LARGE- N RENORMALIZATION GROUP ON FUZZY SPHERE

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We study the large- N renormalization group of scalar field theory on a fuzzy sphere. We carry out perturbative analysis and formulate the renormalization group equation. We then search for fixed points and investigate their properties.

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1. Introduction

The large- N renormalization group (RG) method, initiated by Brezin and Zinn-Justin,¹ is a useful method to explore the critical behavior of matrix models. For a matrix model of a given size, a map to a similar matrix model of smaller size is defined through performing path-integral for a part of the matrix elements. The change is encapsulated in an analog of RG equation which dictates the change of the parameters of the model. This is called the large- N RG equation, with which it is possible to investigate the critical behavior of matrix models in the large- N limit. For exactly soluble matrix models, this has been shown to give quantitatively good approximations of the critical points and the critical exponents.

We apply this technique to scalar field theory on the fuzzy sphere, where we can naturally introduce the notion of high-energy modes, and then the large- N RG enjoys also an Wilsonian sense.

In the pursuit of this study, we hope to develop the method of analyzing the critical behavior of various matrix models, especially supersymmetric ones, in which the matrix elements also possesses a notion of high/low energy through D -brane interpretation. We expect this study to be a step toward the understanding of the continuum limit of supersymmetric matrix models for critical superstring, such as IIB matrix model.

2. The Model and the Results

We consider scalar field theory on the fuzzy S^2 of radius ρ_N

$$S_N = \frac{\rho_N^2}{N} \text{tr}_N \left(\frac{\mathcal{L}_N^2}{2\rho_N^2} \phi_N^2 + \frac{m_N^2}{2} \phi_N^2 + \frac{g_N}{4} \phi_N^4 \right). \quad (1)$$

ϕ_N is an $N \times N$ hermitian matrix whose basis is spanned by T_{lm} , a fuzzy analog of the spherical harmonics with $0 \leq l \leq (N-1)/2$. $\mathcal{L}_N^2$ is the Laplacian and has the eigenvalue $2l(2l+1)$ for the mode l . By integrating out the highest energy mode $l = (N-1)/2$, we can obtain an $(N-1) \times (N-1)$ matrix model with the different parameters m_{N-1}^2 and g_{N-1} . This change is interpreted as a flow in the parameter space, and we determine the flow (RG) equation by perturbation theory to the quadratic order in g_N . We then look for the fixed points of this flow and investigate their properties. The brief summary of the results is in order (in the preprint,² further analysis has been done and some new results are cited here):

- (1) A kind of locality emerges; for example, generated terms are organized in a derivative expansion. This is due to introducing the fuzzy sphere structure and then the large- N RG is compatible with the Wilsonian RG viewpoint.
- (2) In the procedure the radius ρ_N , and correspondingly the noncommutativity $\theta_N \sim \rho_N^2/N$, will be rescaled to keep the physical length scale in a correlated way with the large- N limit. A couple of choices of the rescaling are discussed; one keeping the fuzzy sphere structure (fuzzy sphere (FS) limit) and one with noncommutative parameter fixed (noncommutative field theory (NCFT) limit).
- (3) The Gaussian fixed point is confirmed and the scaling dimensions of the operators are calculated for FS and NCFT limits. The dimensions are found to coincide with the ones of conventional scalar field theory in two dimensions.
- (4) A nontrivial fixed point is also found and the scaling dimensions are calculated as well. The dimensions are different from the Gaussian case at this order; 0 and -2 for FS limit, for example. A possible relation to a phase transition of the matrix model is also discussed in the preprint.²
- (5) After the perturbative calculation, we find terms involving a phase factor $(-1)^l$. This phase can be understood as an effect of geometrical mapping of the field to the opposite point on the sphere, and we call it the antipode field. In the current study, we do not include the effect of these terms. The preprint² argues a possible relation the noncommutative anomaly³ and a future direction.

For more detailed results and discussion, please refer to the preprint.²

References

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