THE CLASSICAL EQUIVALENCE OF MONODROMY MATRICES IN SQUASHED SIGMA MODEL

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We show Yangian and quantum affine symmetries are realized in two-dimensional non-linear sigma models with squashed spheres target spaces. According to these symmetries, there are two descriptions for the classical dynamics. In each description, a Lax pair and the corresponding monodromy matrix are constructed. Both Lax pairs reproduce the same equations of motion and hence they are equivalent. We show the gauge-equivalence by constructing a map between monodromy matrices with the relation of spectral parameters.

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One of the interesting subjects is to consider deformations of AdS/CFT.¹ Among of them, we are here concerned with warped AdS₃ spaces, which appear, for example, as the gravity duals of condensed matter systems in the presence of magnetic field² and as the near horizon geometries of extremal Kerr black holes³ that played an important role in the context of Kerr/CFT.⁴ In the recent, as a toy model of Kerr/CFT, the warped AdS₃/dipole CFT₂ correspondence has been proposed.⁵

In this duality, the Virasoro $\times$ Virasoro symmetry is argued to be realized in the same way as the usual AdS₃/CFT₂. This circumstance is curious from the viewpoint of integrable structure of two-dimensional non-linear sigma models defined on warped AdS₃. The existence of Virasoro $\times$ Virasoro symmetry implies that the sigma models belong to the rational class, while a warped AdS₃ is regarded as a

one-parameter deformation of AdS_3 and hence the sigma models should belong to the trigonometric class. This observation suggests that two integrable structures coexist, namely *the hybrid integrability*.

In this presentation, we discussed the classical hybrid integrable structure of non-linear sigma models defined on squashed S^3 , in order to get an insight into the above feature (Note that AdS_3 is obtained from S^3 via a double Wick rotation so the (at least) classical result for S^3 is valid also for AdS_3). The isometry of squashed S^3 is $SU(2)_L \times U(1)_R$. In the sigma models on squashes S^3 , $su(2)_L$ is enhanced to the Yangian $Y(su(2))^6$ and $u(1)_R$ is enhanced to the quantum affine algebra $U_q(\widehat{su(2)})$.⁷ In addition, we can construct two kinds of Lax pairs and the associated monodromy matrices: one is the rational Lax pair, based on $SU(2)_L$ and the other is the trigonometric one, based on $U(1)_R$. The Yangian $Y(su(2))$ is obtained from the rational monodromy matrix and the quantum affine algebra $U_q(\widehat{su(2)})$ is obtained from the trigonometric one.

In each description, the equations of motion are obtained from the Lax pair. This means just a local equivalence of the two descriptions and the global equivalence, namely the agreement of classical moduli spaces, has not been revealed yet. In the recent paper⁸ we have shown the gauge-equivalence of the two Lax pairs and monodromy matrices. It is worth noting that the Yangian $Y(su(2))$ and the quantum affine algebra $U_q(\widehat{su(2)})$ are *also* obtained from the trigonometric and rational monodromy matrices respectively. Thus one description is enough to obtain the generators of Yangian and quantum affine algebras. We also have determined a map between the spectral parameters in the two descriptions so that the expansion points of Yangian and quantum affine algebras coincide. The map shows that the trigonometric description is equivalent to a composite of two rational descriptions. In addition, this equivalence gives rise to a unifying picture based on a Riemann sphere with four punctures. This map enables us to show the gauge-equivalence of the two descriptions.

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