PHYSICS OF GLUON HELICITY*

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The total gluon helicity in a polarized proton is shown to be a matrix element of a gauge-invariant but nonlocal, frame-dependent gluon spin operator $\vec{E} \times \vec{A}_\perp$ in the large momentum limit. The operator $\vec{E} \times \vec{A}_\perp$ is fit for the calculation of the total gluon helicity in lattice QCD. This calculation also implies that parton physics can be studied through the large momentum limit of frame-dependent, equal-time correlation functions of quarks and gluons.

Keywords: Gluon helicity; parton physics; lattice QCD.

1. Introduction

Ever since the EMC “spin crisis” [1], the gluon polarization has been one of the most important pursuits of the hadron physics community. The total gluon helicity $\Delta G = \int dx \Delta g(x)$, as measured in high energy scattering experiments,^{2–5} is nonzero and accounts for a considerable part of the proton spin. The latest constraint on gluon polarization from the analysis of the RHIC data in 2009 is $\int_{0.05}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = 0.13$ [6]. The factorization theorems in quantum chromodynamics (QCD) indicate that ΔG is the matrix element of a nonlocal operator involving light-cone correlation, which reduces to the textbook definition of gauge particle spin $\vec{E} \times \vec{A}$ in the light-cone gauge $A^+ = 0$ [7, 8]. Nevertheless, a nonperturbative calculation of ΔG based on such understanding is not feasible since it’s hard to calculate time-dependent correlations on a Euclidean lattice. The difficulty of calculating ΔG has also raised fundamental questions on the gauge-invariance of the gluon spin.⁹

In this talk, we report a breakthrough in understanding the physics of the gluon helicity, and also propose a practical way to calculate this quantity in lattice QCD.¹⁰

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We will show that ΔG is the large momentum limit of a matrix element of a gauge-invariant but nonlocal, frame-dependent operator $\vec{E} \times \vec{A}_\perp$, where $\vec{A}_\perp$ is the transverse part of the gauge potential. We will prove that $\vec{E} \times \vec{A}_\perp$, when boosted to the infinite momentum frame (IMF), coincides with the gluon spin operator from factorization theorems.⁸ The operator $\vec{E} \times \vec{A}_\perp$ was proposed as the gauge-invariant gluon spin,¹¹ but was criticized as physically uninteresting due to its frame dependence.¹² The physics behind the IMF we propose here is that at large momentum the Weizsäcker-Williams equivalent photon picture becomes true⁹ and the charged partons can be regarded as free radiation, so that $\vec{E} \times \vec{A}_\perp$ acquires clear physical meaning. However, the matrix element of $\vec{E} \times \vec{A}_\perp$ has a singular dependence on the nucleon momentum at infinity, and the anomalous dimension of the operator at finite momentum does not agree with that in the light-cone gauge.¹³ Therefore, we will provide a well-defined procedure for taking the large momentum limit, and give an explicit example on how to obtain ΔG from the IMF limit of a frame-dependent, time-independent matrix element. This example indicates that ΔG can be predicted in lattice QCD by studying the large momentum limit of a frame-dependent matrix element of the Euclidean operator $\vec{E} \times \vec{A}_\perp$. Moreover, we will extend this approach to direct calculation of parton physics on a Euclidean lattice,¹⁴ which was considered otherwise infeasible for light-cone correlations.

2. Gauge-Invariant Gluon Spin Operator

According to factorization theorems, the total gluon helicity ΔG is a matrix element of a gauge-invariant, nonlocal operator involving light-cone correlation,⁸

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^+(0) | PS \rangle, \quad (1)$$

where $|PS\rangle$ is a proton plane-wave state with momentum P^μ and polarization S^μ , $\tilde{F}^{\alpha\beta} = (1/2)\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$, and $\mathcal{L}(\xi^-, 0) = P \exp[-ig \int_0^{\xi^-} \mathcal{A}^+(\eta^-, 0_\perp) d\eta^-]$ with $\mathcal{A}^+ \equiv T^c A_c^+$ is a light-cone gauge link defined in the adjoint representation. The light-front coordinates are defined as $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$. It is usually difficult to identify the above operator as the gluon spin or helicity. However, in the light-cone gauge $A^+ = 0$, the whole operator collapses into $\vec{E} \times \vec{A}$, the textbook definition of the gauge particle spin,⁹ which is gauge dependent, and ΔG can be interpreted as the number of gluon partons with helicity in the direction of the proton helicity minus that with opposite helicity. The explicit presence of the real time in ξ^- renders it impossible to evaluate Eq. (1) directly in lattice QCD. An early attempt to calculate the gluon helicity on lattice was to calculate the matrix element of $F_{\mu\nu} \tilde{F}^{\mu\nu}$ [15], but there is no demonstrated connection between this and ΔG .

To find the physics of ΔG without committing to the light-cone gauge, let us examine the operator structure a bit further. For simplicity, we first consider the U(1) gauge theory (quantum electrodynamics, or QED) so that the gauge link is

absent. Carrying out the integration over the longitudinal momentum, the gauge-invariant photon “spin” operator becomes

$$\begin{aligned}
 \hat{S}_\gamma^{\text{inv}}(0) &= i \int \frac{dx}{x} \int \frac{d^2 k_\perp}{(2\pi)^3} \int d\xi^- d^2 \xi_\perp e^{-i(xP^+ \xi^- - \vec{k}_\perp \cdot \vec{\xi}_\perp)} \\
 &\quad \times \left[ixP^+ A^i(\xi^-, \vec{\xi}_\perp) - ik_\perp^i A^+(\xi^-, \vec{\xi}_\perp) \right] \tilde{F}_i^+(0) \\
 &= - \int \frac{dk^+ d^2 k_\perp}{(2\pi)^3} \left[\tilde{A}^i(k^+, \vec{k}_\perp) - \frac{k_\perp^i}{k^+} \tilde{A}^+(k^+, \vec{k}_\perp) \right] \tilde{F}_i^+(0) \\
 &= \left[\vec{E}(0) \times \left(\vec{A}(0) - \frac{1}{\nabla^+} \vec{\nabla} A^+(\xi^-) \right) \right]^3, \tag{2}
 \end{aligned}$$

where $\nabla^+ = \partial/\partial\xi^-$, $E^i = F^{i+}$, $k^+ = xP^+$, and the third component of a vector is interpreted in the usual sense of the cross product. The ξ^- coordinate in A^+ is taken to 0 after operating with the inverse derivative, which is independent of the boundary condition because $\vec{\nabla} A^+(\infty) = 0$.

The above operator is just the IMF limit of $\vec{E} \times \vec{A}_\perp$, where $E^i = F^{i0}$ is the electric field in an ordinary frame, and $A_\perp^i = (\delta^{ij} - \nabla^i \nabla^j / \nabla^2) A^j$ is the transverse part of the gauge potential. The rule of taking the IMF limit of an operator is as follows: For any vector V^μ , define $V^\pm = (V^0 \pm V^3)/\sqrt{2}$. If the boost is along the 3-direction and characterized by $\Lambda \gg 1$, then the components of the vector go like $V^+ \rightarrow V^+ \Lambda$, $V^- \rightarrow V^-/\Lambda$, and $V_\perp \rightarrow V_\perp$. Thus, $\nabla^2 \rightarrow (\nabla^+)^2 \Lambda^2$ and $\vec{\nabla} \cdot \vec{A} \rightarrow \nabla^+ A^+ \Lambda^2$ for the leading order components. Using these rules, one finds that $\vec{E} \times \vec{A}_\perp \rightarrow \vec{E} \times (\vec{A} - (1/\nabla^+) \vec{\nabla} A^+)$, which is exactly the operator in Eq. (2).

In electromagnetic theory the vector potential can be uniquely separated into the longitudinal and transverse parts, $\vec{A} = \vec{A}_\parallel + \vec{A}_\perp$, and the transverse part is gauge invariant [16]: Given $\vec{A}$, $\vec{A}_\perp$ can be uniquely constructed as a functional of $\vec{A}$ with an appropriate boundary condition. Therefore, $\vec{E} \times \vec{A}_\perp$ can be regarded as the gauge-invariant part of the photon spin.¹¹

However, it must be pointed out that separating $\vec{A}$ and $\vec{E}$ into transverse and longitudinal parts is generally not physically meaningful. First of all, the physics of $\vec{E}$ is to exert electric force, and there is no charge that responds separately to $\vec{E}_\parallel$ and $\vec{E}_\perp$. Second, although $\vec{A}_\perp$ is gauge invariant, it does not transform properly under a Lorentz boost. A Lorentz-transformed $\vec{A}$ or $\vec{E}$ has different decompositions in different frames, and in this sense $\vec{E} \times \vec{A}_\perp$ is not a physical observable.¹² Actually the frame dependence of the transverse and longitudinal parts is dynamical. For a static electric field there is only $\vec{E}_\parallel$, and $\vec{E}_\perp$ is generated from $\vec{E}_\parallel$ under a Lorentz boost of the source. In the IMF, $\vec{E}_\parallel \ll \vec{E}_\perp$, and $\vec{B}$ is the same order of $\vec{E}_\perp$, so $\vec{E}_\perp$ becomes physically meaningful, and can be regarded as free radiation along with $\vec{B}$. This has been recognized long ago by Weizsäcker and Williams, in the name of equivalent photon approximation.⁹ Only then, $\vec{A}_\perp$ acquires physical meaning since it generates the physical fields, and $\vec{E} \times \vec{A}_\perp$ can be understood as a physical quantity, where $\vec{E}$ can also be replaced by $\vec{E}_\perp$.

The case for QCD is a bit more complicated. Separating $\vec{A}$ into longitudinal and transverse parts requires generalizing the observations in QED to similar conditions in QCD, which was considered long ago¹⁷ (see also Ref. [11]). First, we would like to have $\vec{A}_\perp$ transform homogeneously under gauge transformation,

$$\vec{A}_\perp \rightarrow U(x)\vec{A}_\perp U^\dagger(x), \quad (3)$$

where $\vec{A}_\perp \equiv T_a \vec{A}_\perp^a$, so it is easy to construct gauge-invariant quantities with $\vec{A}_\perp$. Second, we require $\vec{A}_\parallel$ to produce null magnetic field, as it does in QED. This condition is¹⁷

$$\partial^i A_\parallel^{j,a} - \partial^j A_\parallel^{i,a} - g f^{abc} A_\parallel^{i,b} A_\parallel^{j,c} = 0, \quad (4)$$

which is a nonlinear equation to solve for $A_\parallel^i$ as a functional of A^i . Moreover, the transverse part of the gauge potential satisfies a generalized Coulomb condition¹⁷

$$\partial^i A_\perp^i = ig[A^i, A_\perp^i]. \quad (5)$$

We can then go through a similar derivation as in the QED case and find¹⁸

$$\hat{S}_g^{\text{inv}}(0) = \left[\vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+} (\vec{\nabla} A^{+,b}) \mathcal{L}^{ba}(\xi^-, 0) \right) \right]^3. \quad (6)$$

It is a bit involved to show that the expression in the parentheses above is indeed $\vec{A}_\perp$ in the IMF. Since $\vec{A}_\perp = \vec{A} - \vec{A}_\parallel$, we just need to solve for $\vec{A}_\parallel$. After solving Eqs. (4) and (5) order by order in g , we find that $A_\perp^+$ vanishes, and thus $A_\parallel^+ = A^+$ in the IMF. Substituting this into Eq. (4), we obtain a first-order inhomogeneous linear equation for $A_\parallel^i$,

$$\partial^+ A_\parallel^{i,a} - g f^{abc} A^{+,b} A_\parallel^{i,c} = \partial^i A^{+,a}. \quad (7)$$

Its solution is easy to construct as a geometric series expansion

$$A_\parallel^{i,a} = \frac{1}{\nabla^+} \left[1 + \left(-ig A^+ \frac{1}{\nabla^+} \right) + \cdots + \left(-ig A^+ \frac{1}{\nabla^+} \right)^n + \cdots \right]^{ab} (\partial^i A^{+,b}). \quad (8)$$

By commuting $\partial^i A^+$ systematically to the front of the expression, one finds

$$A_\parallel^{i,a}(\xi^-) = \frac{1}{\nabla^+} \left((\partial^i A^{+,b}) \mathcal{L}^{ba}(\xi'^-, \xi^-) \right), \quad (9)$$

where the coordinate ξ'^- in the gauge link is taken to ξ^- after operating with the inverse derivative.

Alternatively, one can multiply a gauge link $\mathcal{L}$ on both sides of Eq. (7) and find after some manipulations

$$\partial^+ (A_\parallel^{i,a} \mathcal{L}^{ad}) = (\partial^i A^{+,a}) \mathcal{L}^{ad}. \quad (10)$$

It is then straightforward to see that the solution for $A_\parallel^{i,a}$ is

$$A_\parallel^{i,a}(\xi^-) = \left[\frac{1}{\nabla^+} \left((\partial^i A^{+,b}) \mathcal{L}^{bd}(\xi''^-, \xi'^-) \right) \right] (\mathcal{L}^{-1})^{da}(\xi^-, \xi'^-). \quad (11)$$

Since the inverse derivative does not affect ξ'^- , one can insert the inverse gauge link into the squared brackets and get the same solution as in Eq. (9).

Clearly $\vec{A}_{\parallel}^a(0)$ is just the part subtracted from $\vec{A}^a(0)$ in the gluon-helicity operator in Eq. (6). Therefore, we established the same conclusion in QCD that the gluon-helicity operator is the IMF limit of the gauge-invariant, nonlocal gluon spin operator $(\vec{E} \times \vec{A}_{\perp})^3$.

3. Matrix Elements in Two Different Limiting Procedures

In this section we give an explicit example on how to calculate the large momentum limit of a frame-dependent matrix element. Let us consider the photon spin $\hat{S}_{\gamma}^{\text{inv}} = (\vec{E} \times \vec{A}_{\perp})^3$ in a free electron state $|p, s\rangle$.

A simple perturbative calculation of Fig. 1 yields¹⁰

$$\langle p, s | \hat{S}_{\gamma}^{\text{inv}} | p, s \rangle = \frac{\alpha_{\text{em}}}{4\pi} \left[\frac{5}{3}D + \frac{4}{3} \ln \frac{\vec{p}^2}{m^2} - \frac{1}{9} + \frac{8}{3} \ln 2 \right] \bar{u}(p, s) \Sigma^3 u(p, s) + \mathcal{O} \left(\frac{m^2}{\vec{p}^2} \right), \quad (12)$$

where $p^{\mu} = (p^0, 0, 0, p^z)$, $D = 1/\epsilon - \gamma_E + \ln 4\pi + \ln(\mu^2/m^2)$, and m is the mass of the electron used to regularize the collinear divergence. The result has a nontrivial dependence on $\vec{p}^2$, which makes its physical interpretation not straightforward. In particular, the result is divergent as $\vec{p}^2$ goes to infinity. The ultraviolet (UV) part of the matrix element is consistent with that found in Ref. [13]. This result also shows that the statement in Ref. [19] about the frame independence is incorrect.

However, the measurable photon helicity can be obtained by the same matrix element by going to the IMF limit first before doing the loop-momentum integration. In this limit, the external momentum dependence is dropped out, and the matrix element becomes

$$\langle p, s | \hat{S}_{\gamma}^{\text{inv}} | p, s \rangle = \frac{\alpha_{\text{em}}}{4\pi} (3D + 7) \bar{u}(p, s) \Sigma^3 u(p, s). \quad (13)$$

This is exactly the same as that calculated using the factorization expression in Eq. (1) or $\vec{E} \times \vec{A}$ (notice the full $\vec{A}$) in the light-cone gauge $A^+ = 0$ [20]. The UV property of the matrix element is the same as that derived by Altarelli and Parisi (AP) [21]. In fact, in the original derivation of the AP evolution equation, a finite frame result was used to obtain parton physics in the IMF limit. Our result also

Fig. 1. Matrix element of $\vec{E} \times \vec{A}_{\perp}$ in an asymptotic electron state.

indicates the claim that gluons carry only about $1/5$ of the nucleon momentum in Ref. [11] is incorrect, because the matrix elements of the quark and gluon momentum operators were calculated in the finite momentum frame. We have verified that if they are boosted to the IMF, we can get the standard mixing matrix in Ref. [22]. Meanwhile, it should be pointed out that the result in Ref. [23] is standard because they used the light-cone gauge condition.

4. The Matching Calculation of ΔG in Lattice QCD

The last section shows that the IMF and UV limits are not exchangeable. Since the collinear divergent part is the same, the difference between the two procedures is perturbatively calculable. Thus, one can relate them through a perturbative matching condition, which turns out to be the key for a nonperturbative computation of ΔG .

Since lattice QCD cannot handle real-time correlations, it's hard to calculate Eq. (1) directly. Instead, one can study the matrix element of $\vec{E} \times \vec{A}_\perp$ as a function of the external momentum $\vec{P}$. The largest momentum attainable on a lattice is of order $1/a$, with a being the lattice spacing. Meanwhile, the matrix element also has UV dependence on $1/a$, and can be calculated in lattice perturbation theory.²⁴ Thus, we can calculate the matrix element at the largest momentum $\sim 1/a$, matching the results of the two different limits in a perturbative way. For instance, in the above one-loop example with a finite ϵ , one can match the two results by setting

$$\ln \vec{p}^2 = (D + \ln m^2) + \frac{16}{3} - 2 \ln 2. \quad (14)$$

The result will be accurate up to controllable power corrections of type M^2/P^2 , where M is the nucleon mass. On the lattice, $D + \ln m^2$ is replaced by $-\ln a^2$, and the matching condition becomes $\ln(pa)^2 = \text{const}$. To get a large momentum nucleon, one has to have fine lattice spacing in the z -direction, and to separate excited states of the nucleon one also needs fine lattice spacing in the time direction. The issue of lattice calculation and matching conditions will be discussed in more detail in a separate publication.²⁵

5. Parton Physics on Euclidean Lattice

The calculation of ΔG in lattice QCD opens a new window for the computation of parton properties on Euclidean lattice. Instead of committing to the light-cone gauge, one can obtain the same light-cone result by taking the large moment limit of the matrix elements of time-independent, frame-dependent operators, as long as one can prove their equivalence to the relevant light-cone correlation operators in the IMF. There is a class of gauge conditions, including the Coulomb gauge, that suffice to providing us with the desired light-cone correlations in the IMF limit. The axial gauge is the simplest one that we can choose.

To demonstrate this idea, let us consider the quark distribution function

$$q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle PS | \bar{\psi}(\xi^-) \gamma^+ \exp \left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right) \psi(0) | PS \rangle, \quad (15)$$

where the nucleon momentum $P^\mu = (P^0, 0, 0, P^z)$, μ^2 is the renormalization scale, and ψ is a quark field of flavor q . The gauge link is defined in the fundamental representation. It is argued in Ref. [14] that in the large P^z limit,

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle PS | \bar{\psi}(z) \gamma^z \exp \left(-ig \int_0^z dz' A^z(z') \right) \psi(0) | PS \rangle + \mathcal{O}(\Lambda_{\text{QCD}}^2 / (P^z)^2, M^2 / (P^z)^2), \quad (16)$$

where $x = k^z / P^z$. An intuitive understanding of the above result is that an infinite Lorentz boost will tilt the line segment connecting $(0, 0, 0, z)$ with the origin to the light-cone direction. Of course, the line segment remains space-like for any amount of boost, but this slight off-light-cone-ness only introduces power corrections which vanish asymptotically. For the parton distribution in Eq. (15) the nucleon can be regarded as in the rest frame while the probe is traveling at the speed of light, while the distribution in Eq. (16) is obtained from a static probe on a nucleon traveling at large momentum. The latter is related to deep-inelastic scattering in a frame where the virtual photon momentum contains only the spatial components $q^\mu = (0, Q)$, and the nucleon momentum is $P^z \sim Q/2x$. One can also obtain a similar expression for the quark helicity distribution with the replacement of γ^z by $\gamma^z \gamma_5$. As for the gluon distribution, we have in the large P^z limit

$$g(x, \mu^2, P^z) = \int \frac{dz}{4\pi k^z} e^{izk^z} \langle PS | F^{3i}(z) \exp \left(-ig \int_0^z dz' A^z(z') \right) F_i{}^3(0) | PS \rangle, \quad (17)$$

where A^z is a matrix in the adjoint representation and i runs over the transverse directions. Similarly, the polarized gluon distribution is obtained by replacing $F_i{}^3(0)$ in Eq. (17) with $\tilde{F}_i{}^3(0)$.

To calculate the exact parton distribution as given by Eq. (16) one needs to renormalize it. As we have encountered in the last section, taking the large P^z limit before and after the UV regularization yield different results. If the large P^z limit is taken first then one obtains the standard light-cone result; if one does the UV-regularization first, then the result will contain large logarithms $\ln P^z$, which can be translated into standard renormalization scale dependence in the light-cone distribution through a matching condition. One can see the interplay of the different limiting procedures by calculating the one-loop correction in the $A^z = 0$ gauge.¹⁴ The connection of the two limiting procedures is reflected by the following factorization theorem up to power corrections:

$$q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z \left(\frac{x}{y}, \mu^2, P^z \right) q(y, \mu^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2 / (P^z)^2, M^2 / (P^z)^2), \quad (18)$$

where the evolution kernel Z has a perturbative expansion in α_s ,

$$Z(x, \mu^2, P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu^2, P^z) + \cdots \quad (19)$$

If one takes the IMF limit first, then $Z^{(1)}$ is the AP evolution kernel [21]; if one does the UV regularization first at finite momentum P^z , then $Z^{(1)}$ will be dependent on $\ln P^z$. Thus the large logarithmic dependence on P^z in $q(x, \mu^2, P^z)$ can be transformed into the renormalization scale dependence through the above matching condition. On the lattice, the matching condition must be recalculated up to a constant accuracy using lattice perturbation theory.²⁴

The above approach can be generalized to any light-cone correlations in hadron physics. A partial list of interesting physical quantities includes [14]:

- Generalized parton distributions (GPD's).
- Transverse-momentum-dependent (TMD) parton distribution.
- Wigner distributions, which are generating functions for both GPD's and TMD's.
- Light-cone amplitudes, which are the matrix elements between the hadron states and the QCD vacuum, of the light-cone correlations of the quark and gluon interpolating fields.
- Light-cone wave functions.
- Higher-twist parton distributions.

6. Summary

We have shown that in the IMF limit the gauge-invariant, frame-dependent operator $\vec{E} \times \vec{A}_\perp$ has a clear physical meaning and is just the gluon-helicity operator from factorization theorems. The total gluon helicity ΔG can be predicted in lattice QCD by studying the IMF limit of the matrix element of the time-independent operator $\vec{E} \times \vec{A}_\perp$. We have also provided a well-defined procedure for taking the large momentum limit, and explained how to obtain matching conditions to recover the light-cone physics.

The calculation of ΔG in lattice QCD also provides a new window for computing light-cone correlations of quarks and gluons on a Euclidean lattice. This calculation can be done to an accuracy of $\Lambda_{\text{QCD}}^2/(P^z)^2$ and $M^2/(P^z)^2$. With a fine lattice spacing along the z direction, one can study the x -dependence of the parton distribution functions to an accuracy that's controlled by the lattice size L , and especially the small- x behavior of these distribution functions. Within two years from now, we would expect a lot of meaningful results coming out from lattice QCD.

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