STOKES PHENOMENA AND QUANTUM INTEGRABILITY IN NON-CRITICAL STRING/M THEORY

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Non-critical string/M theory is a solvable model which has been studied to reveal various non-perturbative aspects of string theory with providing new key concepts to the next developments of string theory. Here we show some recent progress in study of Stokes phenomenon in non-critical string theory of the multi-cut two-matrix models. In particular, we argue that it is Stokes phenomenon which allows us to know concepts of non-perturbative completion with analytic study of string-theory landscape from the first principle.

Keywords: Non-perturbative string theory; matrix models; integrable systems.

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1. Non-Perturbative Completion of String Theory

String theory is proposed as the ultimate theory of our universe. This is one of the major motivations for studying string theory, and string physicists have tried to confirm this fact from theoretical point of view. However, in order to achieve this fundamental objective, we should take a step toward completing non-perturbative understanding of string theory. “Completion” means that all the non-perturbative information can be extracted in principle from the knowledge. For instance, *true vacuum*, *meta-stability*, *decay rates*, therefore the key of *vacuum selection rule*, should be calculable theoretically. Here we would like to report recent developments on this topic. In particular, we now have all the control over the non-perturbative aspects of string theory at least in the non-critical dimension.

2. Information for the Completion, and the String Landscape

For discussions of *non-perturbative complete string theory*, we study *reconstruction problem* from perturbative string theory. With perturbation theory (which includes non-perturbative soliton/instantons), observables are obtained as an asymptotic expansion in string-coupling g . For instance, the free-energy is generally given as:

$$\mathcal{F}(g) \simeq \sum_{n=0}^{\infty} g^{2n-2} \mathcal{F}_n + \sum_I \theta_I g^{\gamma_I} \exp \left[g^{-a_I} \sum_{n=0}^{\infty} g^n \mathcal{F}_n^{(I)} \right] + \mathcal{O}(\theta_I^2). \quad (1)$$

All the coefficients of the expansion are calculable except for the overall weight $\{\theta_I\}_I$ of non-perturbative effects, called *instanton fugacity*. The fugacity is best controlled by *Stokes phenomena of isomonodromy systems*.¹ The point is that, with information of the fugacity, we can reconstruct a full analytic theory. For example, recently, it is explicitly shown² that instability and the true vacuum of minimal string theory is calculable with this information. Therefore this is the information to complete string theory.

3. What is the Principle of String Theory?

In quantum field theory or quantum mechanics, it is not difficult to know the vacuum structure and meta-stability of perturbative vacua, because we already know *path-integral* of the system. One might be easily convinced that a natural information of “path-integral” in string theory is provided by *matrix models*. In fact, we have shown^{3,4} that the fugacity satisfies quantum integrability (i.e. T-systems) as a necessary condition of matrix models. In this sense, matrix models provide a new integrability in non-perturbative completion of string theory.

In future investigation, we would like to take a step further. Now that we have all the control over non-perturbative completion of string theory, we wish to reveal the primary reason for being matrix models, and then wish to complete the principle of string theory.

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