Hyperon-hyperon interaction based on quark-model baryon-baryon interactions

Kenji Fukukawa

*RIKEN Nishina Center, RIKEN, Hirosawa 2-1
Wako, Saitama 351-0114, Japan
Istituto Nazionale di Fisica Nucleare, Sez. di Catania
Via S. Sofia 64, Catania 95123, Italy
kenji.fukukawa@ct.infn.it*

Published 16 April 2014

Energy-independent nonlocal Gaussian potential based on the quark-model baryon-baryon interaction is derived by using the Gauss-Legendre quadrature and the Bargmann algebra. The reliability of this potential is examined with respect to the NN , YN and YY phase shifts. This potential reproduces the phase shifts predicted by quark-model baryon-baryon interaction fss2.

Keywords: Quark-model baryon-baryon interaction; phase shifts.

PACS Numbers: 13.75.Cs, 13.75.Ev

1. Introduction

The baryon-baryon (BB) interaction is the most fundamental ingredient in all the many-body nuclear systems. Compared with nucleon-nucleon (NN) and hyperon-nucleon (YN) interactions, we have a lot of ambiguity about hyperon-hyperon (YY) interactions since there is no scattering data. In order to understand the properties of $\Lambda\Lambda$ interactions, it is planned to perform the systematic search of $\Lambda\Lambda$ hypernuclei at the J-PARC facility.

The quark-model (QM) BB interactions developed by Kyoto-Niigata group have achieved an accurate description of available NN and YN experimental data.¹ These interactions are constructed within the framework of resonating group method (RGM) for two $(0s)^3$ quark clusters. One of the main features of these interactions is the treatment based on the SU_6 spin-flavor symmetry. The results

This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 3.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

predicted by lattice QCD calculations² are consistent with those by QM BB interactions. The advanced versions of QM BB interactions, FSS³ and fss2,⁴ have been successfully applied to light ordinary nuclei and hypernuclei (${}^3\text{H}$, ${}^3_\Lambda\text{H}$,⁵ ${}^9_\Lambda\text{Be}$ ($= 2\alpha + \Lambda$)⁶ and ${}^6_{\Lambda\Lambda}\text{He}$ ($= \alpha + 2\Lambda$)⁷) through Faddeev calculations.

However, the quark structure of the baryon makes QM BB interactions hard to deal with on the applications to more complex systems. QM BB interactions are nonlocal and have the linearly energy-dependent term as the result of RGM formalism. The energy dependence is not favorable since the energy of subsystems is not well-defined in few-body scattering problems. This energy dependence can be eliminated by the off-shell transformation.⁸ The energy-independent version of fss2 is applied to the three-body bound-state⁵ and nucleon-deuteron scattering systems.⁹

The purpose of this work is to construct BB interaction based on QM BB interaction. We have already constructed the energy-dependent nonlocal Gaussian potential¹⁰ based on the QM NN interaction fss2. In this paper, we will present the energy-independent version of this Gaussian potential and extend it to YN and YY channels. In the next section, we recapitulate the QM BB interaction fss2 and the formalism for the energy-independent Gaussian potential is briefly explained. The accuracy of the energy-independent potential is also discussed. The final section is devoted to the summary and future works.

2. The Energy-Independent Gaussian Potential Based on QM BB Interactions

As mentioned in the introduction, QM BB interactions are formulated in the RGM. The quark-quark interaction is composed of a confinement potential with a quadratic power law, the full Fermi-Breit interaction with explicit quark-mass dependence and the effective meson exchange potentials (EMEPs).⁴ The RGM equation for the parity-projected relative wave function $\chi_\alpha^\pi(\mathbf{R})$ is derived from $\langle \delta\Psi | E - H | \Psi \rangle = 0$ and it reads³

$$\left[\varepsilon_\alpha + \frac{\hbar^2}{2\mu_\alpha} \left(\frac{\partial}{\partial \mathbf{R}} \right)^2 \right] \chi_\alpha^\pi(\mathbf{R}) = \sum_{\alpha'} \int d\mathbf{R}' G_{\alpha\alpha'}(\mathbf{R}, \mathbf{R}'; E) \chi_{\alpha'}^\pi(\mathbf{R}'). \quad (1)$$

The subscript α stands for a set of quantum numbers of the channel wave function. For simplicity, we will fix the channels and omit the subscripts α and α' hereafter. The energy-dependent RGM kernel $V_{\text{RGM}}(\mathbf{q}_f, \mathbf{q}_i; E)$ is defined through

$$\begin{aligned} V_{\text{RGM}}(\mathbf{q}_f, \mathbf{q}_i; E) &= \langle e^{i\mathbf{q}_f \cdot \mathbf{R}} | G(\mathbf{R}, \mathbf{R}'; E) | e^{i\mathbf{q}_i \cdot \mathbf{R}} \rangle \\ &= \sum_{\Omega, (\beta)} \sum_{x, T} M_{xT}^{\Omega, (\beta)}(\mathbf{q}_f, \mathbf{q}_i) O^\Omega(\mathbf{q}_f, \mathbf{q}_i) - \varepsilon N(\mathbf{q}_f, \mathbf{q}_i), \end{aligned} \quad (2)$$

where ε is related to the total energy of the system in the center of mass system through $\varepsilon = E_{\text{total}} - E_{\text{int}}$, $N(\mathbf{q}_f, \mathbf{q}_i)$ is the normalization kernel and $O^\Omega(\mathbf{q}_f, \mathbf{q}_i)$ stands for various space-spin invariants for nine interaction species; Ω = central, spin-spin, LS , $LS^{(-)}$, $LS^{(-)\sigma}$ and three types of tensor forces.¹ The RGM interaction

type is denoted by $\mathcal{T}$ and x is the number of quarks exchanged. In the EMEP terms, the meson β is exchanged between quarks. The each components of Born kernel $M_{x\mathcal{T}}^{\Omega,(\beta)}(\mathbf{q}_f, \mathbf{q}_i)$ is explicitly given in Refs. 4 and 11.

The energy-independent renormalized RGM kernel $V^{\text{RGM}}(\mathbf{q}_f, \mathbf{q}_i)$ for a two-cluster system reads⁸

$$V^{\text{RGM}}(\mathbf{q}_f, \mathbf{q}_i) = \sum_{\Omega,(\beta)} \sum_{x,\mathcal{T}} M_{x\mathcal{T}}^{\Omega,(\beta)}(\mathbf{q}_f, \mathbf{q}_i) O^{\Omega}(\mathbf{q}_f, \mathbf{q}_i) + W(\mathbf{q}_f, \mathbf{q}_i). \quad (3)$$

The nonlocal kernel W is given by¹²

$$\begin{aligned} W &= \Lambda \frac{1}{\sqrt{N}} h \frac{1}{\sqrt{N}} \Lambda - h = \mathcal{K}h + h\mathcal{K} + \mathcal{K}h\mathcal{K} \quad \text{with} \\ \mathcal{K} &= \Lambda \left(\frac{1}{\sqrt{N}} - 1 \right) \Lambda = \sum_{r=1}^{r_{\max}} \frac{(2r-1)!!}{(2r)!!} [\Lambda(1-N)\Lambda]^r, \end{aligned} \quad (4)$$

where $h = h_0 + \sum_{\Omega,(\beta)} \sum_{x,\mathcal{T}} M_{x\mathcal{T}}^{\Omega,(\beta)} O^{\Omega}$ with h_0 being the kinetic energy for the two-cluster relative motion, $\Lambda = 1 - |u\rangle\langle u|$ is a two-cluster Pauli projection operator, and $|u\rangle$ is a Pauli-forbidden state satisfying $N|u\rangle = 0$.

For the direct terms of EMEPs, the original form is employed. For the symmetric exchange terms, $M_{x\mathcal{T}}^{\Omega,(\beta)}(\mathbf{q}_f, \mathbf{q}_i)$ is expanded as

$$\begin{aligned} M_{x\mathcal{T}}^{\Omega,(\beta)}(\mathbf{q}_f, \mathbf{q}_i) &\sim \sum_i \left\{ C_{1i} + C_{2i}(\mathbf{q}_f + \mathbf{q}_i)^2 + C_{3i}(\mathbf{q}_f - \mathbf{q}_i)^2 + C_{4i}(\mathbf{q}_f^2 - \mathbf{q}_i^2)^2 \right. \\ &\quad \left. + C_{5i}(\mathbf{q}_f \times \mathbf{q}_i)^2 \right\} \times e^{-\alpha_i(\mathbf{q}_f^2 + \mathbf{q}_i^2) + \gamma_i(\mathbf{q}_f \cdot \mathbf{q}_i)} \end{aligned} \quad (5)$$

For asymmetric exchange kernels, we need slight modifications. The coefficients ($C_{1i}, \dots, C_{5i}$, α_i , and γ_i (and also β_i for non-symmetric kernels)) are determined by using the Gauss-Legendre integration quadrature to the special functions which appear in $M_{x\mathcal{T}}^{\Omega,(\beta)}(\mathbf{q}_f, \mathbf{q}_i)$. If no Pauli forbidden state exists, we expand $\mathcal{K}$ in the nonlocal kernel W as

$$\begin{aligned} \mathcal{K} &\sim \sum_{r=1}^{r_{\max}} (-1)^r \left(-\frac{1}{4\gamma}(\mathbf{q}_f^2 + \mathbf{q}_i^2) \right) \left[x_r^d \widetilde{M} \left(\mathbf{q}_f, \mathbf{q}_i; \left(\frac{1}{3} \right)^r \right) - x_r^e \widetilde{M} \left(\mathbf{q}_f, \mathbf{q}_i; \left(-\frac{1}{3} \right)^r \right) \right] \\ &\quad + (A + B(\mathbf{q}_f \cdot \mathbf{q}_i)) \exp \left(-\frac{1}{4\gamma}(\mathbf{q}_f^2 + \mathbf{q}_i^2) \right), \end{aligned} \quad (6)$$

with

$$\widetilde{M}(\mathbf{q}_f, \mathbf{q}_i; \tau) = \left(\frac{1}{1 - \tau^2} \right)^{\frac{3}{2}} \exp \left(\frac{1}{2\gamma} \frac{\tau^2}{1 - \tau^2} (\mathbf{q}_f^2 + \mathbf{q}_i^2) + \frac{\tau}{\gamma(1 - \tau^2)} \mathbf{q}_f \cdot \mathbf{q}_i \right), \quad (7)$$

where $\gamma = \mu\nu$ with μ and ν being the reduced mass and the harmonic-oscillator width parameter of each cluster, respectively. In the strangeness sector, there are Pauli-forbidden states in some channels. If they exist, we have to subtract corresponding infinite term appearing at A and B in Eq. (7). By using the Bargmann

Fig. 1. Calculated the phase shifts for (a) NN ($I = 0$) channels and for (b) $S = -2 YY \ ^1S_0$ coupled channels. Calculations are made in the isospin basis. The NN ($I = 0$) phase shifts are compared with the phase-shift analysis by Arndt et al. (Ref. 13, squares and crosses). The differences between phase shifts ($\Delta = |\delta_{\text{fss2}} - \delta_{\text{Gauss}}|$) predicted by fss2 and the energy-independent Gaussian potential are also shown in the lower panels.

algebra, the coefficients (x_r^d, x_r^e, A and B) can be calculated analytically from the spin-flavor-color factor.

Figure 1 shows the phase shifts predicted by fss2 for (a) NN ($I = 0$) channels up to the energy $T_{\text{lab}} = 300$ MeV and for (b) $S = -2 YY \ ^1S_0$ coupled channel up to $p_{\text{lab}} = 1000$ MeV/c.¹ The differences between phase shifts ($\Delta = |\delta_{\text{fss2}} - \delta_{\text{Gauss}}|$) predicted by fss2 and the energy-independent Gaussian potential are also shown. We take r_{max} in Eq. (6) as $r_{\text{max}} = 5$ in these calculations. These calculations are made in the isospin basis. The energy-independent nonlocal Gaussian potential reproduces the phase shifts predicted by fss2 with an accuracy of better than 0.04° , reserving the nonlocality which fss2 possesses.

3. Summary and Future Works

We have presented the nonlocal energy-independent Gaussian potentials for the quark-model kernels, using the Gauss-Legendre quadrature and the Bargmann algebra. We calculated the phase shifts in the NN , YN and YY channels and confirmed that this energy-independent potential reproduces the phase shifts predicted by fss2 very accurately, although this potential is rather complicated.

As a next step, we will test this potential in the few-baryon problems and plan to transform it to the coordinate-space potential, and to apply it to light ordinary and hypernuclei. We would also like to obtain more tractable form of this potential in order to apply it the variational calculations by using the Gaussian expansion method¹⁴ or the stochastic variational method¹⁵ *etc.*

Acknowledgments

This work is supported by Grants-in-Aids for Scientific Research (S) (Grant No. 23224006) from the Japan Society for the Promotion of Science (JSPS). This work is also supported by the interdisciplinary Theoretical Science Research Group (iTHES). The numerical calculations were carried out on SR16000 at Yukawa Institute for Theoretical Physics (YITP) in Kyoto University, on the high-performance computing system Intel Xeon X5680 at Research Center for Nuclear Physics (RCNP) in Osaka University, and on the RIKEN Integrated Cluster of Clusters (RICC).

References

1. Y. Fujiwara, Y. Suzuki and C. Nakamoto, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
2. HAL QCD Collab. (T. Inoue, N. Ishii, S. Aoki *et al.*), *Prog. Theor. Phys.* **124**, 591 (2010).
3. Y. Fujiwara, C. Nakamoto and Y. Suzuki, *Phys. Rev. Lett.* **76**, 2242 (1996); *Phys. Rev. C* **54**, 2180 (1996).
4. Y. Fujiwara, M. Kohno, C. Nakamoto and Y. Suzuki, *Phys. Rev. C* **64**, 054001 (2001); Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto and Y. Suzuki, *Phys. Rev. C* **65**, 014002 (2002).
5. Y. Fujiwara, Y. Suzuki, M. Kohno and K. Miyagawa, *Phys. Rev. C* **77**, 027001 (2008).
6. Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, D. Baye and J.-M. Sparenberg, *Phys. Rev. C* **70**, 024002 (2004).
7. Y. Fujiwara, M. Kohno, K. Miyagawa, Y. Suzuki and J.-M. Sparenberg, *Phys. Rev. C* **70**, 037001 (2004).
8. Y. Suzuki, H. Matsumura, M. Orabi, Y. Fujiwara, P. Descouvemont, M. Theeten and D. Baye, *Phys. Lett. B* **659**, 160 (2008).
9. Y. Fujiwara and K. Fukukawa, *Few-Body Syst.* (Online first), DOI: 10.1007/s00601-012-0501-4 (2012).
10. K. Fukukawa and Y. Fujiwara, *Mod. Phys. Lett. A* **24**, **11–13**, 1035 (2009).
11. Y. Fujiwara, M. Kohno, T. Fujita, C. Nakamoto and Y. Suzuki, *Prog. Theor. Phys.* **103**, 755 (2000).
12. Y. Fujiwara and K. Fukukawa, *Prog. Theor. Phys.* **124**, 433 (2010).
13. R. A. Arndt, Scattering Analysis Interactive Dial-up (SAID), Virginia Polytechnic Institute, Blacksburg, Virginia.
14. E. Hiyama, Y. Kino and M. Kamimura, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
15. K. Varga and Y. Suzuki, *Phys. Rev. C* **52**, 2885 (1995).