PRINCIPLES OF GRAVITATIONAL WAVES DETECTION THROUGH ATOM INTERFEROMETRY

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The output of a simple Mach-Zehnder atom interferometer (with light field beam splitters) is studied in order to obtain sensitivity curves for GW signals in the paraxial approximation by using the ABCD matrices techniques and first order perturbation theory for mirroratom interaction; order of magnitude of relevant physical parameters for a realistic GW detector through atom interferometry is deduced, both for single- and coupled-interferometers configurations. Finally a synthetic overview of ongoing activities of the Florence-Urbino group in this field is presented.

Keywords: Gravitational waves; atom interferometry.

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1. Introduction

The direct detection of Gravitational Waves (GW) is one of the most exciting challenges of current scientific researches: presently ground based (optical) interferometric antennas such as Virgo [1] and LIGO [2] are already running, while space-based detectors are considered for the future [3]. The very impressive development undergone in matter wave interferometry with neutral atoms [4–6] stimulated a revival of the idea to use Atom Interferometers (AI) to detect GW [7–13]: looking at this promising field, in this paper the ABCD matrices approach is used to obtain the difference of the phase at the output of an atom interferometer (with Mach-Zehnder geometry) with light field beam splitters, subjected to a monochromatic GW, looking at the sensitivity curve as a performance test, both in single and coupled differential configuration. Alternative approaches in which the atom optics elements are

material nanostructures have been discussed elsewhere [8]: however, they are not considered here.

The paper is organized as follows. In Section 2 the science case is analysed, determining the frequency range where AI could play a relevant role. Sections 3 and 4 are devoted to the study of the Frequency Response Function and of the sensitivity curve for a Mach-Zehnder AI under suitable hypotheses about noise. Finally, in section 5 a short review about ongoing activities in the Florence-Urbino group is presented.

2. The Science Case

LIGO [2] and Virgo [1] detectors are well tuned to the observation of signals emitted by low-mass systems, like NS-NS binaries, and their advanced version will in a few years open up the band down to 10 Hz, thus making it possible to observe well also low-mass binary black holes, up to a few tens of solar masses. There is a great interest into extending the frequency band towards even lower frequencies, for the reason that phenomena involving more massive objects will emit most of the GW energy at frequencies outside the LIGO and Virgo band. The proposed LISA space-based detector, recently being reformulated as an ESA-only mission [14] (the New Gravitational wave Observatory, NGO) would open up the $[10^{-4} - 1]$ Hz band, with a sensitivity better than $10^{-22} \text{ Hz}^{-1/2}$ in the more restricted $[10^{-3} - 10^{-1}]$ Hz band; LISA could thus target signals emitted by massive black-holes, or by binary systems with a lower mass still orbiting far away from the coalescence. However, the $[10^{-1} - 10]$ Hz band would require other instruments to be covered; this role would be played in part by the Einstein Telescope (ET), which aims at extending the band of conventional light-interferometry detectors down to 1 Hz; ET could be complemented by one or more atom-interferometry instruments, which would also allow to observe the same sources from different locations on Earth, thus improving the source localization, somewhat limited with the single-site ET configuration [15]. The $[10^{-1} - 10]$ Hz frequency band, or even portions of it, could offer a great wealth of astrophysical observations [16], not hampered by the so-called confusion noise, due to the signal emitted by a great number of unresolved galactic binary sources, that in this band would be very low. For instance, it would be possible to study the low-frequency GW emission from pulsars which are known to exist in this frequency range; to observe the merger phase of massive black hole systems, during which general relativity would be seen at work in extreme, but in clean, pure field conditions. And the same binary systems observed in LIGO and Virgo could be followed at earlier stages of their evolution, contributing with a large SNR, since the signal amplitude scales as $f^{-7/6}$. Such a complementary observation would improve the determination of the physical parameters; for instance, the extended observation of the inspiral phase at lower frequencies would allow an accurate determination of the mass, orbital and spin parameters of the compact objects, which are key

ingredients in the analysis of the merger phase observed in the LIGO and Virgo band.

3. The Frequency Response Function

For a system under linearity and time invariance hypotheses, in the (angular) frequency domain we have^a

$$\Delta\phi(\Omega) = F(\Omega)I(\Omega) \quad (1)$$

where the output $\Delta\phi(\Omega)$ is related to the input $I(\Omega)$ through the Frequency Response Function (FRF), $F(\Omega)$. The sensitivity of the antenna may be defined as the smallest amplitude wave that can be detected at a fixed Signal to Noise Ratio (SNR). To calculate the FRF of an AI to a GW, we use the ABCD matrices approach, described elsewhere [7, 9, 12, 19–21]. Assume that the Hamiltonian of the motion is at most quadratic in momentum and position operators

$$\mathcal{H} = \sum_{n,r=1}^3 \left[\frac{1}{2M} p_n \beta_{nr}(t) p_r + \frac{1}{2} p_n \alpha_{nr}(t) q_r - \frac{1}{2} q_n \delta_{nr}(t) p_r - \frac{M}{2} q_n \gamma_{nr}(t) q_r \right] \quad (2)$$

where $\mathbf{p}$ and $\mathbf{q}$ are row (column) vectors of momentum and position, respectively, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\delta}$ are suitable square matrices (with $\boldsymbol{\delta} = -\boldsymbol{\alpha}^T$, where the apex T indicates the transposed matrix); M is the mass. The non relativistic case is here shown [see Ref. 12 for the relativistic approach]. Consider an atoms beam (a Gaussian packet under paraxial approximation [7, 9, 19, 21]) which is divided and recombined through a sequence of R light-field beam splitters, supplied by a laser: from the first beam splitter to the last one (the output port) we may identify two paths, conventionally indicated by s and i . By using the tt theorem [9] for the atoms/beam splitter interactions, and the mid-point property of Gaussian beam [7], the phase difference at the output port of the interferometer can be written as:

$$\Delta\phi = \sum_{j=1}^R \left[\left(k_s^j - k_i^j \right) \frac{q_s^j + q_i^j}{2} + \left(\omega_s^j - \omega_i^j \right) t^j + \left(\theta_s^j - \theta_i^j \right) \right] \quad (3)$$

where $k_{s(i)}^j$ is the momentum transferred to the atoms by the j^{th} beam splitter along the $s(i)$ arm, $\omega_{s(i)}^j$ is the angular frequency of the laser beam and $\theta_{s(i)}^j$ is the phase of the laser beam at the j^{th} interaction, $q_{s(i)}^j$ is the distance of j^{th} interaction point from the laser source. Equal masses for the atoms along s and i paths are assumed. The expression in Eq. (3) is manifestly gauge-invariant [12, 21]. Consider now a Mach - Zehnder geometry (see Fig. 1), assuming a plane GW with + polarization and amplitude h , propagating along the $x_3 = z$ axis and perpendicular to the plane of the interferometer. Using for instance Fermi Coordinates [22] we can evaluate

^aFor statistical & system theory concepts referred to GW antennas see e.g. Ref. 17; for terminology and general properties of GWs see e.g. Ref. 18.

Fig. 1. Single atom interferometer with Mach – Zehnder geometry. Dotted arrows: laser beams; bold continuous arrows: relevant momentum transferred to the atoms; g = ground, e = excited internal states of the atoms; k : momentum ($\hbar$ units)

$\Delta\phi$ by the proper ABCD matrices in Eq. (3) for a single Fourier component $h(\Omega)$ of the GW, at the first order in h (weak field approximation). Indicating by k_1 the unperturbed wave vector of the laser beam, q_1 the unperturbed distance of the first interaction point from the laser, p_1 the unperturbed momentum of the atoms just before the first interaction with the laser beam, $\hbar$ being the reduced Planck constant, we obtain for the phase difference at the output [12, 21]:

$$\begin{aligned} \Delta\phi(\omega) = & -\Omega h(\Omega) T^2 k_1 \frac{1}{M} \left(p_1 + \frac{k_1 \hbar}{2} \right) \\ & \times \left\{ \left[\sin(\Omega T) \left(\frac{\sin(\Omega T/2)}{\Omega T/2} \right)^2 + \frac{\cos(2\Omega T) - \cos(\Omega T)}{\Omega T} \right] \right. \\ & + i \left[\frac{\sin(2\Omega T) - \sin(\Omega T)}{\Omega T} - \cos(\Omega T) \left(\frac{\sin(\Omega T/2)}{\Omega T/2} \right)^2 \right] \Big\} \\ & + \frac{\Omega^2 \hbar(\Omega)}{2} T^2 k_1 q_1 \left(\frac{\sin(\Omega T/2)}{\Omega T/2} \right)^2 [\cos(\Omega T) + i \sin(\Omega T)] + \theta_1 - \theta_2 - \theta_3 + \theta_4. \end{aligned} \quad (4)$$

4. Shot Noise Limited Sensitivity

Apart from the proper laser phases, in Eq. (5) we have two contributions: the first one is related to the momentum, and the second one is related to the position of the first interaction of atoms with the laser beam. Neglecting the recoil term $\hbar k_1^2/(2M)$, the first contribution reads

$$\Delta\phi(\Omega) = -4\pi\psi \frac{L}{\lambda_{MW}} C(\Omega) h(\Omega) \quad (5)$$

where ψ is the opening angle between the s, i paths and λ_{MW} is the de Broglie wavelength of the atoms. The first contribution has the same form of the input/output relation for optical interferometers [1], where the geometrical term L (length of the arms, here defined as the distance between interactions of the atoms with the laser beam) is now completed by the opening angle; the "probe term is the wavelength (de Broglie waves instead of optical waves), and suitable configuration term $C(\Omega)$ (essentially the reciprocal of the FRF) characterizes the analytical behaviour. So the kinetic term (i.e. the term related to the atom momentum p_1) shows the

same structure as in the optical interferometer. Assuming the atom interferometer as shot-noise-limited [12, 21], at the level of $\text{SNR} = 1$ we have for the sensitivity function

$$h(\Omega) = \frac{1}{\sqrt{N}} \frac{\lambda_{MW}}{4\pi\psi L} \frac{1}{C(\Omega)}. \quad (6)$$

Detailed study of $C(\Omega)$ was done elsewhere [21]: here we recall only that the behaviour near the origin is $|C(\Omega)| \sim \Omega^2$; and when $\Omega \rightarrow \infty$, $|C(\Omega)|$ oscillates more and more rapidly between 0 and $1 + \epsilon$, where $\epsilon \sim 1/\Omega$; the bandwidth of different branches (i.e. distance between adjacent zeroes of FRF) is determined by the Time of Flight (ToF) T of the atoms inside the interferometer:

$$\Delta\Omega = \frac{2\pi}{T} = \frac{2\pi p_1}{LM^*} = \frac{2\pi v_1}{L}. \quad (7)$$

The behaviour of $|C(\Omega)|$ determines the shape and branches of the sensitivity curves (6): if we want to have a quantitative evaluation of possible performances of the atom interferometer shown in Fig. 1, we may rewrite Eq. (6) by using the relations $L = v_1 T$, $\psi \sim \tan \psi = \frac{p_{tr}}{p_1}$ where p_{tr} is the transversal momentum (i.e. the momentum transferred to the atoms by the laser beam), and the definition of the de Broglie matter wave. The expression for $h(\Omega)$ becomes

$$h(\Omega) = \frac{\hbar}{p_{tr} v_1 T \sqrt{N}} \frac{1}{2|C(\Omega)|} = \frac{5 \times 10^{-35}}{C(\Omega)\Sigma}. \quad (8)$$

So, the larger is $\Sigma \equiv p_{tr} v_1 T \sqrt{N}$ the better is the interferometer sensitivity. However T is not a totally free parameter, since the bandwidth goes as $1/T$, and $L (= v_1 T)$ is the longitudinal dimension of the interferometer which is necessarily limited as well as the transferred momentum p_{tr} . An interesting example is related to the medium-low frequency range, just below the interval accessible to the ground-based optical interferometer [1, 2]. If we look at an AI which has to cover the (0.55) Hz range, by a two parameters fit on Eq. (8), we obtain the plot in Fig. 2 with the values

$$ToF \simeq 0.4s; \quad \Sigma \simeq 1.1 \times 10^{-13} Js. \quad (9)$$

In order to have some reference numbers, assume for the AI a linear dimension of a few km (typical length of the arms of ground based optical GW antennas), which means a velocity of the atoms in the main beam of about $10^4 m/s$; assuming a flow of 10^{16} atom/s, the required transferred transversal momentum is of about $2.7 \times 10^{-25} kg m/s$ (transversal velocity in the range of $1.6 \times 10^2 m/s$ for Hydrogen atoms), which is equivalent to few tens of UV Lyman- α photons. These numbers are very high, but not unrealistic if we look at some results in producing continuous atom flows [23, 24], and at the recent, very promising techniques for large momentum transfer for Cs or Rb atoms [25–27]. It is worth noting that the curve in Fig. 2 has been obtained under the approximation of shot-noise limited performance. If we consider optical GW antennas, such as Virgo or LIGO, their shot-noise limited

Fig. 2. Shot-Noise- limited sensitivity for the Atom Interferometer in Fig. 1 with best sensitivity around 1.3 Hz.

curves [1] are better than the one in Fig. 2: in the region under few hundred Hz, the curves are flat and around the value of few $10^{-23} \text{ Hz}^{-1/2}$. As a matter of fact, the limiting noise in this region for Virgo and LIGO is thermal noise (of mirrors and suspensions) [1, 2]; for atom interferometers, if we want to have a real evaluation of their possible use for GW detection, we need an estimation of the overall noise budget in order to have a reliable comparison in the region of interest. As an example, if in this low frequency region the limiting noise for an atom interferometer is the shot noise, this detector is better than the optical ones. Coming back to Eq. (5), consider now the second contribution. This term, already introduced in a different context [28] and recently presented in this general form [12, 21], is a kind of clock term taking into account the influence of GW onto the laser beam along its path from the source to a well defined physical point. This term was analyzed in recent papers [13, 29, 30] and the most relevant new property is the introduction of q_1 (path of laser beam) in place of L (path of atom beam); so, in order to improve the sensitivity, enlarging q_1 seems in principle easier than enlarging L ; however, the request of measuring the distance from the laser, and strong requirements about coherence and stability with enough power density for laser beam are not simple problems, preventing too optimistic conclusions. Anyway the suggestion for adopting a two-interferometers differential configuration [13, 26, 29, 30] sounds very interesting to overcome the first of the cited problems; furthermore it may result in a good common-modes rejection, allowing to face in an efficient way the problem of some typical noise sources (e.g. seismic and/or general vibrations, laser fluctuations, also of thermal origin). In the hypothesis of common lasers for two identical Mach-Zehnder atom interferometers in differential configuration for which the reciprocal distance L' satisfies the condition $\Omega L'/c \ll 1$, where c is the speed of light in vacuum, from the equation (5) the total difference between the two partial differences of phase from the output port of the two interferometers I and II, for a

single Fourier component, is

$$\Delta\phi_{Total} = 2kL' \sin^2\left(\frac{\Omega T}{2}\right) h(\Omega)e^{i\Omega T} \quad (10)$$

where $L' = q_1^{II} - q_1^I$. The behaviour of the FRF and of the related sensitivity is very similar to the one previously discussed, where the dimension L of the single interferometer (the trip of the atoms) is substituted by the distance L' between the two interferometers (the trip of the laser beam) which in principle can be increased in an easier way [29, 30].

5. Ongoing Activities of the Florence-Urbino Group

Since many years at the Physics Department of Florence University the group developed MAGIA [31] (Italian acronym for "Accurate Measurement of G with Atom Interferometry", see Fig. 3), which is an atom interferometry experiment to measure the gravitational constant G . It basically consists in a gravity gradiometer with ultracold Rb atoms, that measures the change in gravity gradient induced by well characterized source masses. MAGIA provided the most accurate measurement of the gravitational constant with atom interferometry techniques [32]. At the same time the group is applying atom interferometry techniques to test the Newtonian gravitational law at micrometric distances [33], and is developing compact atom interferometry accelerometers for space research and geophysical applications [34].

Fig. 3. Left: scheme of the MAGIA apparatus; two clouds of Rb atoms are sequentially loaded into a magneto-optical trap in the lower chamber, then launched upwards in a magnetically shielded tubes; simultaneously Raman interrogation occurs around the apogee of the atomic trajectories, then the atoms are detected by fluorescent spectroscopy in the chamber below the tube; right: a picture of the experimental setup.

Experimental activities are ongoing, based on the MAGIA apparatus, to demonstrate one or more possible schemes to detect GWs in a frequency band inaccessible to optical interferometers, i.e. between about 0.1 Hz and 10 Hz, with a smaller and simpler apparatus. For the atomic sources a two-dimensional magneto-optical trap (2D-MOT) capable of an extremely large flux of cold atoms, in the range of 10^{12} atoms/s or more, is under study. At the same time, the group is implementing methods for multiphotons splitting, with the target of coherently transferring several tens of photon momenta, scalable to several hundreds. This is still far from the requirements for GW detection, but techniques involving accelerating optical lattices, such as the BBB beam splitter [25], may achieve the required splitting. Atom detection methods, with sufficient SNRs to detect large fluxes of atoms (in the range of 10^{12} atoms/s) at the shot noise limit, are under development. These involve high efficiency fluorescence light detection and processing, laser frequency and intensity stabilization, and modulation techniques to overcome technical noises.

6. Conclusions

The shot noise limited sensitivity for a Mach-Zehnder AI appears interesting, even if dedicated technological developments are needed in order to obtain the planned performance. Furthermore, a detailed study of the noise budget for a realistic detector is mandatory to have a reliable comparison between optical and atom interferometers. Florence-Urbino group, among others [35–37], is developing a facility as a preliminary fundamental step to test the feasibility of this new GW detector.

References

1. <http://www.virgo.infn.it>
2. <http://www.ligo.caltech.edu>
3. <https://gwic.ligo.org/roadmap>
4. F. Riehle *et al.*, *Phys. Rev. Lett.* **67**, 177 (1991); M. Kasevich and S. Chu, *Phys. Rev. Lett.* **67**, 181 (1991).
5. P. Berman (ed.), *Atom Interferometry*, (Academic Press, New York, 1997).
6. C.J. Bordé, *Metrologia* **39**, 435 (2002).
7. 2004 Aspen Winter Conference on GW and their Detection, http://www.ligo.caltech.edu/LIGO_web/Aspen2004/pdf/vetrano.pdf
8. R. Y. Chiao and A.D. Spiliotopoulos, *J. Mod. Optics* **51**, 861 (2004).
9. C.J. Bordé, *Gen. Relativ. Gravit.* **36**, 475 (2004).
10. A. Roura, D.R. Brill, B.L. Hu, C.W. Misner and W.D. Phillips, *Phys. Rev. D* **73**, 084018 (2006).
11. P. Delva, M.C. Angonin and P. Tournenc, *Phys. Lett. A* **357**, 249 (2006).
12. G.M. Tino and F. Vetrano, *Class. Quantum Grav.* **24**, 2167 (2007).
13. S. Dimopoulos, P.W. Graham, J.M. Hogan, M. Kasevich and S. Rajendran, *Phys. Rev. D* **78**, 122002 (2008).
14. <http://sci.esa.int/lisa>
15. A. Freise, S. Hild, K. Somiya, A. Viceré, M. Barsuglia and S. Chelkowski, *Gen. Relativ. Gravit.* **43**, 537 (2010).

16. B.Sathyaprakash *et al.*, Scientific Potential of the Einstein Telescope, in *Proceedings of Moriond 2011*, <http://arxiv.org/abs/1108.1423v1>
17. P.R. Saulson, *Fundamentals of Interferometric GW Detectors*, (World Scientific, Singapore, 1994).
18. M. Maggiore, *Gravitational Waves*, vol. 1 (Oxford University Press, Oxford, 2008).
19. J.F. Riou, Y. Le Coq, F. Impens, W. Guerin, C.J. Bordé, A. Aspect and P. Bouyer, *Phys. Rev. A* **77**, 033630 (2008).
20. C.J. Bordé, *Eur.Phys.J.* **163**, 315 (2008).
21. G.M. Tino and F. Vetrano, *Gen. Relativ. Gravit.* **43**, 2037 (2011).
22. F.K. Manasse and C.W. Misner, *J.Math.Phys.* **4**, 735 (1963).
23. G. Scoles (Ed), *Atomic and Molecular Beam Methods* (Oxford University Press, New York, 1988).
24. D.E. Keith, C.R. Ekstrom, Q.A. Turchette and D.E. Pritchard, *Phys.Rev.Lett.* **66**, 2693 (1991).
25. H. Müller, S. Chiow, Q. Long, S. Hermann and S. Chu, *Phys.Rev.Lett.* **100**, 180405 (2008).
26. H. Müller, S. Chiow, S. Hermann and S. Chu, *Phys.Rev.Lett.* **102**, 240403 (2009).
27. S. Chiow, T. Kovachy, H. Chien and M. Kasevich, *Phys.Rev.Lett.* **107**, 130403 (2011).
28. C.J. Bordé, J. Sharma, P. Tourenco and T. Damour, *J.Physique Lett.* **44**, L983 (1983).
29. S. Dimopoulos, P.W. Graham, J.M. Hogan and M. Kasevich, *Phys.Rev. D* **78**, 042003 (2008).
30. S. Dimopoulos, P.W. Graham, J.M. Hogan and M. Kasevich, *Phys.Lett. B* **678**, 37 (2009).
31. F. Sorrentino, Y.H. Lien, G. Rosi, L. Cacciapuoti, M. Prevedelli and G.M. Tino, *New J. Phys* **12**, 095009 (2010).
32. G. Lamporesi, A. Bertoldi, L. Cacciapuoti, M. Prevedelli and G.M. Tino, *Phys. Rev. Lett* **100**, 050801 (2008).
33. M. De Angelis, A. Bertoldi, L. Cacciapuoti, A. Giorgini, G. Lamporesi, M. Prevedelli, G. Saccorotti, F. Sorrentino and G.M. Tino, *Measurement Science and Technology* **20** (2), 022001 (2009).
34. F. Sorrentino, K. Bongs, P. Bouyer, L. Cacciapuoti, M. De Angelis, H. Dittus, W. Ertmer, M. Hauth, S. Herrmann, M. Inguscio, E. Kajari, T. Knemann, C. Lmmerzuhl, A. Landragin, G. Modugno, F. Pereira dos Santos, A. Peters, M. Prevedelli, E.M. Rasel, W.P. Schleich, M. Schmidt, A. Senger, K. Sengstok, G. Stern, G.M. Tino, R. Walser, *Microgravity Sci. Tech.* **22** (4), 551 (2010).
35. M. Hoensee, S. Lan, R. Houtz, C. Chan, B. Estey, G. Kim, P. Kuan and H. Muller, *Gen. Relativ. Gravit.* **43**, 1905 (2011).
36. J.M. Hogan, D.M.S. Johnson, S. Dickerson, T. Kovachy, A. Sugarbaker, S. Chiow, P.W. Graham, M. Kasevich, B. Saif, S. Rajendran, P. Bouyer, B.D. Seery, L. Feinberg and R. Keski-Kuha, *Gen. Relativ. Gravit.* **43**, 1953 (2011).
37. L. Zhou, Z.Y. Xiong, W. Yang, B. Tang, W.C. Peng, K. Hao, R.B. Li, M. Liu, J. Wang and M.S. Zhan *Gen. Relativ. Gravit.* **43**, 1931 (2011).