LARGE- N_C UNIVERSALITY IN QCD AND QCD-LIKE THEORIES

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A direct application of the lattice Monte Carlo method to QCD with a finite baryon chemical potential is difficult due to the sign problem. We introduce a novel approach to avoid the sign problem, which is inspired by the string theory.

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1. Introduction

Consider the N_f -flavor QCD at a finite baryon chemical potential,

$$\mathcal{L} = \frac{1}{4g^2} \text{Tr}(F_{\mu\nu})^2 + \sum_{f=1}^{N_f} \bar{\psi}_f (\gamma^\mu D_\mu + m_f + \mu \gamma^4) \psi_f, \quad (1)$$

where the gauge group is $SU(3)$, ψ_f are quarks with mass m_f in the fundamental representation, and μ is the quark chemical potential which is related to the baryon chemical potential μ_B as $\mu_B = 3\mu$. It is difficult to study this system by the Monte Carlo simulation because of the *fermion sign problem* – the fermion determinant $\prod_{f=1}^{N_f} \det(\gamma^\mu D_\mu + m_f + \mu \gamma^4)$ is complex in general.

In order to circumvent this difficulty, several gauge theories which do not suffer from the sign problem have been studied. The main examples are QCD with an

isospin chemical potential μ_I (i.e. $N_f = 2$, $\mu_1 = -\mu_2 = \mu_I/2$) and degenerate mass $m_1 = m_2$, and $\text{SO}(2N_c)$ and $\text{Sp}(2N_c)$ Yang-Mills theories. However, the relationship between those theories and $N_c = 3$ QCD is not clear a priori. It is important to understand *what we can learn from these theories*, or in other words, *in what sense they are similar to real QCD with baryon chemical potential*.

2. Main Results

An answer to this problem can be given by considering the large- N_c limit.^{1–4} The basic statements are^{1,2}

- At large N_c , $\text{SO}(2N_c)$ YM with μ_B , $\text{Sp}(2N_c)$ YM with μ_B , $\text{SU}(N_c)$ QCD with μ_I and $\text{SU}(N_c)$ QCD with μ_B are equivalent outside the BEC/BCS crossover region of the former three theories.
- More generally, $\text{SO}(2N_c)$, $\text{Sp}(2N_c)$ and $\text{SU}(N_c)$ theories with fermion mass $m_1, \dots, m_{N_f}$ and chemical potential $\mu_1, \dots, \mu_{N_f}$ are equivalent.

These statements have been derived by using the *orbifold equivalence*.⁵ As shown in Ref. 1 and 2, there are orbifold projections relating $\text{SO}(2N_c)$ and $\text{Sp}(2N_c)$ theories with μ_B , QCD with μ_B and QCD with μ_I . At large N_c , the orbifold equivalence guarantees these theories are equivalent in the sense a class of correlation functions (e.g. the expectation value of the chiral condensate) and the phase diagrams determined by such quantities coincide.

The parameter region where the equivalence holds is relevant for the physics of the heavy ion collisions. Thanks to the equivalence, one can study this parameter region by using the sign-free theories. In the case of the two-flavor theory, QCD with μ_I is nothing but the phase-quenched version of QCD with μ_B . Therefore, the sign problem is merely an illusion, up to the $1/N_c$ correction. The equivalence holds rather nicely already at $N_c = 3$,⁷ and the same equivalence holds in various effective models and holographic models.^{6,7} Applications of this equivalence include the Monte Carlo simulations with sign-problem-free ensembles and a robust statement based on the positivity of the fermion determinant.⁸

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