$\eta(T)/s(T)$ AND COLLECTIVE FLOW IN CHIRAL HYDRODYNAMICS

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Using relativistic conformal hydrodynamics coupled to the linear σ model we study the evolution of matter created in heavy-ion collisions. We focus the study on the influence of the dynamics of the chiral fields on the charged-hadron elliptic flow v_2 for a temperature-independent as well as for a temperature-dependent η/s that is calculated from kinetic theory. We find that v_2 is not very sensitive to the coupling of chiral fields to the hydrodynamic evolution, but the temperature dependence of η/s plays a much bigger role on this observable.

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1. Introduction

In any attempt of describing experimental results of ultrarelativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) via a viscous fluid dynamic model, crucial physics input reflecting the properties of the flowing nuclear matter must be supplemented, like the equation of state (EOS) and transport coefficients such as the shear (η) and bulk (ζ) viscosities¹.

The EOS of nuclear matter created at RHIC and LHC can be obtained from Lattice QCD simulations and is fairly known in the low chemical potential regime of heavy ion collisions. In contrast, the temperature dependence of the transport coefficients describing the flow of this matter is not precisely known. Although it is expected that for the Quark–Gluon Plasma (QGP) η and ζ will depend strongly on temperature, usually in hydrodynamic simulations temperature-independent values for these coefficients are assumed throughout the entire evolution. The impact of

the temperature dependence of transport coefficients on momentum anisotropies as obtained from fluid dynamic models has been investigated only recently^{2,3,4}.

The comparison of hydrodynamic models to data would allow, in principle, to pinpoint the location of the QCD phase transition or of a crossover from hadronic matter to the QGP. In this context, in addition to the hydrodynamic degrees of freedom related to energy-momentum conservation, degrees of freedom associated with order parameters of broken continuous symmetries must be considered as well since they are all coupled to each other (see e.g. Refs.^{5,6}).

We study, in the context of second-order dissipative relativistic fluid dynamics – see Refs.^{1,7,8,9,10} for details, the influence of the long-wavelength modes of chiral fields on the expansion of the fireball created in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. In particular, we aim at quantifying the effect of the coupled evolution of chiral degrees of freedom on the flow asymmetry characterized by v_2 , when shear finite viscosity is taken into account within a simple microscopic model for the chiral condensate.

2. Conformal fluid dynamics coupled to the linear σ model

In this section we describe the chiral-hydrodynamic model used to compute v_2 . The quark fluid is described by second-order conformal fluid dynamics^{9,10}, while the chiral dynamics is obtained from the linear σ model (LSM). Further details on the model and its numerical implementation can be found in Ref.⁶.

The classical equations of motion of the LSM are given by

$$D_\mu D^\mu \phi_a + \frac{\delta U}{\delta \phi^a} = -g\rho_a, \quad (1)$$

where $\phi = (\sigma, \vec{\pi})$ and

$$\rho_a = g\phi_a d_q \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{p^2 + g^2 \sum_a \phi_a^2}} f_0(p^\mu, x^\mu), \quad (2)$$

with $f_0(p^\mu, x^\mu)$ being the Fermi-Dirac function and U the potential exhibiting chiral symmetry breaking. The quarks act as a thermal bath, leading to an effective potential

$$V_e(\phi_a, T) = U(\phi_a) - d_q T \int \frac{d^3p}{(2\pi)^3} \ln[1 + e^{-E/T}], \quad (3)$$

where $d_q = 24$ and $E = \sqrt{p^2 + m_q^2}$ with $m_q^2 = g^2 \sum_a \phi_a^2$.

The classical equations of motion of the chiral fields are solved selfconsistently together with the conservation equations for the quark fluid

$$\begin{aligned} D\epsilon &= -(\epsilon + p)\nabla_\mu u^\mu + \Pi^{\mu\nu}\sigma_{\mu\nu} + g(\rho_s D\sigma + \vec{\rho}_{ps} \cdot D\vec{\pi}), \\ (\epsilon + p)Du^i &= \frac{1}{3}(g^{ij}\partial_j\epsilon - u^i u^\alpha \partial_\alpha\epsilon) - \Delta_\alpha^i D_\beta \Pi^{\alpha\beta} + g(\rho_s \nabla^i \sigma + \vec{\rho}_{ps} \cdot \nabla^i \vec{\pi}), \end{aligned} \quad (4)$$

where $(\rho_s, \vec{\rho}_{ps}) = \rho_a$, $D = u_\mu D^\mu$ is the comoving time derivative, $\nabla_\mu = \Delta_{\mu\alpha} D^\alpha$ is the spatial gradient, and $\sigma_{\mu\nu}$ is shear tensor. The evolution equation for $\Pi^{\mu\nu}$

contains terms up to second order in velocity gradients and reads

$$\begin{aligned} \partial_\tau \Pi^{i\alpha} = & -\frac{4}{3u^\tau} \Pi^{i\alpha} \nabla_\mu u^\mu - \frac{1}{\tau_\pi u^\tau} \Pi^{i\alpha} + \frac{\eta}{\tau_\pi u^\tau} \sigma^{i\alpha} - \\ & - \frac{\lambda_1}{2\tau_\pi \eta^2 u^\tau} \Pi_\mu^{<i} \Pi^{\alpha>\mu} - \frac{u^i \Pi_\mu^\alpha + u^\alpha \Pi_\mu^i}{u^\tau} D u^\mu - \frac{u^j}{u^\tau} \partial_j \Pi^{i\alpha}, \end{aligned} \quad (5)$$

where η is the shear viscosity and (τ_π, λ_1) are second-order transport coefficients. From Eqs. (1) and (4) it is seen that the evolution of the chiral fields affects the evolution of the quark fluid through the sources in the energy-momentum conservation equations; in turn, the quark fluid affects the evolution of the chiral fields through the densities ρ_s and $\bar{\rho}_{ps}$, that depend on the local values of the fluid dynamic variables T and u^μ .

In order to compute the temperature dependence of the shear viscosity $\eta(T)$ in the LSM, we adapt the method described in Ref.¹¹ and employ the linearized Boltzmann equation in the relaxation time approximation (see Ref.⁶). The shear viscosity is given by

$$\eta = \frac{4\tau}{5T} \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E^2} f_0(1-f_0), \quad (6)$$

where $\tau = \tau(T)$ is the collision time calculated from the averaged cross sections $\bar{\sigma}$ for quark-quark and quark-antiquark scattering processes including $1/N_c$ next to leading order corrections as

$$\tau^{-1} = 6f_0 \left(\bar{\sigma}_{uu \rightarrow uu} + \bar{\sigma}_{ud \rightarrow ud} + \bar{\sigma}_{u\bar{u} \rightarrow u\bar{u}} + \bar{\sigma}_{u\bar{u} \rightarrow d\bar{d}} + \bar{\sigma}_{u\bar{d} \rightarrow u\bar{d}} \right). \quad (7)$$

We refer the reader to Ref.¹¹ for details on the calculation of the $\bar{\sigma}'_s$

The speed of sound c_s^2 and η/s of the LSM with $g = 3.2$ is shown in Fig. 1, together with the c_s^2 corresponding to Lattice QCD¹² (shown for comparison). The value of $g = 3.2$ is chosen because it leads to a smooth crossover with a drop in c_s^2 near the critical temperature^a T_c which is comparable to the one obtained from Lattice QCD simulations. For larger (smaller) values of g , the drop in c_s^2 near T_c is significantly larger (smaller), as shown in Fig. 2. It is seen that decreasing the value of g leads to a softening of the crossover and an increase in T_c , while increasing it leads to a decrease in T_c and a steeper variation of c_s^2 with temperature. We note that a first order phase transition is obtained if $g \simeq 3.8$, but we will not consider this case since Lattice QCD simulations¹² indicate the QGP-hadron transition is a smooth crossover (see Fig. 1).

For the second-order transport coefficients we take⁶: $\tau_\pi = 2(2 - \ln 2) \frac{\eta}{sT}$ and $\lambda_1 = \frac{\eta}{2\pi T}$ corresponding to the $\mathcal{N} = 4$ Super Yang-Mills theory. We use a $13 \text{ fm} \times 13 \text{ fm}$ transverse plane, set the impact parameter to $b = 7 \text{ fm}$ and the initialization time to $\tau_0 = 1 \text{ fm}/c$. As initial conditions we use $u^x = u^y = 0$ and $\Pi^{xx} = \Pi^{xy} =$

^aStrictly speaking, there is no T_c since we are dealing with a crossover and not with a phase transition. However, we will still refer to a T_c as an approximate critical temperature around which c_s^2 varies rapidly

$\Pi^{yy} = 0$. These are typical values for the parameters used as input in hydrodynamic simulations of heavy ion collisions at RHIC, and were used in Refs.^{6,7}. The initial energy density profile is obtained from Glauber's model, with a temperature $T_i = 330$ MeV at the center of the fireball. As a reasonable ansatz we take $\vec{\pi}(\tau_0, \vec{r}) = 0$

Fig. 1. Speed of sound of the LSM and that of the model EOS of Laine and Schröder¹² connecting a high-order weak-coupling perturbative QCD calculation to a hadron resonance gas through a crossover (*upper panel*); and $\eta/s(T)$ in the LSM with $g = 3.2$ (*lower panel*).

Fig. 2. Square of the sound speed c_s^2 of the LSM as a function of temperature, for three values of the chiral coupling: $g = 3, 3.2, 3.4$.

and $\sigma(\tau_0, \vec{r}) = f_\pi[1 - e^{-(\tau/r_0)^2}]$ with $r_0 = 9$ fm as initial values for the chiral fields. We use the isothermal Cooper-Frye freeze-out prescription with a freeze-out temperature $T_F = 130$ MeV.

3. Charged-hadron elliptic flow

We will now discuss the results obtained for the charged-hadron elliptic flow v_2 computed from the chiral-hydrodynamic model described above.

Fig. 3 shows v_2 calculated taking or not taking into account the source terms in the hydrodynamic equations, for either the temperature-dependent computed from kinetic theory or a temperature-independent η/s which is chosen to be the averaged value throughout the evolution of the fireball. The results shown were obtained with a value $g = 3.2$ in the LSM, which corresponds to a smooth crossover between hadronic matter and the QGP as suggested by Lattice QCD calculations (see e.g. Ref.¹²).

It is seen from Fig. 3 that both for a temperature-dependent and a temperature-independent η/s , the elliptic flow does not depend strongly on the chiral sources. More precisely, we see that for a temperature-independent η/s , v_2 turns out to be practically independent of the dynamics of the chiral sources for $p_T < 3$ GeV. In contrast, for the temperature-dependent η/s there are small but visible differences in the elliptic flow calculated with or without the chiral sources. Specifically, v_2 is

Fig. 3. Elliptic flow of charged-hadrons calculated taking or not taking into account the source terms in the hydrodynamic equations, for either a temperature-dependent or a temperature-independent η/s (taken as the averaged value throughout the evolution of the fireball). The value of the coupling constant is $g = 3.2$.

Fig. 4. Elliptic flow of charged-hadrons calculated taking or not taking into account the source terms in the hydrodynamic equations, for $g = 3, 3.2, 3.4$ and a temperature-dependent η/s .

slightly larger at low p_T and saturates at a smaller value of p_T when the chiral fields are taken into account as sources for the hydrodynamic variables.

In order to determine the impact of the value of g on the behavior of v_2 with p_T , we also calculated v_2 for a temperature-dependent η/s and different values of g close to $g = 3.2$. The results are shown in Fig. 4. It is seen that when the value of g increases, v_2 decreases (this is due to the fact that for larger values of g , we find that the average value of η/s throughout the hydrodynamic evolution becomes smaller). Comparing the values of v_2 calculated taking or not taking into account the chiral fields as sources for different values of g , it is seen that the influence of chiral dynamics on v_2 becomes smaller as g increases. This supports our earlier conclusion that the evolution of the chiral fields have only a small impact on v_2 .

It seems clear that there are difficulties in extracting (an average value of) η/s from data on v_2 . Our results show that uncertainties associated with the dependence of η/s on temperature lead to appreciable changes in the curve of v_2 versus p_T , much larger than the changes stemming from the coupling of fluid dynamic variables to evolving classical chiral fields. The uncertainty associated with the temperature-dependence of η/s should be added to the theoretical uncertainty that comes e.g. from the initial conditions (for example using Color Glass Condensate or Glauber initial conditions) and the freeze-out process, that according to recent studies add up to an overall uncertainty which can be roughly estimated in 0.1 (see e.g. Refs.^{1,7,8}).

4. Conclusions

We have found that the values of v_2 do not depend strongly on the evolution of the chiral fields. Specifically, for a temperature-independent η/s this dependence is negligible for $p_T < 3$ GeV, while for a temperature-dependent η/s it is appreciable but still small even at small p_T .

In line with the results of Refs.^{2,3,4}, our results show that not knowing precisely the temperature-dependence of η/s leads to significant uncertainties in attempts of extracting this ratio from data on v_2 , in addition to the uncertainties that stem from the initial conditions and the freeze-out process, among others sources.

It is worth noting that despite the coupling of chiral sources to the hydrodynamic evolution would add further uncertainties to the values of η/s extracted from data, they are not very big and do not affect in a significant way the possible extraction of η/s values from charged-hadron elliptic flow data.

Future work includes improvements such as the consideration of bulk viscosity in the hydrodynamic equations and fluctuations of the chiral fields. This latter effect would act as noise sources in the classical equations of motion^{13,14,15}.

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