DUALITY DOMAIN WALL INDEX ON $S^1 \times S^3$

EUNKYUNG KOH

*Korea Institute for Advanced Study
 Seoul 130-012, Korea
 ekoh@kias.re.kr*

We calculate a superconformal index of the half-BPS duality domain wall inserted at the great S^2 of S^3 in $\mathcal{N} = 4$ $SU(2)$ super Yang-Mills theory on $S^1 \times S^3$.

Keywords: S-duality; index; domain wall.

1. Introduction

The $T[G]$ theory is the theory on the half-BPS (Bogomolnyi-Prasad-Sommerfield) S-duality domain wall between 4d $\mathcal{N} = 4$ SYM (super Yang-Mills) theories with gauge group G and ${}^L G$, where ${}^L G$ is the Langlands dual of G .¹ One important property of $T[G]$ theory is that it has a global symmetry $G \times {}^L G$.

We use two recently developed concepts in our calculation. Firstly, the “generalized” superconformal index for 3d field theories has been introduced by Kapustin and Willett.² Secondly, the “half-index” of 4d SYM has been introduced by Dimofte, Gaiotto, and Gukov.³

2. 3D Index on the Duality Domain Wall

Let us consider a generalized index for $T[SU(2)]$ as a function of fugacities and magnetic charges for $SU(2) \times {}^L SU(2)$ global symmetry. $T[SU(2)]$ is a 3d abelian $\mathcal{N} = 4$ theory with two electron hypermultiplets. The generalized index for $T[SU(2)]$ can be written as

$$I_{T[SU(2)]}(u, m, u', m') = \sum_s \oint \frac{d\zeta}{2\pi i} (u')^{2s} \zeta^{2m'} x^\epsilon \text{PE} \left[\frac{x^{1/2}}{1+x} \sum_{\pm} (x^{|s \pm m|} (u\zeta^\pm + u^{-1}\zeta^{-(\pm 1)})) \right], \quad (1)$$

where m and u are magnetic flux and fugacity for $SU(2)$ symmetry, m' and u' for ${}^L SU(2)$ symmetry, and s and ζ for the $U(1)$ gauge group. m, m', s can be integers or half-integers satisfying the Dirac quantization condition, $s \pm m \in \mathbb{Z}$. ϵ is the Casimir energy given as $\epsilon := \frac{1}{2}|s+m| + \frac{1}{2}|s-m|$. PE denotes the Plethystic exponential. x is

a chemical potential for $H + j$, where H and j are Cartans of $U(1) \times SO(3)$ rotating $S^1 \times S^2$. The index can be regarded as a polynomial of non-negative powers of x , in each sector of (m, m') , with coefficients as functions of u and u' .

Using Mathematica, perturbatively in x , we checked that Eq. (1) is invariant under the exchange of (u, m) and (u', m') for various values of m and m' . It is consistent with the fact that $T[SU(2)]$ is 3d mirror dual to itself.

3. 4D Index with the Duality Domain Wall

The 4d half-index $\Pi_m(x, u)$ for $\mathcal{N} = 4$ $SU(2)$ SYM is given as³

$$\Pi_m(u) = \delta_{m,0} \text{PE}\left[\frac{x}{1+x}(u^2 + 1 + u^{-2})\right]. \quad (2)$$

The 4d index with the insertion of the duality domain wall in $\mathcal{N} = 4$ SYM can be obtained by coupling 3d $T[SU(2)]$ index with the 4d half-indices,

$$I_{\text{wall}}(x) = \sum_{m,m'} \oint \oint [du'] \Pi_m(u)^\dagger I_{T[SU(2)]}(u, m, u', m') \Pi_{m'}(u'). \quad (3)$$

where $[du] := \frac{1}{2} x^{-2m} (1 - x^{2m} u^2) (1 - x^{2m} u^{-2}) du$. We checked perturbatively in x , the following holds

$$\Pi_m(u) = \sum_{m'} \oint [du'] I_{T[SU(2)]}(u, m, u', m') \Pi_{m'}(u'). \quad (4)$$

One implication of Eq. (4) is that the 4d index with the duality domain wall is same with the index of $\mathcal{N} = 4$ SYM itself. It is consistent with the fact that a superconformal index is invariant under continuous deformations, since via S-duality on a hemisphere the duality domain wall can be mapped to a Janus domain wall which in turn can be continuously deformed to $\mathcal{N} = 4$ SYM without a defect.⁴ Eq. (4) also suggests the role of $T[SU(2)]$ index as a duality kernel. In this view, with the presence of line operators $\hat{O}_L$, it can be generalized to

$$\sum_{m'} \oint [du']_m I_{T[SU(2)]}(u, m, u', m') \left(\hat{O}_L \cdot \Pi_{m'}(u') \right) = \hat{O}_{L'} \cdot \Pi_m(u) \quad (5)$$

where $\hat{O}_{L'}$ is S-dual to $\hat{O}_L$. We checked that Eq. (5) holds for several examples.

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