SPACETIME STRUCTURE OF THE BLACK HOLE EVAPORATION

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We study a self-consistent solution of the semi-classical Einstein equation including the back reaction from the Hawking radiation. Our geometry is constructed by connecting flat space and the outgoing Vaidya metric at the locus of the shock wave. In order to prove that this is the self-consistent solution, we first show that the Weyl anomaly is canceled if we take the effects of the fluctuations of the metric into account. We further demonstrate that the Hawking radiation occurs even if the geometry has no horizon. Then, the energy-momentum tensor is found to be consistent with the semi-classical Einstein equation. Since our geometry has neither horizon nor singularity, all matters inside the black hole finally come back to infinity. Therefore, no information is lost by the black hole evaporation. Furthermore, we take into account the gray-body factor. We construct a stationary solution for a black hole in the heat bath and estimate the entropy. The entropy-area law is reproduced by the volume integration of the entropy density over the inside of the horizon, and the black hole can be treated as an ordinary thermodynamic object.

Keywords: Black hole evaporation; black hole entropy; back reaction from evaporation.

We first consider the case without the back reaction. In the Schwarzschild geometry, each partial wave of matter fields feels the effective potential: $V_l(r) = f(r) \left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r} + m_0^2 \right)$, where $f(r) = 1 - \frac{a}{r}$, a is the Schwarzschild radius, and l is angular momentum. Compared with the square of the temperature $T_H \sim \frac{1}{a}$, the second and third terms can be neglected near the horizon $r \sim a$, and the field behaves as a massless field. Here, as the simplest model, we assume that the angular momentum obeys $l \leq l_c$, where the cutoff l_c can be large but finite.¹ By using the Eikonal approximation, we can explicitly check $T_\mu^\mu = 0$ and $T_\theta^\theta = 0$.

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Now we take into account the back reaction. Those conditions for the back reaction are satisfied by the outgoing Vaidya metric: $ds^2 = -\left(1 - \frac{a(u)}{r}\right) du^2 - 2du dr + r^2 d\Omega^2$, where $m(u) = \frac{a(u)}{2}$ is the Bondi mass and the only Einstein tensor is $G_{uu} = -\frac{\dot{a}(u)}{r^2}$. If we consider collapsing processes by a shock wave, the self-consistent solution is obtained by connecting the Vaidya metric and the flat space with the shock wave.¹ This geometry has neither horizon nor singularity.

Since the energy-momentum tensor for the Vaidya metric must be traceless, the Weyl anomaly would break its consistency. Fortunately, the Weyl anomaly is canceled if we take the quantum fluctuations of the metric into account.^{1,2}

We emphasize that the Hawking radiation occurs even if the geometry has no horizon. All that is necessary is that the affine parameters on the null generators of past and future null infinity are related exponentially.³ We demonstrate that as long as the adiabatic condition $a \gg 1$ holds, the Hawking radiation will occur with the temperature which slowly evolves as a function of time, $T(u) = \frac{1}{4\pi a(u)}$, where $a(u)$ is determined by $\dot{a}(u) \sim -\frac{1}{a(u)^2}$.¹

By using the conservation law $\nabla_\mu \langle T^\mu_\nu \rangle = 0$ and an appropriate boundary condition, the renormalized energy-momentum tensor $\langle T_{\mu\nu} \rangle$ can be determined completely.¹ We can also construct the consistent surface energy-momentum tensor T^μ_Σ on the locus of the shock wave.¹ Thus the consistency has been proven.

Next we discuss the gray-body factor, which leads to ingoing flow. In order to introduce it, we construct the general metric that satisfies $G^\mu_\mu = G^\theta_\theta = 0$:

$$ds^2 = -\partial_r p(u, r) \left(\frac{p(u, r)}{r} du^2 + 2du dr \right) + r^2 d\Omega^2, \quad (1)$$

and replace the Vaidya metric with it. p behaves like $p \sim r - a$ for $r > a$, curves around $r \sim a$ and remains a very small constant value for $r < a$. Therefore (1) has no coordinate singularity. This case also has no horizon and leads to the radiation.¹

We here consider the case that the black hole is put in a box filled with the thermal radiation. Then we can show that, after a sufficient time, the energy of the shock wave is converted to the radiation completely, it disappears, and the black hole finally comes to an equilibrium, in which the region $0 < r < a$ is filled with the radiation from the heat bath.¹ By using (1) without u -dependence for $r < a$ and assuming the local equilibrium thermodynamic relation $\rho + P = Ts$ and $P = 0$, the entropy can be estimated as $S = \int_\Sigma d^3x \sqrt{g_\Sigma} s = \frac{1}{4} A$. This result may indicate that microstates that contribute to the entropy live not on the surface but inside.¹ We can also reproduce the ADM mass from the inside.

References

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