CLASSICAL SOLUTIONS IN THE LORENTZIAN MATRIX MODEL FOR SUPERSTRING THEORY

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Recent Monte Carlo study on the Lorentzian matrix model for superstring theory revealed that an expanding $(3+1)d$ universe appears dynamically from $(9+1)d$. The mechanism for the spontaneous breaking of rotational symmetry relies crucially on the noncommutative nature of the three expanding spaces. As a complementary approach to possible future beyond the numerical result, we discuss classical solutions for the Lorentzian matrix model and their properties.

Keywords: Expanding universe; emergent spacetime; IIB matrix model; Lorentzian matrix model.

1. Early Universe as an Unique Nonperturbative Solution of the Lorentzian Matrix Model

Implications of quantum gravity on cosmology is an important frontier of high energy particle physics and studied in various respects.^{1–4} Interesting questions are how we can explain the accelerated expansion or the emergence of $(3+1)d$ spacetime. To study these questions from the first principle, we need a nonperturbative tool which can resolve the cosmic initial singularity. As lattice QCD is for QCD, matrix model is proposed as a nonperturbative formulation for superstring theory, where dynamic generation of spacetime can be described.^{5–7}

Recently we found a regularization for IIB matrix model in Lorentzian signature by two infrared cutoffs instead of a Wick rotation.⁸ This allows us to study real time dynamics with full quantum corrections by a Monte Carlo method. Two cutoffs play a similar role as Dirichlet boundary conditions for $(9+1)d$ spacetime. This opens a

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possibility to overcome the criticisms in string theory by a numerical method, since the model has an unique solution at the nonperturbative level.

The Monte Carlo study revealed that this unique solution describes expanding 3d spaces emerged from 9d spaces after some critical time. This can be easily understood if we drastically increase temporal cutoff for fixed matrix size. In this limit, there is a mechanism which prefers $su(2)$ algebra. Three generators of $su(2)$ give rise to expansion of 3 spaces from higher dimension. The expanding behavior is consistent with exponential, which remind us of a beginning of the inflation. Supersymmetry plays an important role for continuity and infinitely extended property of dynamic time.

2. Candidate for Future Universe as Various Classical Solutions of the Lorentzian Matrix Model

Direct simulation is rather restricted to the very beginning of the universe due to the numerical cost, while there are a lot of interesting questions in the late time. If we assume that a classical equation of motion is dominant at late time, an analytic study on classical solutions can provide a complementary method.⁹ With two constraints for cutoffs in the Lorentzian model, we consider

$$\begin{aligned} -[A_0, [A_0, A_i]] + [A_j, [A_j, A_i]] - \lambda A_i &= 0, \\ -[A_j, [A_j, A_0]] - \tilde{\lambda} A_0 &= 0, \end{aligned} \quad (1)$$

where λ and $\tilde{\lambda}$ are corresponding Lagrange multipliers.

Among many solutions satisfying $[A_i, A_j] = 0$, we are interested in

$$\begin{aligned} A_0 &= T_0 \otimes 1_k, \\ A_i &= T_1 \otimes \text{diag}(x_i^{(1)}, x_i^{(2)}, \dots, x_i^{(k)}), \end{aligned} \quad (2)$$

where $i = 1, \dots, d$ and $A_{d+1}, \dots, A_9 = 0$. $x_i^{(j)}$ are arbitrary constants and T_i are generators of $su(1, 1)$ algebra

$$[T_0, T_1] = iT_2, \quad [T_1, T_2] = -iT_0, \quad [T_2, T_0] = iT_1. \quad (3)$$

Note that time evolution is described by T_i , while diagonal part $x_i^{(j)}$ can describe any d -dimensional comoving commutative spaces as $k \rightarrow \infty$.

After some straightforward calculation we can obtain expanding universes with $R(t) \propto \sqrt{t^2 + c}$, where equation of state parameter for cosmology is $w = -2c/(3t^2) - 1/3$. Depending on the sign of a constant c , this classical solution can describe either an accelerated expansion with cosmological constant decrease as time, or an evolution from radiation domination to matter domination era. We hope a study in this direction will uncover various mysteries in cosmology such as dark energy and future fate of our universe.

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