LINE OPERATOR INDEX ON $S^1 \times S^3$

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We derive a general formula for an index for $\mathcal{N} = 2$ superconformal field theories. In the computation, 1-loop and monopole bubbling effects are taken into account. Our computation is compatible with S -dualities in the $\mathcal{N} = 4$ theories and the holographic principle which maps Wilson line operators in the $\mathcal{N} = 4$ SYMs to macroscopic objects in $AdS_5 \times S^5$.

Keywords: S-duality; superconformal index; line operators.

1. Line Operator Index: Definition and Computation

We are considering $\mathcal{N} = 2$ superconformal field theories (SCFTs) on $\mathbb{R} \times S^3$. These theories can be obtained from the $\mathcal{N} = 2$ SCFTs on flat space time ($\mathbb{R}^4$) via conformal mapping. In this note we only consider the simplest case, $\mathcal{N} = 4$ supersymmetric Yang-Mills theories (SYMs). Generalization to $\mathcal{N} = 2$ theories is straightforward.¹ The bosonic symmetries for the $\mathcal{N} = 4$ SCFTs are

$$SO(2, 4) \times SU(4)_R. \quad (1)$$

These SCFTs also have 16 fermionic symmetries whose generators are denoted as

$$Q^{\alpha, A}, \bar{Q}_{\dot{\alpha}}^A, S_{\alpha, A}, \bar{S}_{\dot{\alpha}}^A. \quad (2)$$

Here $(\alpha, \dot{\alpha})$ are $(\mathbf{2}, \mathbf{0})$ and $(\mathbf{0}, \mathbf{2})$ for $SU(2)_L \times SU(2)_R = SO(4) \subset SO(2, 4)$, rotational symmetry on S^3 . $A = 1, \dots, 4$ are fundamental indices of $SU(4)$ R-symmetry.

One can introduce two types of line operators to the $\mathcal{N} = 4$ theories. One is Wilson line operators which is labelled by electric charge (representation) R of the gauge group G . The other one is 't Hooft line operators labelled by magnetic charge (coweight) B of G . Combining with an adjoint scalar in the theory, the line operator can be supersymmetric. We will consider half BPS line operators stretched along the time direction $\mathbb{R}$ in $\mathbb{R} \times S^3$ and located at the north/south poles of S^3 .

To define an 'index' which is compatible with line operators, we choose a supercharge Q as

$$Q = Q^{\alpha=1, A=1} + \bar{Q}_{\dot{\alpha}=1, A=2}. \quad (3)$$

With proper choice of an adjoint scalar appearing in the definition of supersymmetric line operators, line operators preserve the supercharge Q . We introduce line

operator index which is defined as follows

$$I_L(x) := \text{Tr}_{\mathcal{H}_L}(-1)^F x^{\epsilon+j_L+j_R} . \quad (4)$$

Here $\mathcal{H}_L$ denote the Hilbert-space of the theory on S^3 with insertion of line operator L . $(-1)^F$ gives 1 for bosonic states and -1 for fermionic states. ϵ denotes the energy associated with the time direction in $\mathbb{R} \times S^3$. j_L, j_R denote the Cartan generators of $SU(2)_L, SU(2)_R$ respectively. The quantum number $\epsilon + j_L + j_R$ is chosen so that it commutes with the chosen supercharge Q .

Using the localization technique,² the line operator index defined above can be exactly calculable. For general $\mathcal{N} = 4$ SCFTs of gauge group G , the line operator index formula can be written as

$$\begin{aligned} I_{W=R} &= \int [dU]_{\nu=0} Z_{\nu=0}^{1-loop} [U, x] (\text{tr}_R U) (\text{tr}_R U^\dagger) , \quad \text{Wilson line index} \\ I_{T=B} &= \sum_{\nu \leq B} \int [dU]_{\nu} Z_{mono}^{S^3} [U, x; B, \nu] Z_{\nu}^{1-loop} [U, x] , \quad \text{'t Hooft line index.} \end{aligned} \quad (5)$$

where $(\lambda = \sum_a \lambda_a H^a : \text{Cartan subalgebra}, \alpha : \text{roots of } G)$

$$\begin{aligned} \int [dU]_{\nu} &= \frac{1}{(\text{symmetry})} \int_0^{2\pi} \left(\prod_a \frac{d\lambda_a}{2\pi} \right) \prod_{\alpha \neq 0, \alpha(B)=0} (1 - e^{i\alpha(\lambda)}), \\ Z_{\nu}^{1-loop} &= \prod_{\alpha(\nu) \neq 0} (1 - e^{i\alpha(\nu)}) \frac{\prod_{\alpha} \prod_{n=0}^{\infty} (1 - x^{3/2+2n+|\alpha(\nu)|} e^{i\alpha(\lambda)})}{\prod_{\alpha} \prod_{n=0}^{\infty} (1 - x^{1/2+2n+|\alpha(\nu)|} e^{i\alpha(\lambda)})}. \end{aligned} \quad (6)$$

Non-perturbative monopole bubbling contribution $Z_{mono}^{S^3}$ turned out to be the square of monopole bubbling index on $\mathbb{R}^3$, $Z_{mono}^{S^3} = |Z_{mono}^{\mathbb{R}^3}|^2$, which is studied in Ref. 3.

2. Consistency Checks

Using the exact calculation of the line operator indices, one can perform several non-trivial checks for various kinds of dualities involving $\mathcal{N} = 2$ SCFTs. S-duality is an electric/magnetic duality and it exchanges Wilson/'t Hooft line operators. By comparing the indices for two line operators related by S -duality, we confirm that they give the exactly same indices. Another type of duality one can consider is gauge/gravity duality. For $U(N)$ $\mathcal{N} = 4$ SYM theory, the theory is believed to be equivalent to type IIB string theory on $AdS_5 \times S^5$. Under the duality, fundamental/anti-symmetric Wilson line operators in the gauge theory is mapped to fundamental string/ $D5$ -brane on $AdS_5 \times S^5$.⁴⁻⁶ By comparing the large N limit of fundamental/anti-symmetric Wilson line indices in the $U(N)$ gauge theory with the indices obtained by analyzing the fluctuations around fundamental/ $D5$ -branes on $AdS_5 \times S^5$, we also found the exact matches.

References

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