F-THEORY FLUXES, CHIRALITY AND CHERN-SIMONS THEORIES

HIROTAKA HAYASHI

*School of Physics, Korea Institute for Advanced Study
 Seoul, 130-722, Korea
 hayashi@kias.re.kr*

We have established a relation between the four-dimensional chiral index in F-theory compactifications and the three-dimensional Chern-Simons coupling in M-theory compactifications by using F-theory - M-theory duality. The content of this article is based on our recent paper,¹ which is in collaboration with Thomas W. Grimm.

Keywords: F-theory; chirality; Chern-Simons theory.

1. 4D Chiral Index from 3D Chern-Simons Coupling

F-theory compactifications have recently been studied for constructing four-dimensional Grand Unified Theory. One of the important phenomenological interests is the chiral index. In this article, we present a new way to compute the four-dimensional chirality from a compact manifold via F-theory – M-theory duality.¹

A three-dimensional theory from an M-theory compactification on an elliptically-fibered Calabi–Yau fourfold X_4 in the limit where the volume of the elliptic fiber vanishes is dual to a four-dimensional theory on $\mathbb{R}^{1,2} \times S^1_\infty$ from an F-theory compactification on X_4 .² Here S^1_∞ denotes a circle of an infinitely large radius. When X_4 has a singularity to have a non-Abelian gauge group G , one can perform a resolution of the singularity to consider the dimensional reduction of eleven-dimensional supergravity on the resolved Calabi–Yau fourfold $\tilde{X}_4$. Then, the bases of the Kähler form ω_Λ , ($\Lambda = 1, \dots, \text{rank}(G)$) associated to the volume of the exceptional divisors correspond to three-dimensional vector multiplets which contain Abelian gauge fields A^Λ and adjoint scalar fields ξ^Λ as bosonic components.

If one turns on a four-form flux G_4 , the following Chern-Simons term is induced

$$S_{\text{CS}}^{(3)} = \frac{1}{4} \int_{\mathbb{R}^{1,2}} \Theta_{\Lambda\Sigma} A^\Lambda \wedge F^\Sigma, \quad \Theta_{\Lambda\Sigma} = \int_{\tilde{X}_4} G_4 \wedge \omega_\Lambda \wedge \omega_\Sigma. \quad (1)$$

Here G -flux is turned on under the conditions that it does not break the gauge symmetry G nor the four-dimensional Poincaré symmetry in the limit to F-theory.

On the other hand, the Chern-Simons coupling in a general framework of three-dimensional Abelian gauge theories is³

$$(k_{\Lambda\Sigma})_{\text{eff}} = (k_{\Lambda\Sigma})_{\text{class}} + \frac{1}{2} \sum_f (q_f)_\Lambda (q_f)_\Sigma \operatorname{sign} \left(\sum_{\Gamma=1}^{\operatorname{rank}(G)} (q_f)_\Gamma \xi^\Gamma + \tilde{m}_f \right), \quad (2)$$

where $(q_f)_\Lambda$ is a charge of a fermion f under $U(1)_\Lambda$ and the $\tilde{m}_f$ is a classical mass for the fermion. In fact, $(k_{\Lambda\Sigma})_{\text{class}}$ vanishes in the present case.^{4,5} Furthermore, since the fermions become massless in the F-theory limit, one also has to set $\tilde{m}_f = 0$. By performing a part of the sum in (2), one obtains

$$\Theta_{\Lambda\Sigma} = \frac{1}{2} \sum_{\mathbf{r}} n_{\mathbf{r}} \sum_{\mathbf{w} \in \mathbf{r}} q_{\Lambda}^{\mathbf{w}} q_{\Sigma}^{\mathbf{w}} \operatorname{sign} \sum_{\Gamma=1}^{\operatorname{rank}(G)} q_{\Gamma}^{\mathbf{w}} \xi^{\Gamma}, \quad (3)$$

where $\mathbf{r}$ denotes a representation of a fermion and $\mathbf{w}$ is a weight of the representation. Since the three-dimensional theory is obtained from the S^1 reduction of a four-dimensional theory, the zero mode spectrum in the three-dimensional theory should match with that of the four-dimensional theory. Hence, $n_{\mathbf{r}}$ precisely matches with the chirality in the four-dimensional theory of the dual F-theory compactification.

Let us rewrite the right-hand side of (3) by geometric information of $\tilde{X}_4$. The matter multiplets in a weight $\mathbf{w}$ for the massive fermions arise from M2-branes wrapping on a two-cycle $C_{\mathbf{w}}$ labeled by the same weight $\mathbf{w}$ which appears from the resolution. Hence, one obtains

$$q_{\Lambda}^{\mathbf{w}} = \int_{C_{\mathbf{w}}} \omega_{\Lambda}, \quad \operatorname{sign} \sum_{\Gamma=1}^{\operatorname{rank}(G)} q_{\Gamma}^{\mathbf{w}} \xi^{\Gamma} = \operatorname{sign} \int_{C_{\mathbf{w}}} \tilde{J}, \quad (4)$$

where $\tilde{J}$ is the Kähler form normalized by $\operatorname{vol}(\tilde{X}_4)$. Then, the right-hand side of (3) can be written by the geometric data of $\tilde{X}_4$ and one can extract the chiral index $n_{\mathbf{r}}$

$$n_{\mathbf{r}} = t_{\mathbf{r}}^{\Lambda\Sigma} \Theta_{\Lambda\Sigma}. \quad (5)$$

$t_{\mathbf{r}}^{\Sigma\Lambda}$ can be determined systematically for hypersurfaces or complete intersections of Calabi–Yau fourfolds in toric varieties.

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