BARYONS IN A HYBRID QUARK-DIQUARK MODEL*

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One of the main problems in baryon spectroscopy is the presence of many so-called "missing resonances". A possible solution to this problem is to describe two correlated quarks inside the baryons by means of the diquark effective degree of freedom. Three different quark-diquark model are developed and compared. Some theoretical predictions are given and compared with the available experimental data.

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1. Introduction

The three constituent quark model for the baryons has been developed since the very introduction of the quarks. In particular, the light baryons have been ordered according to the approximate $SU(3)_f$ symmetry. However, on one hand many unseen excited resonances are predicted by every three-quark models; on the other hand, states with certain quantum numbers appear in the spectrum at excitation energies much lower than predicted.¹ For example, in the non-strange sector up to an excitation energy of 2.41 GeV, on average about 45 N states are predicted, but only 12 have been established (four- or three-star) and 7 are tentative (two- or one-star).¹ This is the so-called "missing resonances" problem. One possible solution to this problem is the introduction of a new effective degree of freedom: the diquark. In this case the number of states predicted is considerably reduced.

Baryon resonances problems have already been treated with other quark-diquark models,²⁻⁴ unquenching quark models,⁵⁻¹⁰ and hypercentral models¹¹⁻³⁰ but in the end baryon resonances still remain an open problem.³¹

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2. Baryons in the Quark-Diquark Model

We consider baryons as a bound state of two elements, a constituent quark and a constituent diquark. The diquarks in our model have no internal spatial excitations, or at least we hypothesize that their internal spatial excitations will be of higher energy than the scale of masses of the resonances we will consider, i.e. light resonances up to 2 GeV masses. The only diquark states allowed inside a baryon (labeled by their color, flavor, spin quantum numbers) are $|\bar{3}_c, [3]_f, 0 >$, $|\bar{3}_c, [6]_f, 1 >$. The first of the above states is the scalar (or good) diquark; the second is the axial-vector (or bad) diquark.

Then we have to combine these diquark states to the third quark in order to form a baryon. Since, in the quark-diquark model, we freeze one spatial degree of freedom, thus setting one of the two relative angular momenta to zero and letting the other vary, we can have only symmetric (if the relative orbital angular momentum L is even) or mixed (if L is odd) spatial parts. This means that for the quark-diquark system we cannot have all the states in the antisymmetric $[20]_A$ spin-flavor representation that we normally have in the three-quark model.

3. Three Different Quark-Diquark Models

In a quark-diquark model, we consider every baryon as a bound state of a quark (strange or non-strange) and a diquark (scalar or axial-vector, with all the possible strangenesses). In a first and elementary approach, which we will henceforth call the "pure" quark-diquark model, we discard the possibility of a superposition of the two types of diquarks in some baryons. While all the currently known resonances fit well into this scheme, the main problem of this model is that it still predicts several missing resonances, in particular we expect the presence of a $\frac{1}{2}^+$ octet close in energy to the ground state octet featuring the nucleon. To avoid the necessity of this second ground state octet, we can introduce a variation to our model. In this case the nucleon (and all the other components of the ground state octet) results from the superposition of the scalar diquark and of the axial-vector diquark. This model has been called the "mixed" quark-diquark model. In the hypothesis that the weight of the two diquarks inside the nucleon (and in the rest of its octet) is approximately 1/2, the two ground state octets would have the same composition and thus would again overlap and solve the problem of the second "embarrassing" octet. In the present paper we have also considered a further possibility, which we have called the "hybrid" model. In this model we make the assumption that the diquarks do not form inside the ground-state, non-excited baryons: these baryons would be compressed to provide the usual three-quark $SU(6)$ -symmetry structure, while the excited states would be spatially separated to give rise to the quark-diquark string-like structure. In this case we would have only one ground-state three-quark octet, and the quark-diquark scheme would be valid only starting from the spatially excited resonances. In brief, this model results from the deformation

of a three-quark model for the ground states into a quark-diquark model for the excited states; hence the name "hybrid".

4. The Algebraic Mass Formula

In constructing the mass formulas for our quark-diquark models, we follow the methodology developed in the algebraic models for mesons and baryons.^{32–35} We need two mass formulas, one to be applied to the "pure" and "mixed" models and the other to the "hybrid" model. The main features of both mass formulas are the presence of rotational and vibrational Regge trajectories and the explicit dependence on quark and diquark masses.

The mass formula for the "hybrid" model is

$$\begin{aligned}
 M^2 = & b L + n \nu + \delta_{L,0} \delta_{\nu,0} \left(\sum_{i < j} \frac{c' \mathbf{S}_i \cdot \mathbf{S}_j + g' \mathbf{F}_i \cdot \mathbf{F}_j}{M_i M_j} + d' J + e' I(I+1) + M_{3q}^2 \right) \\
 & + (1 - \delta_{L,0} \delta_{\nu,0}) \left(\Lambda_1 + \frac{1}{M_q M_{diq}} (g \mathbf{F}_q \cdot \mathbf{F}_{diq} + h \mathbf{G}_q \cdot \mathbf{G}_{diq} + e \mathbf{I}_q \cdot \mathbf{I}_{diq}) \right. \\
 & \left. + (M_q + M_{diq})^2 \right). \tag{1}
 \end{aligned}$$

For the explanation of the terms of these formulas, see Ref. 36. Here we do not give the mass formula for the "pure" and "mixed" models since, as already discussed in Ref. 36, they give worse results and they were not used for the predictions.

The parameters of the mass formulas have been determined through a fit to a set of well known three- or four-star baryon, according to the PDG classification. The results can be seen in Tables IV-VII of Ref. 36. In Table 1, we add some other predictions for one- or two-star states or higher energy states that were excluded from the aforementioned Tables. All the theoretical prediction have been done with the "hybrid" model, which have been proved according to Ref. 36 to be more accurate.

Among the states shown in Table 1, there are many one- or two-star resonances that need experimental confirmations. Some of these states are reproduced quite well in our scheme, such as $\Sigma(1620)S_{11}$, $\Sigma(1770)P_{11}$, $\Sigma(1840)P_{13}$, $\Sigma(2080)P_{13}$, $\Sigma(2070)F_{15}$, $N(1890)^{37}$ (also called $N(2000)F_{15}$ in the PDG), $N(1880)$, and $N(1900)P_{13}$. On the other hand there are some resonances that fit less than ideally in our scheme. $\Xi(1690)$ (with unknown J^P quantum numbers) is best included in the multiplet of $N(1440)P_{11}$ with a predicted mass of 1.807 GeV. $\Delta(1900)S_{31}$ and $\Delta(1940)D_{33}$ are two resonances that we can assign $L = 1$ and $\nu = 1$ quantum numbers and predict a mass around 2.080 GeV. However the latest GWU analysis³⁸ finds no evidence for these resonances. Thus we suggest to look for them at higher energies. See Ref. 36 for a more complete discussion of our results.

With our model we can as well investigate the higher spin resonances. The resonances $N(2220)H_{19}$ and $\Delta(2420)H_{3,11}$ are well established and we manage to reproduce their masses in a satisfactory way. $\Delta(2750)I_{3,13}$, $\Delta(2390)F_{37}$, $\Delta(2300)H_{39}$ and $\Delta(2350)D_{35}$ are one- or two-star resonances which have not been seen by the latest analysis of Ref.38. Our predictions are somewhat higher than the experimental

Table 1. Predicted values for the masses of some one- or two-star resonances and higher spin resonances. All the predictions are done with the "hybrid" mass formula (Eq. 1). All the masses are in GeV.

Resonances	J^P	L	ν	S	S_{diq}	$M(exp)$	$M(hybrid)$
$\Sigma(1620)S_{11}$	$\frac{1}{2}^-$	1	0	$\frac{1}{2}$	0	1.698	~ 1.620
$\Sigma(1770)P_{11}$	$\frac{1}{2}^+$	0	1	$\frac{1}{2}$	1	1.714	~ 1.770
$\Sigma(1840)P_{13}$	$\frac{3}{2}^+$	2	0	$\frac{1}{2}$	0	1.893	~ 1.840
$\Sigma(2080)P_{13}$	$\frac{3}{2}^+$	2	0	$\frac{3}{2}$	1	2.064	~ 2.080
$\Sigma(2070)F_{15}$	$\frac{5}{2}^+$	2	0	$\frac{3}{2}$	1	2.064	~ 2.070
$N(1890)F_{15}$	$\frac{5}{2}^+$	2	0	$\frac{1}{2}$	1	1.866	$1.950 - 2.150$
$N(1880)$	$\frac{1}{2}^+$	0	2	$\frac{1}{2}$	0	1.903	1.870 ± 0.035
$N(1900)P_{13}$	$\frac{3}{2}^+$	2	0	$\frac{1}{2}$	1	1.866	~ 1.900
$\Xi(1690)$	$\frac{1}{2}^+$	0	1	$\frac{1}{2}$	1	1.807	1.690 ± 0.010
$\Delta(1900)S_{31}$	$\frac{1}{2}^-$	1	1	$\frac{3}{2}$	1	2.080	$1.840 - 1.920$
$\Delta(1940)D_{33}$	$\frac{3}{2}^-$	1	1	$\frac{3}{2}$	1	2.080	$1.940 - 2.060$
$N(2220)H_{19}$	$\frac{9}{2}^+$	4	0	$\frac{1}{2}$	0	2.262	$2.200 - 2.300$
$\Delta(2420)H_{3,11}$	$\frac{11}{2}^+$	4	0	$\frac{3}{2}$	1	2.534	$2.300 - 2.500$
$\Delta(2750)I_{3,13}$	$\frac{13}{2}^-$	7	0	$\frac{1}{2}$	1	3.095	~ 2.750
$\Delta(2390)F_{37}$	$\frac{7}{2}^+$	4	0	$\frac{3}{2}$	1	2.490	~ 2.390
$\Delta(2300)H_{39}$	$\frac{9}{2}^+$	4	0	$\frac{3}{2}$	1	2.490	~ 2.300
$\Delta(2350)D_{35}$	$\frac{5}{2}^-$	3	1	$\frac{1}{2}$	1	2.565	~ 2.350
$\Delta(2950)K_{3,15}$	$\frac{15}{2}^+$	6	0	$\frac{3}{2}$	1	2.907	~ 2.950
$\Delta(2400)G_{39}$	$\frac{9}{2}^-$	5	0	$\frac{1}{2}$	1	2.706	$\sim 2.643 \pm 0.141$

measurements. Since these measures are very old, we urge new experiments and data analysis to confirm the existence of the resonances and to determine precisely their masses. Finally we have the two-star resonances $\Delta(2950)K_{3,15}$ and $\Delta(2400)G_{39}$, for which we predict masses around 2.907 and 2.706 GeV respectively. $\Delta(2400)G_{39}$ has been seen in Ref. 38 with a mass of 2.643 ± 0.141 GeV.

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