we end up with the value of κ

$$\kappa \simeq -1.97. \quad (24)$$

The similar method has been employed recently in Refs. 30 and 31. The physical significance of κ will be discussed in the next section.

3. Concluding Remarks

The jet quenching is an important signal of the quark-gluon plasma created via heavy ion collisions. At the energy scale of RHIC, the QCD running coupling constant is not weak, the perturbative analysis may not be appropriate, nor the lattice is available. In terms of the QCD running coupling constant g_{QCD} and $\alpha_{\text{QCD}} = \frac{g_{\text{QCD}}^2}{4\pi}$, the 't Hooft coupling $\lambda_{\text{QCD}} = 4\pi N_c \alpha_{\text{QCD}} = 12\pi \alpha_{\text{QCD}}$ and is strong when $\alpha_{\text{QCD}} = O(1)$. This motivates the application of AdS/CFT to the $\mathcal{N} = 4$ super Yang-Mills plasma as an important reference. To the leading order of large N_c and strong 't Hooft coupling λ , the jet quenching parameter in the super Yang-Mills plasma has been calculated. The sub-leading term in the strong coupling expansion, which dominates the $O(\lambda^{-\frac{3}{2}})$ term in the literature, is provided in this paper.

To figure out the leading order correction, we have expanded the Nambu-Goto action around the world sheet underlying the order jet quenching in the quadratic order in the fluctuation coordinates and carried out. The negative sign of κ is consistent a monotonic behavior from strong coupling to weak coupling. For $\alpha_{\text{SYM}} = 1/2$ and $N_c = 3$, we have $\lambda = 18.8$, which gives rise to 32% reduction of $\hat{q}_{\text{SYM}}$ from the leading order amount,

$$\hat{q}_{\text{SYM}}(\lambda) = \hat{q}_{\text{SYM}}^{(0)}[1 - 1.97\lambda^{-1/2} + O(\lambda^{-1})]. \quad (25)$$

In order to compare the SYM results to the extracted values of $\hat{q}$, one should take into account the different number of degrees of freedom of the $N = 4$ SYM and the QCD. The former is almost three times of that of 3-flavor QCD. Therefore, for a the same energy density the reduced temperature for SYM from T_{qcd} satisfies $3T^4 = T_{\text{qcd}}^4$. Considering this effect, our Next leading-order jet quenching parameter $\hat{q}$ turns out to be in line with that people extracted from data, ranging from 0.67–1.07 GeV²/fm for $\hat{q}$ at $T = 300$ MeV, and 5–7.8 for the ratio $\hat{q}/T^3$ for $\alpha_s = 0.3$ and 0.5.³² One can also match the corresponding entropy density and consider the reduced temperature, then our next-leading order jet quenching parameter also falls within the range of $\hat{q}$ extracted from experimental data although the leading-order is significantly above this range.³²

The jet quenching parameter has also been calculated within the AdS/CFT correspondence via the dragging force with a slightly different outcome to the leading order. As this calculation also involves the minimum area under the Nambu-Goto action, it remains to examine the order $\lambda^{-\frac{1}{2}}$ along this line. This discrepancy from the experimental result may also stem from other difference between QCD and the super Yang-Mills at large N_c and large 't Hooft coupling, such as finite N_c and infrared cutoff of the former. We hope to report our progress in this regard.

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