Revisiting $\bar{K}N$ scattering and relevant baryon resonances

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We give an updated study on meson-baryon scattering within the framework of unitary chiral perturbation theory. We focus on S -wave scattering with strangeness -1 channel. After fixing the unknown couplings through fitting to experimental data, we then analyze the baryon spectroscopy and the sub-threshold behavior of the $\bar{K}N$ amplitude. We end with a summary and a brief outlook.

Keywords: $\Lambda(1405)$; $\bar{K}N$ scattering length; unitary chiral perturbation theory.

1. Introduction

The measurement on the energy shift and width of the kaonic hydrogen state is important in understanding the complicated dynamics near the $\bar{K}N$ threshold, where the famous sub-threshold resonance of $\Lambda(1405)$ is involved. Recently, it was also confirmed experimentally by the CLAS Collaboration¹ the prediction performed in Ref. 2 from leading order Unitary Chiral Perturbation Theory ($U\chi PT$) on the existence of an isovector partner of the $\Lambda(1405)$ in the same energy region. The results of the kaonic hydrogen state from the KEK³ and DEAR⁴ collaborations show clear inconsistency on their measurements. Very recently, the SIDDHARTA collaboration releases their new results,⁵ which turns out to be consistent with the previous ones from KEK³ but with much smaller uncertainties. In Ref. 6, we exploit the new measurement from SIDDHARTA, in addition to other scattering and event distribution data, in our study of S -wave meson-baryon scattering with strangeness

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-1 under the framework of $U\chi$ PT at next-to-leading order (NLO). We review the main results from that reference in the following.

2. Theoretical Setups and Phenomenological Discussions

Our theoretical starting point is the simplified algebraic N/D solution for a partial-wave two-body scattering amplitude

$$T(W) = [1 + N(W) \cdot g(W^2)]^{-1} \cdot N(W), \quad (1)$$

with W the energy in the center of mass frame. The function $g(s)$, $s = W^2$, encodes the right-hand cuts from the two-particle intermediate states required by unitarity, while $N(W)$ does not include any right-hand cut and collects the crossed-channel cuts. One can calculate the function $g(s)$ by using the dimensional regularization and then replace the divergence by a constant, which corresponds to the subtraction constant in the once-subtracted dispersion relation.⁷ The subtraction constants will be fitted to data. Up to and including NLO the $N(W)$ function corresponds to the perturbative partial-wave amplitudes directly calculated from baryon χ PT.^{2,8} The explicit perturbative calculations can be also found in Ref. 9. In the multiple-channel case, the N/D ansatz in Eq. (1) can be straightforwardly extended and $T(W)$, $g(W)$, $N(W)$ in this equation should be then understood as matrices.² In the present work, we consider the S -wave amplitudes and include ten coupled channels: $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^+\Sigma^-$, K^-p , $\bar{K}^0n$, $\eta\Lambda$, $\eta\Sigma^0$, $K^0\Xi^0$ and $K^+\Xi^-$.

With the key ingredient presented in Eq. (1), the experimentally measurable cross section $(\phi B)_i \rightarrow (\phi B)_j$ takes the form

$$\sigma[(\phi B)_i \rightarrow (\phi B)_j] = \frac{1}{16\pi s} \frac{|\vec{p}_j|}{|\vec{p}_i|} |T_{(\phi B)_i \rightarrow (\phi B)_j}|^2, \quad (2)$$

being $\vec{p}_i$ and $\vec{p}_j$ the initial and final three momenta of the baryons in the center of mass frame, respectively. The cross sections from eight processes are included in our fit: $\sigma(K^-p \rightarrow K^-p)$, $\sigma(K^-p \rightarrow \bar{K}^0n)$, $\sigma(K^-p \rightarrow \pi^+\Sigma^-)$, $\sigma(K^-p \rightarrow \pi^-\Sigma^+)$, $\sigma(K^-p \rightarrow \pi^0\Sigma^0)$, $\sigma(K^-p \rightarrow \pi^0\Lambda)$, $\sigma(K^-p \rightarrow \eta\Lambda)$ and $\sigma(K^-p \rightarrow \pi^0\pi^0\Sigma^0)$. The $\pi^-\Sigma^+$ event distribution from $K^-p \rightarrow \Sigma^+(1660)\pi^- \rightarrow \Sigma^+\pi^-\pi^+\pi^-$ and $\pi^0\Sigma^0$ distribution from $K^-p \rightarrow \pi^0\pi^0\Sigma^0$ are also taken into account by introducing two additional free parameters r, r' .⁶ At the K^-p threshold, three ratios of different cross sections are considered in the fit:

$$\begin{aligned} \gamma &= \frac{\sigma(K^-p \rightarrow \pi^+\Sigma^-)}{\sigma(K^-p \rightarrow \pi^-\Sigma^+)}, \quad R_c = \frac{\sigma(K^-p \rightarrow \text{charged particles})}{\sigma(K^-p \rightarrow \text{all})}, \\ R_n &= \frac{\sigma(K^-p \rightarrow \pi^0\Lambda)}{\sigma(K^-p \rightarrow \text{all neutral states})}. \end{aligned} \quad (3)$$

The width and energy shift of the kaon hydrogen 1s state are connected with the $\bar{K}N$ scattering length, see Ref. 10 for details. This allows us to take into account

the new measurement from SIDDHARTA collaboration. The K^-p scattering length a_{K^-p} is related to the unitarized partial-wave amplitude in Eq. (1) through

$$\mathcal{F}(\sqrt{s}) \equiv \frac{T_{K^-p \rightarrow K^-p}(\sqrt{s})}{8\pi\sqrt{s}},$$

$$a_{K^-p} = \mathcal{F}(M_{K^-} + m_p).$$
(4)

Not only the scattering lengths, but also the $\bar{K}N$ amplitudes in the sub-threshold energy region, are interesting, since they are crucial to understand the properties of kaons in nuclear matter. Therefore, we also study the behavior of the quantity $\mathcal{F}(\sqrt{s})$, defined in the equation above, in the following discussions. Moreover, additional experimental inputs are included in our fit, such as the pion-nucleon sigma term $\sigma_{\pi N}$, pion-nucleon isoscalar scalar scattering length a_{0+}^+ and the masses of N , Λ , Σ and Ξ at NLO of baryon χ PT. The $\pi\Lambda$ phase shift at the mass of Ξ^- is fitted as well.

When fitting the unknown parameters to the experimental data, we employ two different strategies. In one case, we do not distinguish the decay constants of the light pseudoscalar mesons and we use only one common constant for all the channels, which is fitted to the data. While in the other case, we use the physical values for decay constants of π , K , and η . The former case will be referred as Fit I and the latter as Fit II.

We notice that in the chiral community there are two usual ways for defining the χ^2 in fits to meson-baryon scattering data. In addition to the standard definition for the χ^2 , such as in Refs. 2, 8, it is also common to use the number of the data points in an individual measurement to weight its chi-square.^{11–13} In order to test the stability of the fits, we employ both definitions of χ^2 in Fit I and Fit II. Generally speaking, we find that both fits can reproduce well the experimental input and no significant changes in the reproduction of data stem from switching between Fit I and Fit II, regardless of which definitions of the χ^2 we use in the fit.^a We refer to Ref. 6 for a more detailed discussion of the fit results.

After fixing the free parameters through the fit procedure, we are ready to present our predictions for the sub-threshold extrapolation of the $\bar{K}N$ amplitudes and also relevant baryon resonance spectroscopy.

Our final results for the $\bar{K}N$ scattering lengths (in units of fm) read

$$\begin{aligned} a_{K^-p} &= (-0.69 \pm 0.16) + i(0.94 \pm 0.11), \\ a_{I=0} &= (-1.74 \pm 0.34) + i(1.31 \pm 0.20), \\ a_{I=1} &= (0.36 \pm 0.12) + i(0.56 \pm 0.12), \end{aligned}$$
(5)

which are compatible with the determinations from Refs. 11, 12, 13 that also take into account the SIDDHARTA measurement. However, we observe that the

^aIn Ref. 6, we have also performed an $O(p)$ -Fit, where we only consider the leading order χ PT Lagrangian. Except the cross section of $K^-p \rightarrow \eta\Lambda$, it turns out that the $O(p)$ -Fit can reasonably reproduce other observables.

Fig. 1. Sub-threshold extrapolation of the $K^-p \rightarrow K^-p$ amplitude $\mathcal{F}(\sqrt{s})$ defined in Eq. (4) both for Fit I (solid red line) and Fit II (blue dashed line). We show the real part in the left panel, and the imaginary part in the right one. The shaded and hatched areas surrounding the different lines stand for the statistical error bands from the fit of Ref. 6. At the K^-p threshold point, we plot the scattering lengths with error bars directly obtained from the SIDDHARTA measurement by combining the modified Deser formula in Ref. 10.

consistency of the scattering lengths from different fits does not necessarily guarantee the same conclusion on the sub-threshold extrapolations, as we show in Fig. 1. It is clear that the uncertainties caused by employing different theoretical expressions, i.e. to set a common decay constant or to distinguish between the decay constants for π, K, η , are already larger than the statistical error bars (that correspond to the shaded and hatched areas in Fig. 1). This tells us that a higher order calculation, together with experimental input in the $\pi\Sigma$ energy threshold, would have a clear impact to settle down the ambiguity in the sub-threshold extrapolation of the $\bar{K}N$ amplitude.

Another important prediction obtained from the fits is the baryon resonance spectroscopy. We have obtained resonance poles in the proper unphysical Riemann sheets both for isoscalar and isovector baryon resonances. See Ref. 6 for the definitions of different Riemann sheets. For the $\Lambda(1670)$, its pole positions from Fit I and Fit II are found to be compatible

$$\begin{aligned}\sqrt{s} &= (1674_{-3}^{+3} - i 11_{-3}^{+7}) \text{ MeV}, & (\text{Fit I}) \\ \sqrt{s} &= (1677_{-3}^{+5} - i 11_{-3}^{+5}) \text{ MeV}, & (\text{Fit II})\end{aligned}\quad (6)$$

where we only show the poles in the fifth Riemann sheet. Similar poles are also found in other sheets.⁶ For the $\Lambda(1405)$, we confirm its double-pole structure discovered in Ref. 2, which is also supported by the recent measurement from the CLAS collaboration.¹⁴ Furthermore, we also confirm that one of the poles is broad, with a large imaginary part, and the other is narrow, with a small imaginary part. The pole positions in the third Riemann sheet from the two fits read

$$\begin{aligned}\sqrt{s_1} &= (1436_{-10}^{+14} - i 126_{-28}^{+24}) \text{ MeV}, & \sqrt{s_2} &= (1417_{-4}^{+4} - i 24_{-4}^{+7}) \text{ MeV}, & (\text{Fit I}) \\ \sqrt{s_1} &= (1388_{-9}^{+9} - i 114_{-25}^{+24}) \text{ MeV}, & \sqrt{s_2} &= (1421_{-2}^{+3} - i 19_{-5}^{+8}) \text{ MeV}, & (\text{Fit II}).\end{aligned}\quad (7)$$

Table 1. The couplings $|\beta_i|_{(I)}$ of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances from Fit I and Fit II. All of the numbers are given in units of GeV. The isospin label I is given when more than one isospin channel exists.

Pole	$ \beta_{\pi\Lambda} $	$ \beta_{\pi\Sigma} _0$	$ \beta_{\pi\Sigma} _1$	$ \beta_{\pi\Sigma} _2$	$ \beta_{\bar{K}N} _0$	$ \beta_{\bar{K}N} _1$	$ \beta_{\eta\Lambda} $	$ \beta_{\eta\Sigma} $	$ \beta_{K\Xi} _0$	$ \beta_{K\Xi} _1$
$\Lambda(1405)$										
Fit I: $\sqrt{s}_1$	$0.0^{+0.0}_{-0.0}$	$8.8^{+0.9}_{-0.4}$	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$7.7^{+1.3}_{-0.7}$	$0.0^{+0.1}_{-0.0}$	$1.4^{+0.4}_{-0.3}$	$0.0^{+0.1}_{-0.0}$	$2.1^{+0.8}_{-0.7}$	$0.0^{+0.0}_{-0.0}$
$\sqrt{s}_2$	$0.1^{+0.0}_{-0.0}$	$5.0^{+1.5}_{-0.8}$	$0.1^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$7.7^{+1.2}_{-0.6}$	$0.1^{+0.0}_{-0.0}$	$1.4^{+0.4}_{-0.3}$	$0.1^{+0.0}_{-0.0}$	$1.5^{+0.7}_{-0.5}$	$0.1^{+0.0}_{-0.0}$
$\Lambda(1405)$										
Fit II: $\sqrt{s}_1$	$0.0^{+0.0}_{-0.0}$	$8.2^{+0.8}_{-0.5}$	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$6.1^{+1.1}_{-0.6}$	$0.1^{+0.0}_{-0.0}$	$2.2^{+0.6}_{-0.3}$	$0.0^{+0.0}_{-0.0}$	$1.9^{+0.2}_{-0.1}$	$0.1^{+0.0}_{-0.0}$
$\sqrt{s}_2$	$0.2^{+0.1}_{-0.1}$	$4.2^{+1.5}_{-0.9}$	$0.2^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$6.2^{+1.2}_{-0.5}$	$0.3^{+0.1}_{-0.1}$	$2.8^{+0.5}_{-0.3}$	$0.4^{+0.2}_{-0.1}$	$0.7^{+0.4}_{-0.3}$	$0.4^{+0.1}_{-0.1}$
$\Lambda(1670)$										
Fit I	$0.0^{+0.0}_{-0.0}$	$0.9^{+0.4}_{-0.2}$	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$1.6^{+0.4}_{-0.2}$	$0.0^{+0.0}_{-0.0}$	$1.7^{+0.5}_{-0.3}$	$0.0^{+0.0}_{-0.0}$	$11.1^{+0.3}_{-0.3}$	$0.1^{+0.0}_{-0.0}$
$\Lambda(1670)$										
Fit II	$0.0^{+0.0}_{-0.0}$	$0.8^{+0.1}_{-0.1}$	$0.1^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$1.6^{+0.4}_{-0.4}$	$0.1^{+0.0}_{-0.0}$	$1.8^{+0.2}_{-0.2}$	$0.1^{+0.0}_{-0.0}$	$10.5^{+0.2}_{-0.2}$	$0.1^{+0.0}_{-0.0}$

So it is clear that the narrow pole is quite model-independent, while the broad one is sensitive to the detailed theoretical model. Not only the pole positions but also the resonance couplings are important to characterize the resonances. We provide in Table 1 the couplings of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances obtained as the residues of the amplitudes in the pole position.⁶

For the $I = 1$ case, we find that the resonance contents from Fit I and Fit II are quite different.⁶ For example, we observe two narrow Σ resonance poles near the $\bar{K}N$ threshold in Fit II, while these kinds of poles are completely absent in Fit I. In view of the results from leading order $U\chi$ PT, where an $I = 1$ resonance in this region results,² as well as from the recent experimental verification of this resonance,¹ we can conclude that Fit I is disfavored. In addition, Fit I is also the one that provides a mass for the wider $\Lambda(1405)$ pole above the $\bar{K}N$ threshold, which is also at odds with Ref. 2 and the explicit detection of the two $\Lambda(1405)$ poles by the CLAS Collaboration.¹⁴

3. Summary and Outlook

Within the framework of unitary chiral perturbation theory up to next-to-leading order, we have exploited a large amount of data to explore the nonperturbative chiral dynamics in the strangeness -1 meson-baryon scattering. We propose two different strategies to perform the fits. The two fits reproduce well data and yield quite compatible results for the $\bar{K}N$ scattering lengths, while this is not the case for the $\bar{K}N$ amplitudes below the $\bar{K}N$ threshold, where clear deviations are observed from the two fits. This indicates that a higher order χ PT calculation might help to get a definitive answer in the sub-threshold extrapolation of meson-baryon amplitudes. Regarding the spectroscopy, two relevant isoscalar baryon resonances appear in our scattering amplitudes and their properties are rather stable by switching from one

fit to another. However, the situation at NLO for the isovector channel is not so clear, with different Σ -type resonances showing up in different fits. This is due to the proliferation of free parameters at NLO compared with LO, which is able to perform no-free-parameter predictions.² Therefore, it seems necessary to consider more experimental input to sharpen the NLO results. For example, such kind of useful input would be the new data on the $\pi\Sigma$ event distributions given in Ref. 1, which we are analyzing in our new project to study the $\pi\Sigma$ photoproduction.¹⁵

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