EXPERIMENTAL STUDY OF VOLUME CALIBRATION OF MASS WEIGHT WITH HYDROSTATIC WEIGHING

HONG MAN HU

Division of Mechanics and Acoustic, National Institute of Metrology, Beijing 100013, P. R. China
hnh@nim.ac.cn

JIAN WANG, YUE ZHANG, CHANG QING CAI, RUI LIN ZHONG,
HONG YAO, JING AN DING

Division of Mechanics and Acoustic, National Institute of Metrology, Beijing 100013, P. R. China

In this paper, the volumes of mass weights ranging from 500 g to 1 g are calibrated with a fused quartz solid density standard using a hydrostatic weighing device developed by Lab of Volume and Density in NIM, China. The stability of liquid temperature is discussed. And also the measurement uncertainty of the densities is evaluated based on the calibration models.

Keywords: Hydrostatic weighing method; solid volume calibration; mass standards.

1. Introduction

The volume of mass standard is needed to be measured in order to correct the air buoyancy during the mass calibration process. Hydrostatic weighing method is one of traditional volume measuring methods with high measuring accuracy. With the development of Avogadro project [1-3], solid density standards are more used for the high repeatability and stability of density value, comparing with the liquid density standards. The volume of mass standard is needed to be measured in order to correct the air buoyancy during the mass calibration process. Hydrostatic weighing method is one of traditional volume measuring methods with high measuring accuracy.[4-6]

In this paper, the solid density standard made of fused quartz is used for the volume calibration of mass weights. The volumes of OIML E₁ mass weights ranging from 500 g to 1 g are tested. And the measurement uncertainty of the densities is also evaluated based on the calibration models.

2. Experimental apparatus

The experiments are carried out using a hydrostatic weighing device developed by Lab of Volume and Density in NIM, China. Fig. 1 shows the photos of the experimental setup. The weighing capability of the mass comparator (XP50S) is 520 g with a resolution of 0.01 mg. The temperature is monitored by a thermometer with an accuracy of 0.001 Celsius. All the experiments are carried out in n-Tridecane ($C_{13}H_{28}$), which has a low surface tension. A commercial temperature control system AV7000 (Thomas) is used for the temperature control of the working liquid temperature. The expected temperature control accuracy is $\pm 0.01\text{ }^{\circ}\text{C}$ with a 1000 W heat power.

The volumes of test weights ranging from 500 g to 1 g are measured with a fused quartz solid density standard showned in Fig. 2, whose density is calibrated using the silicon sphere primary solid density standard.

Fig.1 Photos of Experiment setup.

Fig.2 Photos of fused quartz solid density standard (a) and test weights ranging from 500 g to 1 g (b).

3. Measurement procedure

During the volume calibration process, the liquid bouncy to test standards and the fused quartz solid density standard are different. The calibration process of weights' volume mainly includes 3 steps as follows:

Step1: The mass of the fused quartz solid standards in liquid is measured to calculate the density of liquid by comparing with count reference weight 1 in air.

Step2: Calibration the mass value of test weights in air by comparing with count reference weight 2 in air.

Step3: The mass of test standards in liquid is measured subsequently with the same method in step 2 by comparing with count reference weight 3 in air.

The density of pure water can be calculated using the Eq. (1),

$$\rho_L(t_1) = \frac{m_{\text{quartz}} - m_{\text{weight1}} + \frac{m_{\text{weight1}}}{\rho_{\text{weight1}}} [1 + \beta_{\text{weight1}}(t_1 - 20)] \rho_{a1} - \Delta I_1}{V_{\text{quartz}} [1 + \beta_{\text{quartz}}(t_1 - 20)]} \quad (1)$$

in which, m_{quartz} and V_{quartz} are the mass and volume of the fused quartz solid density standards. m_{weight1} and ρ_{weight1} are the mass and density of count reference weight 1. β_{quartz} and β_{weight1} are the volume expansion coefficient of fused quartz solid standards and count reference weight 1. $\rho_L(t)$ is the liquid density at temperature t_1 . ρ_{a1} is the air density when measurement is proceeded. ΔI_1 is the difference of mass comparator's indication between fused quartz density standard in liquid and count reference weight 1 in air.

The volume of test weight can be calculated with the Eq. (2),

$$V_{\text{test}} = \frac{A - B}{C - D} \quad (2)$$

in which,

$$A = m_{\text{weight3}} + \Delta I_3 - \frac{m_{\text{weight3}}}{\rho_{\text{weight3}}} [1 + \beta_{\text{weight3}}(t_3 - 20)] \rho_{a3} \quad (3)$$

$$B = m_{\text{weight2}} + \Delta I_2 - \frac{m_{\text{weight2}}}{\rho_{\text{weight2}}} [1 + \beta_{\text{weight2}}(t_2 - 20)] \rho_{a2} \quad (4)$$

$$C = [1 + \beta_{\text{test}}(t_2 - 20)] \rho_{a2} \quad (5)$$

$$D = [1 + \beta_{\text{test}}(t_3 - 20)] \rho_L(t_3) \quad (6)$$

in which m_{weight2} and m_{weight3} are the mass of count reference weight 2 and count reference weight 3. ρ_{weight2} and ρ_{weight3} are the density of count reference weight 2 and count reference weight 3. V_{test} are the volume of test weight. β_{test} , β_{weight2} , β_{weight3} are the volume expansion coefficient of test weight, count reference weight 2 and count

reference weight 3. ρ_{a2} and ρ_{a3} are the air densities. ΔI_2 and ΔI_3 are the difference of mass comparator's indications between the test weight and count reference weight.

4. Measurements and results

Temperature control is a fundamental aspect of the measurement device. The temperature stability is evaluated through a series of test. As shown in Fig. 3(a), under the temperature control of AV7000, the variation of liquid temperature can be limited to 3 mK. And after turn off the the temperature control of AV7000, the liquid temperature can be relatively stable in 30 minutes as shown in Fig. 3(b).

Assume that each parameters are disrelated, the uncertainty of liquid $\rho_L(t)$ can be calculated by taking partial derivative of Eq. (1) with respect to each parameters. And the measurement value and uncertainty values are shown in Table 1. The combined uncertainty is 0.0016 kg/m^3 , and the expanded relative uncertainty is $4.3 \times 10^{-6} (k=2)$.

Fig.3 Temperature variation when temperature is on (a) and off (b).

Table 1. Uncertainty budgets for the density of working liquids $\rho_L(t)$.

Source	Value	Value standard uncertainty	Sensitive coefficient	Uncertainty contribution
m_{quartz}	0.32776356 kg	0.000000006 kg	$6720.566 / \text{m}^3$	0.00041 kg/m^3
m_{weight1}	0.215270178 kg	0.000000006 kg	$0.0117 / \text{m}^3$	$7.1 \times 10^{-10} \text{ kg/m}^3$
V_{quartz}	0.000148797 m^3	0.0000000003 m^3	$-5079437.22 \text{ kg/m}^6$	0.00153 kg/m^3

β_{quartz}	0.0000014 /°C	0.000004 /°C	-24.186 °C kg/m ³	9.7×10 ⁻⁵ kg/m ³
β_{weight1}	0.000046 /°C	0.000004 /°C	1.02×10 ⁻⁶ °C kg/m ³	4.1×10 ⁻¹² kg/m ³
ρ_{weight1}	8000kg/m ³	10 kg/m ³	-2.26×10 ⁻⁵	0.000227 kg/m ³
ρ_a	1.1839 kg/m ³	0.0007 kg/m ³	0.1809	0.00013 kg/m ³
t_1	20.032 °C	0.0057 °C	0.00106 kg/m ³ / °C	6.1×10 ⁻⁶ kg/m ³
$u(\rho_L(t))$				0.0016 kg/m ³
$u_{\text{rel}}(\rho_L(t))$				2.1×10 ⁻⁶
$U_{\text{rel}}(\rho_L(t))$ (k=2)				4.3×10 ⁻⁶

The uncertainty of test weight's volume can be calculated by taking partial derivative of Eq. (2) with respect to each parameter. Take the 10 g weight for example, the measured volume is 1.25100×10⁻⁶ m³, the combined uncertainty ($u(V_{\text{est}})$) is 0.00004 cm³ (k=2). And the measurement value and uncertainty values are shown in Table 2. The volumes and uncertainty of other weights ranging from 500 g to 1 g are shown in Table 3 and Fig. 4.

Table 2. Uncertainty budgets for the volume of the 10 g weight.

Source	Value	Value Standard uncertainty	Sensitive coefficient	Uncertainty contribution
$u(\rho_L(t_3))$	756.124994 kg/m ³	4.23×10 ⁻⁶ kg/m ³	1.66×10 ⁻⁹ m ⁶ /kg	7.01×10 ⁻⁹ cm ³
m_{weight2}	0.01000002 kg	1×10 ⁻⁸ kg	0.00133 m ³ /kg	1.33×10 ⁻⁵ cm ³
m_{weight3}	0.00905470 kg	1×10 ⁻⁸ kg	0.00133 m ³ /kg	1.33×10 ⁻⁵ cm ³
ρ_{weight2}	8000 kg/m ³	10 kg/m ³	2.44×10 ⁻¹³ m ⁶ /kg	2.44×10 ⁻⁶ cm ³
ρ_{weight3}	8000 kg/m ³	10 kg/m ³	2.24×10 ⁻¹³ m ⁶ /kg	2.24×10 ⁻⁶ cm ³
β_{test}	0.000046 /°C	0.000004 /°C	2.79×10 ⁻⁷ °C m ³	1.12×10 ⁻⁶ cm ³
β_{weight2}	0.000041 /°C	0.000004 /°C	9.75 ×10 ⁻¹¹ °C m ³	3.90×10 ⁻¹⁰ cm ³
β_{weight3}	0.000043 /°C	0.000004 /°C	3.99 ×10 ⁻¹⁰ °C m ³	1.60×10 ⁻⁹ cm ³
ρ_{a2}	1.1772 kg/m ³	0.0009 kg/m ³	6.11×10 ⁻¹³ m ⁶ /kg	5.48×10 ⁻¹⁰ cm ³
ρ_{a3}	1.1911 kg/m ³	0.0009 kg/m ³	1.50×10 ⁻⁹ m ⁶ /kg	1.35×10 ⁻⁶ cm ³
t_2	20.050 °C	0.0057 °C	3.31×10 ⁻¹⁷ m ³ /°C	1.99×10 ⁻¹³ cm ³
t_3	20.223 °C	0.0057 °C	5.75×10 ⁻¹¹ m ³ /°C	3.45×10 ⁻⁷ cm ³
$u(V_{\text{est}})$				0.00002 cm ³
$U(V_{\text{est}})$ (k=2)				0.00004 cm ³

Table 3. Uncertainty budgets for the volume of the weights ranging from 500 g to 1 g.

Source	Uncertainty contribution (cm ³)							
	1 g	2 g	5 g	20 g	50 g	100 g	200 g	500 g
$u(\rho_L(t_3))$	7.0×10^{-10}	1.4×10^{-9}	3.5×10^{-9}	1.4×10^{-8}	3.5×10^{-8}	7.0×10^{-8}	1.4×10^{-7}	3.5×10^{-7}
m_{weight2}	6.6×10^{-6}	8.8×10^{-6}	1.1×10^{-5}	1.8×10^{-5}	2.2×10^{-5}	3.5×10^{-5}	6.6×10^{-5}	1.8×10^{-4}
m_{weight3}	6.6×10^{-6}	8.8×10^{-6}	1.1×10^{-5}	1.8×10^{-5}	2.2×10^{-5}	3.5×10^{-5}	6.6×10^{-5}	1.7×10^{-4}
ρ_{weight2}	2.4×10^{-7}	4.9×10^{-7}	1.2×10^{-6}	4.9×10^{-6}	1.2×10^{-5}	2.4×10^{-5}	4.9×10^{-5}	1.2×10^{-4}
ρ_{weight3}	2.2×10^{-7}	4.5×10^{-7}	1.1×10^{-6}	4.5×10^{-6}	1.1×10^{-5}	2.2×10^{-5}	4.5×10^{-5}	1.1×10^{-4}
β_{test}	1.1×10^{-7}	2.2×10^{-7}	5.6×10^{-7}	7.6×10^{-7}	1.1×10^{-6}	9.8×10^{-7}	2.2×10^{-5}	5.6×10^{-5}
β_{weight2}	3.9×10^{-11}	7.8×10^{-11}	1.9×10^{-10}	7.8×10^{-10}	1.9×10^{-9}	3.9×10^{-9}	7.8×10^{-9}	1.9×10^{-8}
β_{weight3}	1.6×10^{-10}	3.1×10^{-10}	7.9×10^{-10}	1.1×10^{-9}	1.6×10^{-9}	1.4×10^{-9}	3.1×10^{-8}	7.9×10^{-8}
ρ_{a2}	2.4×10^{-10}	1.2×10^{-9}	1.2×10^{-9}	4.8×10^{-9}	1.2×10^{-8}	5.9×10^{-8}	5.4×10^{-8}	7.0×10^{-8}
ρ_{a3}	1.3×10^{-7}	2.7×10^{-7}	6.7×10^{-7}	2.7×10^{-6}	6.7×10^{-6}	1.3×10^{-5}	2.7×10^{-5}	6.7×10^{-5}
t_2	8.5×10^{-14}	4.2×10^{-13}	4.3×10^{-13}	1.7×10^{-12}	4.2×10^{-12}	2.1×10^{-11}	2.0×10^{-11}	2.5×10^{-11}
t_3	3.5×10^{-8}	6.9×10^{-8}	1.7×10^{-7}	6.9×10^{-7}	1.7×10^{-6}	3.5×10^{-6}	6.9×10^{-6}	1.7×10^{-5}
$u(V_{\text{est}})$	1.0×10^{-5}	1.3×10^{-5}	1.6×10^{-5}	2.6×10^{-5}	3.6×10^{-5}	6.1×10^{-5}	1.2×10^{-4}	3.1×10^{-4}
$U(V_{\text{est}})$ ($k=2$)	2×10^{-5}	3×10^{-5}	4×10^{-5}	6×10^{-5}	8×10^{-5}	1.2×10^{-4}	2.4×10^{-4}	6.2×10^{-4}

Fig.4 Measurement result and uncertainty evaluation of the volume of test weight ranging from 500 g~1 g.

5. Conclusions

In this paper, the volumes of mass weights ranging from 500 g to 1 g are calibrated with a fused quartz solid density standard using a hydrostatic weighing device developed by Lab of Volume and Density in NIM, China. The variation of liquid temperature can be limited under 3 mK. The volume measurement expanded uncertainty of the test weights ranging from 500 g to 1 g is from 0.00002 cm³ to 0.00064 cm³.

References

1. R. A. Nicohus and K. Fujii, *Meas Sci Technology*. 17, 2527-2539 (2006).
2. K. Fujii, *Meas. Sci. Technol.* 17, 2551-2559(2006).
3. K.Fujii, *Metrologia*. 41, S1-S15(2004).
4. B. Christian, Development and realisation of a fully automatic testing facility for determining the volume of E_1 weights up to 50 kg. in *Fundamental and Applied Metrology* (Lisbon, Portugal, 2009).
5. K. H. Chang and Y. J.Lee, *Metrologia*. 41, S95-S99(2004).
6. K. Riski and H.Kajastie, *Meas. Sci. Technol.* 15,1687-1693(2004).