THE GRAVITY DUALS OF SO/USp SUPERCONFORMAL QUIVERS

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We discuss the gravity duals of SO/USp superconformal quiver gauge theories on M5-branes which are localized on top of $\mathbb{R}^5/\mathbb{Z}_2$ and wrapping on a Riemann surface of genus g . We concentrate on Riemann surfaces with no punctures and show that the gravity solutions are classified by the genus g of the Riemann surface and the torsion part of the four-form flux.

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1. Introduction

The physics of multiple M5-branes is still one of the biggest mysteries in M-theory. Recently, there has been remarkable progress in the study of wrapped M5-branes on a Riemann surface C_g of genus g .¹ With a partial topological twist, the resulting four-dimensional theory becomes an $\mathcal{N} = 2$ $SU(N)$ superconformal gauge theory, where the world-volume of the M5-branes are regarded as the Seiberg-Witten curve of the IR four-dimensional theory. In this article, we only consider Riemann surfaces without any punctures on it. Due to the partial topological twist, the Coulomb branch moduli in four dimensions are encoded in holomorphic differentials on C_g . On the other hand, the marginal gauge couplings of the four-dimensional theory are identified with the complex structure moduli of the Riemann surface C_g . In a weakly-coupled region of the gauge couplings, each tube or handle of C_g corresponds to a vector multiplet while each pair of pants of C_g corresponds to the so-called T_N -theory, which is a still mysterious isolated superconformal theory with $SU(N)^3$ flavor symmetry.

In Ref. 2, its generalization to SO/USp superconformal quiver gauge theories are studied. The essential point is to introduce a $\mathbb{Z}_2$ -quotient and consider wrapped $2N$ M5-branes on top of $\mathbb{R}^5/\mathbb{Z}_2$. In this case, the M5-branes near a handle of C_g is approximately wrapping on $\mathbb{R}^5 \times S^1 \times \mathbb{R}^5/\mathbb{Z}_2$. The topology of the three-form potential on the geometry is measured by³

$$H^4(S^1 \times \mathbb{RP}^4, \tilde{\mathbb{Z}}) \simeq \mathbb{Z} \oplus \mathbb{Z}_2, \quad (1)$$

where $\widetilde{\mathbb{Z}}_2$ is a sheaf of integers twisted by the orientation bundle of $\mathbb{RP}^4$. In (1), the first $\mathbb{Z}$ counts the number of M5-branes wrapped on the handle while $\mathbb{Z}_2$ measures the torsion part of the four-form flux G_4 .^a After the integration along S^1 , this torsion part of G_4 reduces to the torsion part of the three-form flux H_3 in type IIA string theory, which is known to give the distinction between $SO(2N)$ and $USp(2N-2)$ gauge groups in four dimensions. This implies that each handle of C_g is now associated with either $SO(2N)$ or $USp(2N-2)$ gauge group. Correspondingly, it was shown in Ref. 2 that a pair of pants in C_g is associated with an isolated superconformal theory with either $SO(2N)^3$ or $SO(2N) \times USp(2N-2)^2$ flavor symmetry. The first isolated theory was called T_{SO} -theory in Ref. 2, and we here call the second isolated theory $\widetilde{T}_{SO}$ -theory. Note that there is no similar isolated theory with $SO(2N)^2 \times USp(2N-2)$ or $USp(2N-2)^3$ flavor symmetry. The effective numbers n_v and n_h of the vector and hypermultiplets of the isolated theories are calculated via S-duality invariance as²

$$\begin{aligned} n_v(T_{SO(2N)}) &= \frac{8N^3}{3} - 7N^2 + \frac{10N}{3}, & n_v(\widetilde{T}_{SO(2N)}) &= \frac{8N^3}{3} - 7N^2 + \frac{16N}{3} - 1, \\ n_h(T_{SO(2N)}) &= n_h(\widetilde{T}_{SO(2N)}) = \frac{8N^3}{3} - 4N^2 + \frac{4N}{3}. \end{aligned} \quad (2)$$

In this article, we discuss the gravity duals of these SO/USp superconformal quivers in the eleven-dimensional supergravity, based on Ref. 4. The gravity duals of $SU(N)$ superconformal quivers were already studied in Ref. 5, and we generalize it to the SO/USp quivers. We argue that the dual gravity is classified by the genus g of the Riemann surface C_g and the torsion part of the four-form flux G_4 .

2. The Gravity Duals of $SO(2N)$ Quivers

We start with the gravity duals of $SU(M)$ superconformal quivers realized by M5-brane wrapping C_g without any $\mathbb{Z}_2$ -quotient. Such gravity duals are studied in Ref. 5 by using the knowledge of a general half BPS solutions in eleven-dimensional gravity.⁶ The explicit form of the metric is

$$\begin{aligned} ds^2 &= (\pi M l_p^3)^{\frac{2}{3}} \frac{W^{\frac{1}{3}}}{2} \left\{ 4ds_{AdS_5}^2 + 2 \left[4 \frac{dr^2 + r^2 d\beta^2}{(1-r^2)^2} \right] + 2d\theta^2 \right. \\ &\quad \left. + \frac{2}{W} \cos^2 \theta (d\psi^2 + \sin^2 \psi d\phi^2) + \frac{4}{W} \sin^2 \theta \left(d\chi + \frac{2r^2 d\beta}{1-r^2} \right)^2 \right\}, \end{aligned} \quad (3)$$

where $W \equiv 1 + \cos^2 \theta$. The parameters θ, ψ, ϕ and χ are the coordinates on S^4 , while r and β parameterize a hyperbolic space. The Riemann surface C_g is obtained as a quotient of the hyperbolic space by some group Γ . The metric (3) has a symmetry

^aSince the $\mathbb{Z}_2$ fixed plane has the M5-charge -1 (counted in the covering space), $[G_4/2\pi]$ is *not* a cohomology class in $H^4(S^1 \times \mathbb{RP}^4, \widetilde{\mathbb{Z}})$. We therefore define $[\widetilde{G}_4/2\pi] \equiv \frac{1}{2}([G_4/\pi] - \chi_E) \in H^4(S^1 \times \mathbb{RP}^4, \widetilde{\mathbb{Z}})$ with the twisted Euler class χ_E .³

Fig. 1. A $SO(2N)$ superconformal quiver gauge theory associated with a Riemann surface of genus g . A grey circle represents $SO(2N)$ gauge group, while a grey triangle expresses $T_{SO(2N)}$ theory.

$SU(2) \times U(1)$ which is identified with the R-symmetry in the dual field theory. This solution correctly reproduces the conformal anomalies in four-dimensions at the leading order of large M .

We now consider its generalization to $SO(2N)$ quivers. What we need to take into account is the presence of the $\mathbb{Z}_2$ -quotient. We now consider $2N$ M5-branes which are localized on top of $\mathbb{R}^5/\mathbb{Z}_2$ and wrapping on a Riemann surface C_g . Since the $\mathbb{Z}_2$ -quotient is acting on the whole normal directions of the M5-brane world-volume, it replaces S^4 with $\mathbb{RP}^4$ in the near horizon geometry (3). Thus, the near horizon geometry is locally described by the same metric (3) with $M = 2N$, but now θ, ψ, ϕ and χ are coordinates on $\mathbb{RP}^4$ rather than S^4 . The holographic calculation of the conformal anomaly then gives

$$c = \frac{\pi R_{AdS_5}^3}{8G_N^5} = \frac{\pi R_{AdS_5}^3 \times \text{Vol}(\Sigma_g \times \mathbb{RP}^4)}{8G_N^{11}} = \frac{4N^3}{3}(g-1). \quad (4)$$

The corresponding four-dimensional gauge theory is now described by the quiver diagram depicted in Fig. 1, which involves $(3g-3)$ $SO(2N)$ gauge groups and $(2g-2)$ $T_{SO(2N)}$ theories. The total effective numbers of vector and hypermultiplets are then given by

$$n_v(\text{total}) = (g-1) \left[\frac{16N^3}{3} - 8N^2 + \frac{11N}{3} \right], \quad (5)$$

$$n_h(\text{total}) = (g-1) \left[\frac{16N^3}{3} - 8N^2 + \frac{8N}{3} \right]. \quad (6)$$

From this, the conformal anomaly c of the field theory is evaluated as

$$c = \frac{2n_v(\text{total}) + n_h(\text{total})}{12} = (g-1) \left[\frac{4N^3}{3} - 2N^2 + \frac{5N}{6} \right], \quad (7)$$

which agrees with (4) at the leading order of large N .

3. Inclusion of $USp(2N-2)$ Gauge Groups

We now take into account $USp(2N-2)$ gauge groups in four dimensions. For that, we need to look at the torsion part of the four-form flux in the bulk. As mentioned

Fig. 2. Left: The “pair of pants” in C_g for the T_{SO} -theory. Right: Its $\tilde{T}_{SO}$ -theory counterpart. When we draw a red line along which $\vartheta \neq 0$, the red line always forms a closed curve in C_g .

before, a handle of the Riemann surface C_g can now be associated with not only $SO(2N)$ but also $USp(2N-2)$ gauge group. The difference between the two gauge groups is the torsion part of the four-form flux G_4 . The torsion part can be measured by integrating the flux along $\mathbb{RP}^3 \times S^1$ where S^1 is a cycle associated with the handle. We denote such an integral by ϑ .^b Since G_4 is a twisted four-form but $\mathbb{RP}^3$ is an *untwisted* cycle, the integral ϑ is well-defined only as an element of $\mathbb{Z}_2$. Then, the gauge group associated with the handle is $SO(2N)$ if $\vartheta = 0$, while it is $USp(2N-2)$ if $\vartheta = 1$.

Now, let us consider a pair of pants in C_g . Each pair of pants has three tubes attached to it, and each such tube is associated with $S^1_{(i)}$ for $i = 1, 2, 3$ (Fig. 2). We denote by ϑ_i the integral of the four-form flux along $\mathbb{RP}^3 \times S^1_{(i)}$. Then the fact $\sum_{i=1}^3 S^1_{(i)} = 0$ implies that $\sum_{i=1}^3 \vartheta_i = 0$ as an element of $\mathbb{Z}_2$. This is consistent with the absence of the isolated superconformal theory with $SO(2N)^2 \times USp(2N-2)$ or $USp(2N-2)^3$ flavor symmetry. This also implies that the torsion part of the four-form flux is always associated with the “B-cycles” of the Riemann surface C_g . Each handle of C_g with $\vartheta \neq 0$ is now associated with a $USp(2N-2)$ gauge group. An example of the quiver diagram of the SO/USp superconformal quivers is depicted in Fig. 3.

Fig. 3. An example of general SO/USp superconformal quivers. A black circle and a black triangle stand for a $USp(2N-2)$ gauge group and a $\tilde{T}_{SO(2N)}$ theory, respectively. This theory is associated with a Riemann surface of genus 7.

^bThe flux here is again the twisted flux $[\tilde{G}_4/2\pi]$ for the same reason as before. To be more specific, $\vartheta = \int_{\mathbb{RP}^3 \times S^1} [\tilde{G}_4/2\pi]$.

We now argue that the metric of the dual gravity is determined only by the M5-charge $2N$ and the genus g of the Riemann surface. This is easily understood when we note that the torsion part of G_4 just freezes two M5-branes at the $\mathbb{Z}_2$ fixed plane.³ Since such freezing of two M5-branes gives rise to only subleading effect in the large N limit, we can expect that the dual geometry is independent of the torsion part of the four-form flux. We can check this observation by calculating the conformal anomalies of a general SO/USp quivers associated with C_g . Since the torsion part is always associated with “B-cycles” of C_g , we can construct a general SO/USp quivers by replacing some number, say k , of $SO(2N)$ gauge groups with k $USp(2N-2)$ gauge groups, as well as k $T_{SO(2N)}$ -theories with k $\tilde{T}_{SO(2N)}$ theories. We can easily see that the conformal anomalies are invariant under this operation because

$$n_v(SO(2N)) + n_v(T_{SO(2N)}) = n_v(USp(2N-2)) + n_v(\tilde{T}_{SO(2N)}), \quad (8)$$

$$n_h(T_{SO(2N)}) = n_h(\tilde{T}_{SO(2N)}). \quad (9)$$

This suggests that the dual geometry is determined by N and g , and independent of the torsion part of the four-form flux.

We now find that the whole dual gravities of the $SO(2N)/USp(2N-2)$ superconformal quivers are fully classified by the genus g of the Riemann surface C_g and the torsion part of the four-form flux associated with the “B-cycles” of C_g .

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